MHD and Oscillatory Flow of Rivlin-Ericksen Fluid Through an Inclined Channel in a Porous Media

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ABSTRACT-*The magneto hydrodynamic unsteady flow of a visco-elastic (Rivlin-Ericksen) fluid through an inclined channel with two parallel flat plates in a porous medium moving with oscillatory motion under the influence of magnetic field with heat transfer including heat generating sources or heat absorbing sinks, while one of these two plates is adiabatic is studied in this paper. Pertubation method is applied to obtain the expression for the velocity and temperature distribution in oscillatory motion. The effect of magnetic parameter, Prandtl number, source or sink, elastic parameter, Froude's number, Grasshoff number, porous medium parameter on velocity distribution is discussed with the aid of graphs. The effect of Prandtl number and source or sink term on the temperature distribution is observed.*

Keywords: *Magnetic field, Porous Medium Froude's number, Grasshoff number, Prandtl number and temperature distributions*

INTRODUCTION

[3] Bhattacharjee.A and Borkakati A. K. (1984) have studied the heat transfer in a hydro magnetic flow between two porous disks one rotating and other at rest, under uniform suction, when the lower plate is adiabatic (which is given and well-known by [2] Schlichting, 1968). [4] Chakraborty S. And Borkakati A. K. (1998) has investigated MHD and flow and heat transfer of a dusty visco-elastic fluid down an inclined channel in porous medium. The MHD flow and heat transfer of dusty Rivlin-Eriksen second order fluids in an inclined channel in porous medium was studied by [5] Chakraborty S. and Borkakati A. K. (1998). [7] S. Ahmed and N. Ahmed (2004) have investigated the two dimensional MHD oscillatory flow along a uniformly moving infinite vertical porous plate bounded by porous medium. The unsteady MHD flow and heat transfer over a continuous porous moving horizontal surface in the presence of an oscillating free stream and heat source was studied by [8] P. R. Sharma, Y. N. Guar and R. P. Sharma (2004). The MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel, with heat sources or sinks when the plates are moving with transient velocity while the one of these two plates is adiabatic was investigated by[6]G. Badosa and A. K. Borkakati (2002).

A study on the magneto hydrodynamic unsteady flow of a visco-elastic fluid through an inclined channel with two parallel flat plates moving with oscillatory motion under the influence of magnetic field with heat transfer including heat generating sources or heat absorbing sinks, while one of these two plates is adiabatic is made in this paper.

MATHEMATICAL FORMULATION

Consider two dimensional incompressible electrically conducting visco-elastic fluid flow in oscillatory motion through an inclined channel between two parallel flat plates which are at a distance 2h apart under the influence of a uniform transverse magnetic field. We assume that the x'-axis along a straight line mid-way between the two plates, the y' –axis perpendicular to it. A magnetic field of uniform strength B_0 is assumed to be applied in y'- direction. Let u' be the velocity component along the direction of the x' – axis and the other components of the velocity be zero.

To write down the governing equations the following conditions are considered:

(i). The plates are infinitely long, so that the fluid velocity u′ is the function of y′ and t only.

(ii). The temperature is uniform with in the fluid particles and the buoyancy force is considered in the equation of motion of the fluid.

(iii). The flow between the plates is fully developed.

(iv).The conductivity of the fluid is assumed to be very small so that the induced magnetic field is neglected.

(v). The Hall Effect and viscous dissipation are assumed to be neglected.

(vi). Only electro-magnetic body force (Lorenz force) is considered.

(vii). Initially i.e., at time t = 0, the plates and the fluid are at zero temperature (i.e., T=0) and there is no flow within the channel. At time t>0 the temperature of the plate y= +h changes to $\frac{\partial T}{\partial y} = 0$, and the temperature of the plate y = -h changes accordingly to $T = T_0 (T_w – T_o)e^{-i\omega t}$, where T_w and T_o are temperature of the plates and ω ≥ 0 is a real number, denoting the decay factor.

If ρ is density of the fluid, B_0 is uniform magnetic field applied transversely to the plate, electrical conductivity of the fluid σ, coefficient of kinematics viscosity υ, specific heat of the fluid c_p, coefficient of thermal expansion β, acceleration due to gravity g, pressure p′ , coefficient of elasticity k_0 , coefficient of viscosity η_0, the source or sink term S' , then under the above assumptions, the governing equations of continuity, motion and energy for the unsteady flow of visco-elastic incompressible electrically conducting fluid between two non-conducting parallel plates in the presence of magnetic field are

$$\frac{\partial u'}{\partial x'} = 0 \longrightarrow (1)$$

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} + \frac{k_0}{\rho}\frac{\partial^3 u'}{\partial t'\partial y'^2} - \sigma\left[\frac{B_0^2}{\rho} + \frac{\upsilon}{k}\right]u' + g\sin\theta + g\beta(T' - T_0) \longrightarrow (2)$$

and $$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_0) \longrightarrow (3)$$

The boundary conditions of the problem are given by

$$\left.\begin{aligned} t' > 0;\quad u' = -u_0,\quad T' = T_0 + (T_w - T_0)e^{-i\omega' t'} \quad \text{at } y' = -h \\ u' = +u_0,\quad \frac{\partial T'}{\partial y'} = 0 \quad \text{at } y' = +h \end{aligned}\right\} \longrightarrow (4)$$

In order to bring out the essential features of the equation of this problem, we now consider the following non-dimensional parameters as given by [5] Shih-I Pai (1961).

$$x = \frac{x' u_0}{\upsilon},\ y = \frac{y' u_0}{\upsilon},\ u = \frac{u'}{u_0},\ t = \frac{t' u_0^2}{\upsilon},\ M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2},\ T = \frac{T' - T_0}{T_w - T_0},\ p_r = \frac{\mu c_p}{k},$$

$$\omega = \frac{\upsilon \omega'}{u_0^2}, \quad S = \frac{S' \upsilon}{u_0^2}, \quad G_r = \frac{\upsilon g \beta (T_w - T_0)}{u_0^3}, \quad p = \frac{p'}{\rho u_0^2}, \quad R_c = \frac{k_0 u_0^2}{\rho v^2}, \quad F_r = \frac{u_0^2}{gh}, \quad \longrightarrow (5)$$

$$R_e = \frac{u_0 h}{\upsilon}, \quad \left(\frac{h u_0}{\upsilon} = 1\right), \text{Da}^{-1} = \frac{v^2}{k u_0^2}$$

Substituting the non-dimensional parameters in the equation (1) to (3), we get

$$\frac{\partial u}{\partial x} = 0 \quad \longrightarrow (6)$$

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + R_c \frac{\partial^3 u}{\partial t \partial y^2} - \left(M + \text{Da}^{-1}\right)u + \frac{\sin\theta}{F_r R_e} + G_r T \longrightarrow (7)$$

and $$\frac{\partial T}{\partial t} = \frac{1}{p_r}\frac{\partial^2 T}{\partial y^2} + ST \longrightarrow (8)$$

Where R_C is the elastic parameter, M is the magnetic field parameter, F_r is the Froude number, p_r is the Prandtl number and S is the source /sink term.

The dimensionless form of the boundary conditions are given by

$t > 0$: u = -1, T = $e^{i\omega t}$ at y = -1

: u= +1, $\frac{\partial T}{\partial y} = 0$ at y = +1 $\longrightarrow$ (9)

SOLUTION OF THE EQUATIONS

The equation (6) shows that u is a function of y and t only or a constant. Also equation (7) shows that the velocity u is independent of x and therefore u is a function of y and t only. Thus, the term $\frac{\partial p}{\partial x}$ must be a constant or the function of t only.

$$\text{Let } \frac{\partial p}{\partial x} = -h(t) \longrightarrow (10)$$

Substituting equation (10) then the equation (7) becomes

$$\frac{\partial u}{\partial t} = h(t) + \frac{\partial^2 u}{\partial y^2} + R_c \frac{\partial^3 u}{\partial t \partial y^2} - \left(M + \text{Da}^{-1}\right)u + \frac{\sin\theta}{F_r R_e} + G_r T \longrightarrow (11)$$

In order to solve the equations (8) and (11) under the boundary conditions (9) we consider

$$u = f(y)\, e^{-i\omega t}, \quad T = g(y)\, e^{-i\omega t}, \quad h = h_0 e^{-i\omega t} \longrightarrow (12)$$

The corresponding boundary conditions are

$$f(-1) = -e^{i\omega t}, \quad g(-1) = 1, \quad f(+1) = e^{i\omega t}, \quad g'(+1) = 0 \longrightarrow (13)$$

Substituting (12) in the equations (8) and (11), we get

$$\frac{\partial^2 g}{\partial y^2} + p_r (S + i\omega) g = 0 \longrightarrow (14)$$

And

$$(1 - i\omega R_c)\frac{\partial^2 f}{\partial y^2} - (M - i\omega + \mathrm{Da}^{-1}) f = -h_0 - \frac{\sin\theta e^{i\omega t}}{F_r R_e} - G_r g \longrightarrow (15)$$

Now, solving the equations (14) and (15) using the boundary conditions (13) and substituting in the equations (12), we get

$$g(y) = \frac{\cos a_1 (1 - y)}{\cos 2a_1} \longrightarrow (16)$$

And

$$T_{RP} = \frac{\cos \omega t \cos a_1 (1 - y)}{\cos 2a_1} \longrightarrow (17)$$

$$T_{IP} = \frac{-\sin \omega t \cos a_1 (1 - y)}{\cos 2a_1} \longrightarrow (18) \qquad (\text{where } a_1 = \sqrt{p_r S}\ )$$

$$f(y) = \left[\begin{array}{l} \frac{M_2 \cos \omega t}{b^2}\left(1 - \frac{\cosh by}{\cosh b}\right) + \frac{G_r}{M_1 \cos 2a_1}\left(-\frac{\sinh by \sin^2 a_1}{\sinh b} - \frac{\cosh by \cos^2 a_1}{\cosh b} + \cos a_1(1-y)\right) \\ + \frac{e^{i\omega t}\sinh by}{\sinh b} + i\left[\frac{\sin\theta \sin \omega t}{\omega F_r R_e}\left(\frac{\cosh dy}{\cosh d} - 1\right) + \left(\frac{e^{i\omega t}\sinh dy}{\sinh d}\right) - \frac{\sin\theta\sin\omega t}{\omega F_r R_e}\right] \end{array}\right]$$

$$\longrightarrow (19)$$

But $U = f(y)\, e^{-i\omega t}$

Substituting eqn. (19) we get

$$U_{RP} = \frac{M_2 \cos^2 \omega t}{b^2}\left(1 - \frac{\cosh by}{\cosh b}\right) + \frac{\sinh by}{\sinh b} + \frac{G_r \cos \omega t}{2M_1 \cos 2a_1}\left(-\frac{\sinh by \sin^2 a_1}{\sinh b} - \frac{\cosh by \cos^2 a_1}{\cosh b} + \cos a_1 (1-y)\right)$$
$$+ \frac{\sin\theta \sin^2 \omega t}{\omega F_r R_{e_1}}\left(\frac{\cosh dy}{\cosh d} - 1\right)$$

$$\longrightarrow (20)$$

$$U_{IP} = \frac{M_2 \cos \omega t \sin \omega t}{b^2}\left(\frac{\cosh by}{\cosh b} - 1\right) - \frac{G_r \sin \omega t}{M_1 \cos 2a_1}\left(\cos a_1(1-y) - \frac{\sinh by \sin^2 a_1}{\sinh b} - \frac{\cosh by \cos^2 a_1}{\cosh b}\right) +$$
$$\frac{\sinh dy}{\sinh d} + \frac{\sin\theta \sin \omega t \cos \omega t}{\omega F_r R_e}\left(\frac{\cosh dy}{\cosh d} - 1\right)$$

(21)

$$U = U_{RP} + iU_{IP} \qquad (22)$$

$$|U| = \sqrt{U_{RP}^2 + U_{IP}^2} \qquad (23)$$

And

$$|T| = \sqrt{T_{RP}^2 + T_{IP}^2} \qquad (24)$$

where $M_1 = p_r s + M - Da^{-1}$,

$$M_2 = \frac{h_0}{\cos \omega t} + \frac{\sin \theta}{F_r R_e},$$

$$a_1 = \sqrt{p_r s},$$

$$b = \sqrt{M - Da^{-1}},$$

$$d_1 = \sqrt{\frac{1}{R_c}}$$

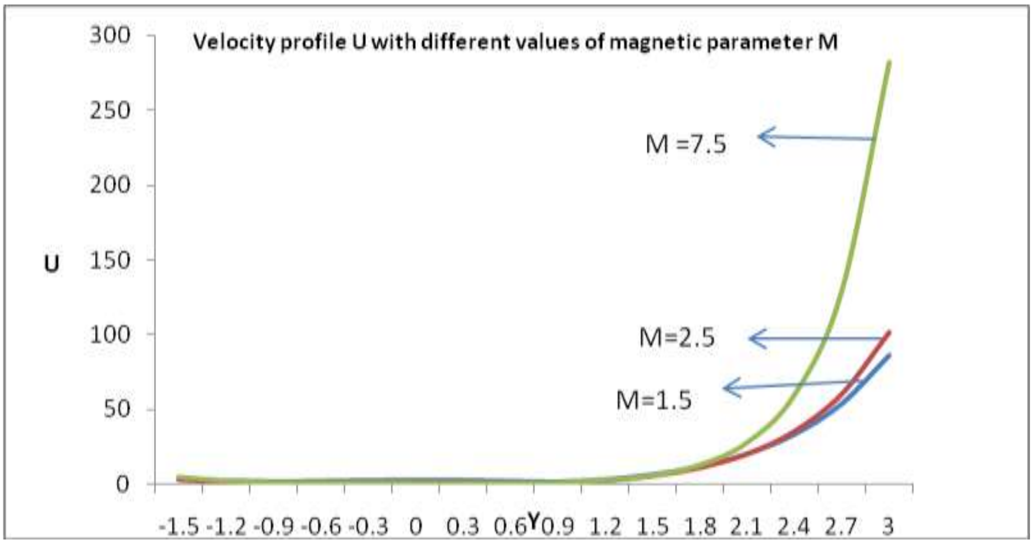

Fig.1

The velocity distribution for the different values of M = 1.5,2.5 and 7.5 against the variable y is plotted by considering the different parameter values as $p_r = 0.5$, S=0.5, $R_c = 0.3$,

$F_r = 3.0, h_0 = 1.0, R_e = 1.0, \omega = 1.0, t = 1.0, G_r = 5.0$ and $\theta = 30^0$

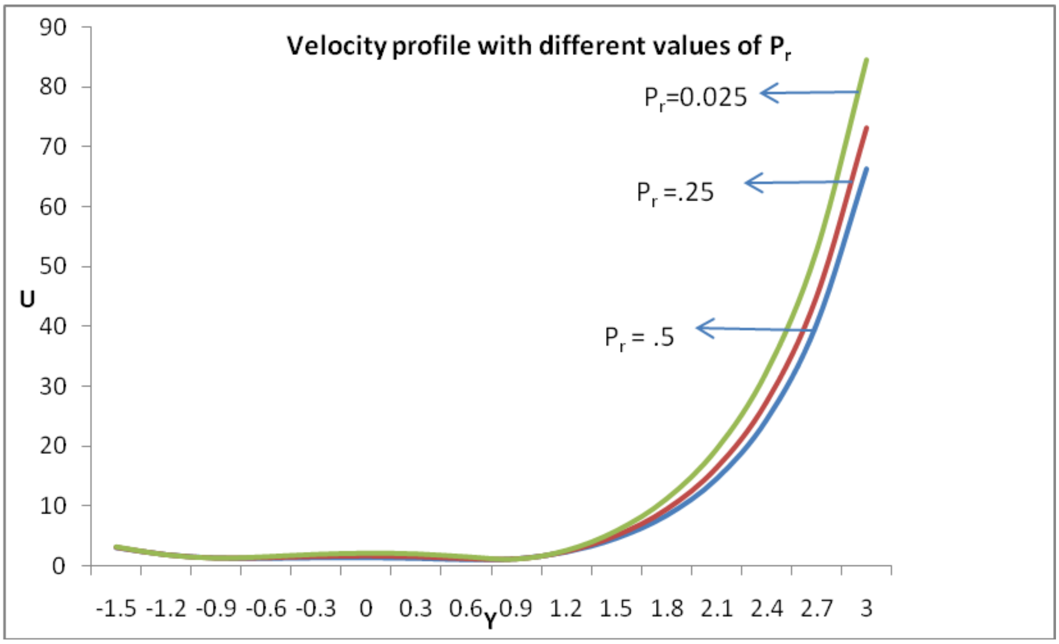

Fig. 2
The velocity profile for different values of Prandtl number $p_r = 0.5$, 0.25, 0.025 for the values of $M = 1.5$ $S = 0.5, R_c = 0.3, F_r = 3.0, h_0 = 1.0, R_e = 1.0, \omega = 1.0, t = 1.0, G_r = 5.0$ and $\theta = 30^0$

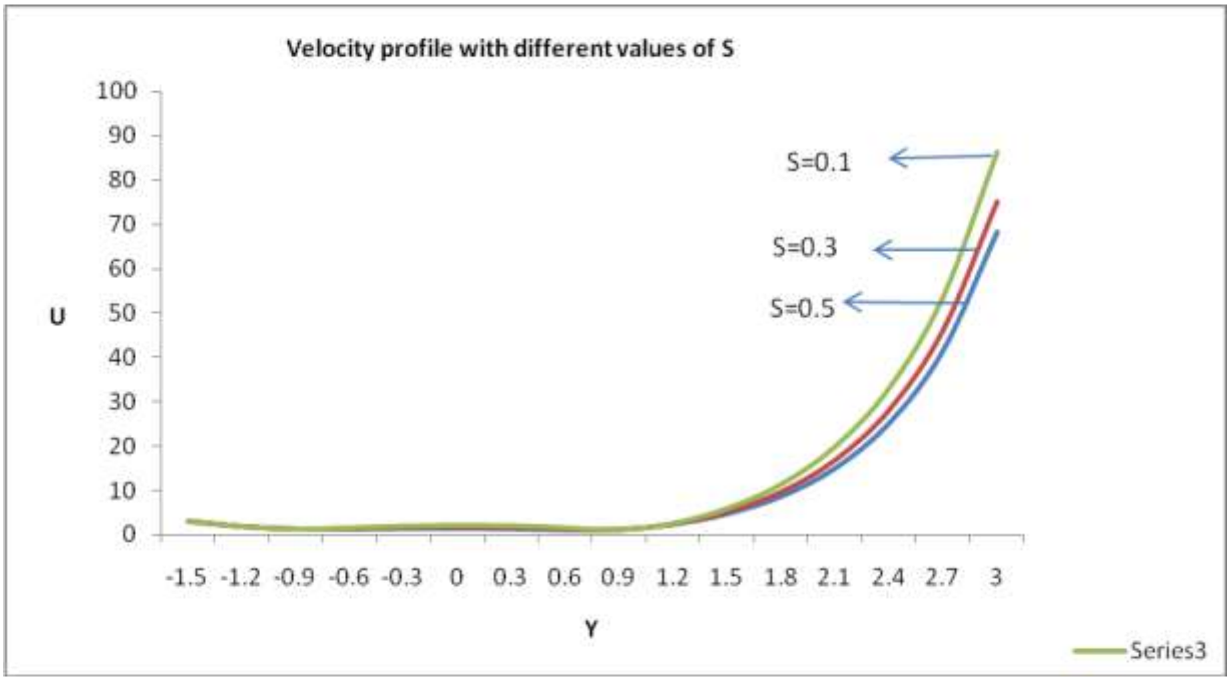

Fig.3
This figure has been found by drawing the velocity distribution of u for various values of source of sink term $S = 0.1, 0.3, 0.5$ when M = 1.5,
$p_r = 0.5, R_c = 0.3, h_0 = 1.0, \omega = 1.0, t = 1.0, R_e = 1.0, S = 0.5, F_r = 3.0, G_r = 5.0,$ and $\theta = 30^0$.

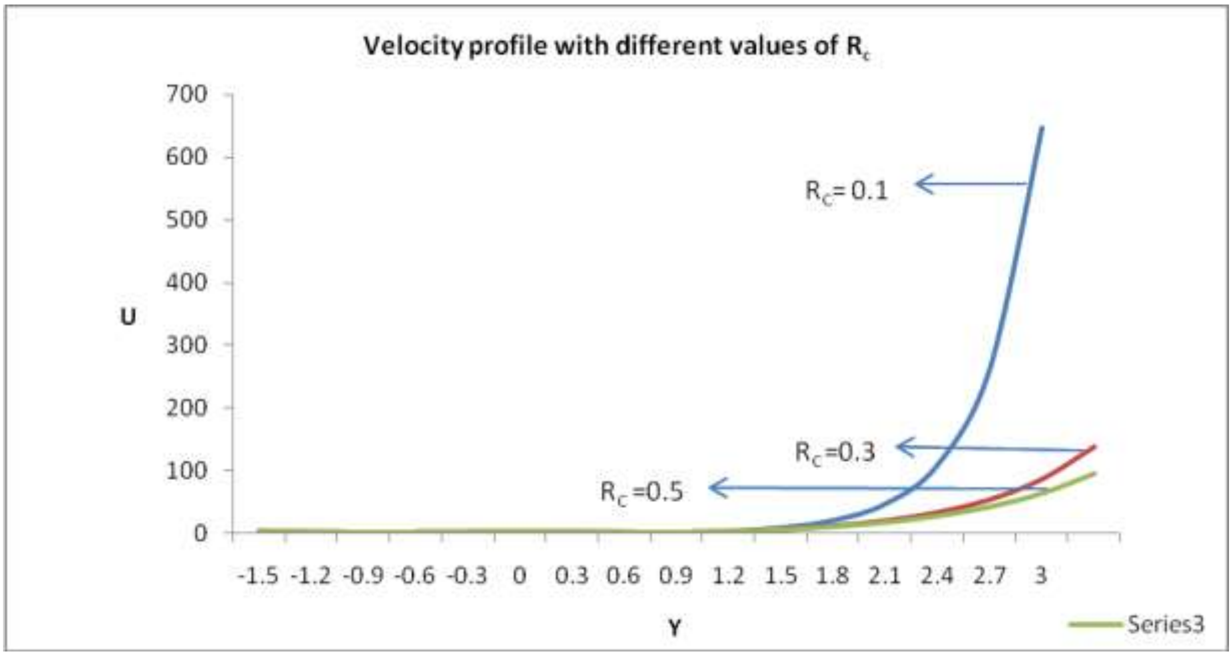

Fig.4 The fluid velocity u against the variable y for different values of elastic parameter $R_c = 0.1$, 0.3, 0.5 when M=1.5, $p_r = 0.5, h_0 = 1.0, \omega = 1.0, t = 1.0, R_e = 1.0, S = 0.5, F_r = 3.0, G_r = 5.0$ and $\theta = 30^0$.

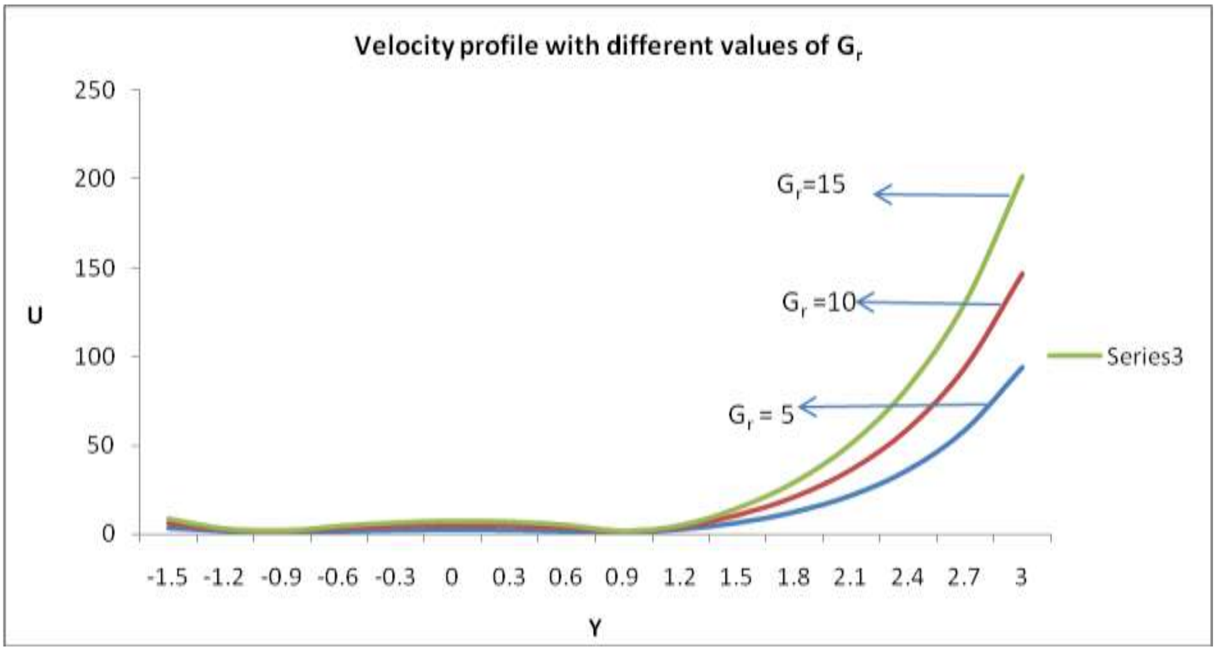

Fig.5

The velocity distribution is plotted against the variable y for different values of Grasshoff number $G_r = 5$,10, 15 when M=1.5

$p_r = 0.5, S = 0.5, h_0 = 1.0, \omega = 1.0, t = 1.0, R_c = 0.3, F_r = 3.0, R_e = 1.0$ and $\theta = 30^0$.

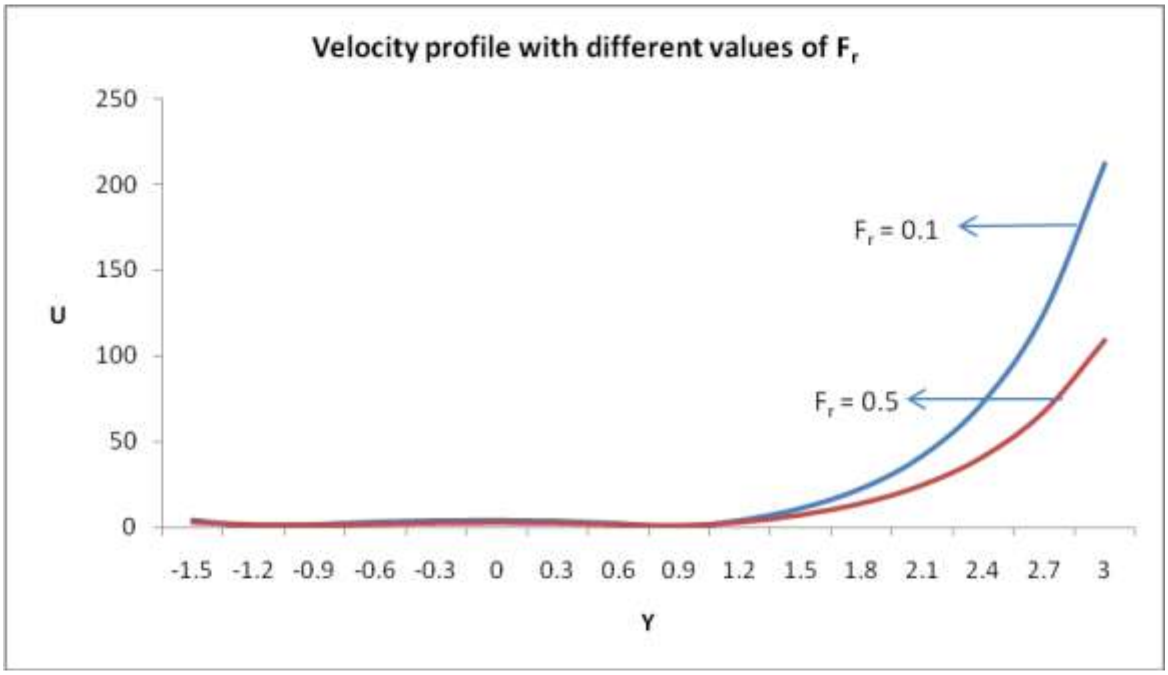

Fig.6 The variation of velocity u is plotted against the variable u for different values of Froude's number $F_r = 0.1, 0.5$ when M=0.5, $p_r = 0.5, S = 0.5, R_c = 0.3, R_e = 1.0, h_0 = 1.0, \omega = 1.0, t = 1.0, G_r = 5.0$ and $\theta = 30^0$.

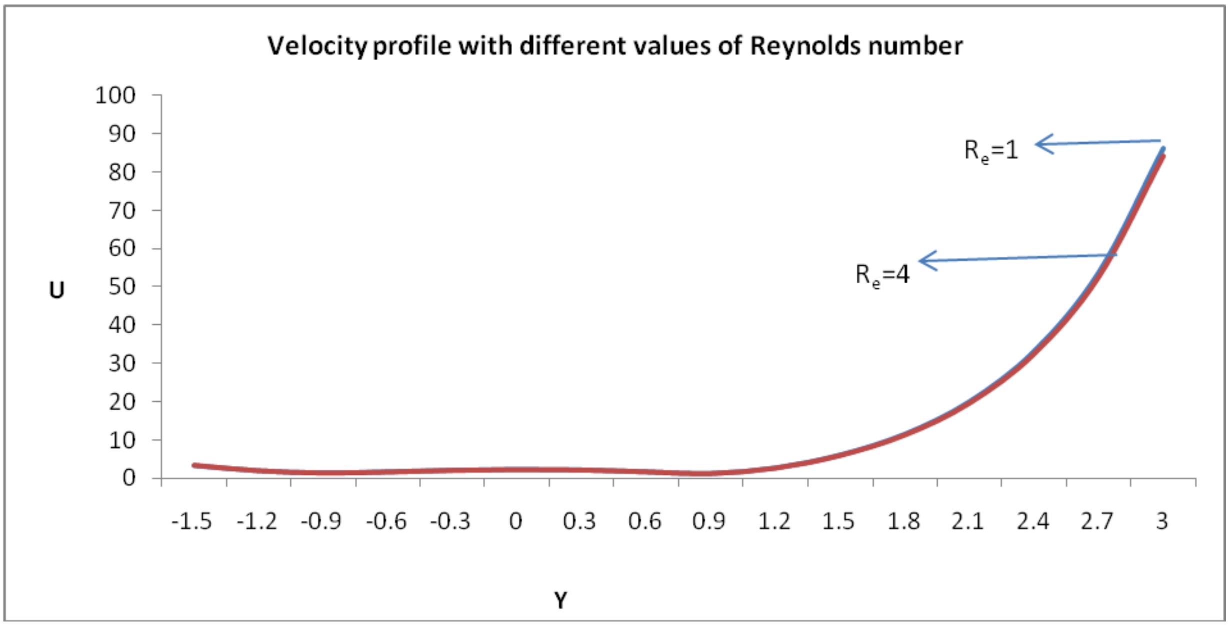

Fig.7
The velocity distribution has been obtained by plotting the graph against the variable y for various values of Reynolds's number $R_e = 1, 2$ when M=1.5,

$p_r = 0.5, S = 0.5, R_c = 0.3, G_r = 5.0, F_r = 3.0, h_0 = 1.0, \omega = 1.0, t = 1.0$ and $\theta = 30^0$.

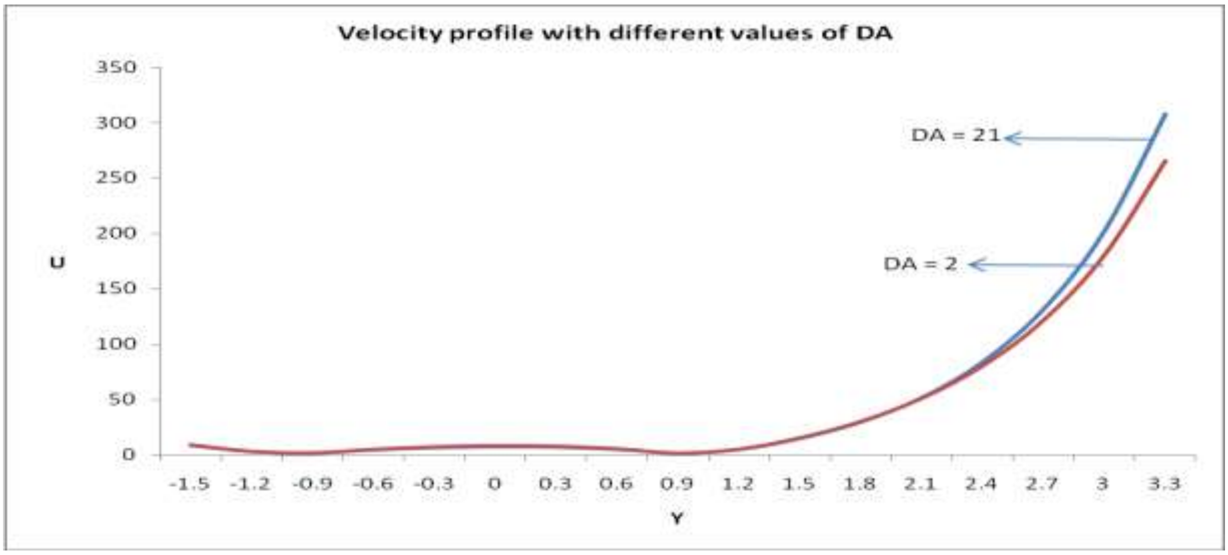

Fig.8
The velocity distribution has been obtained by plotting the graph against the variable y for various values of porous medium parameter Da = 2, 21 when M=1.5,
$p_r = 0.5, S = 0.5, R_c = 0.3, G_r = 15.0, F_r = 3.0, h_0 = 1.0, \omega = 1.0, t = 1.0, R_e = 1.0$ and $\theta = 30^0$.

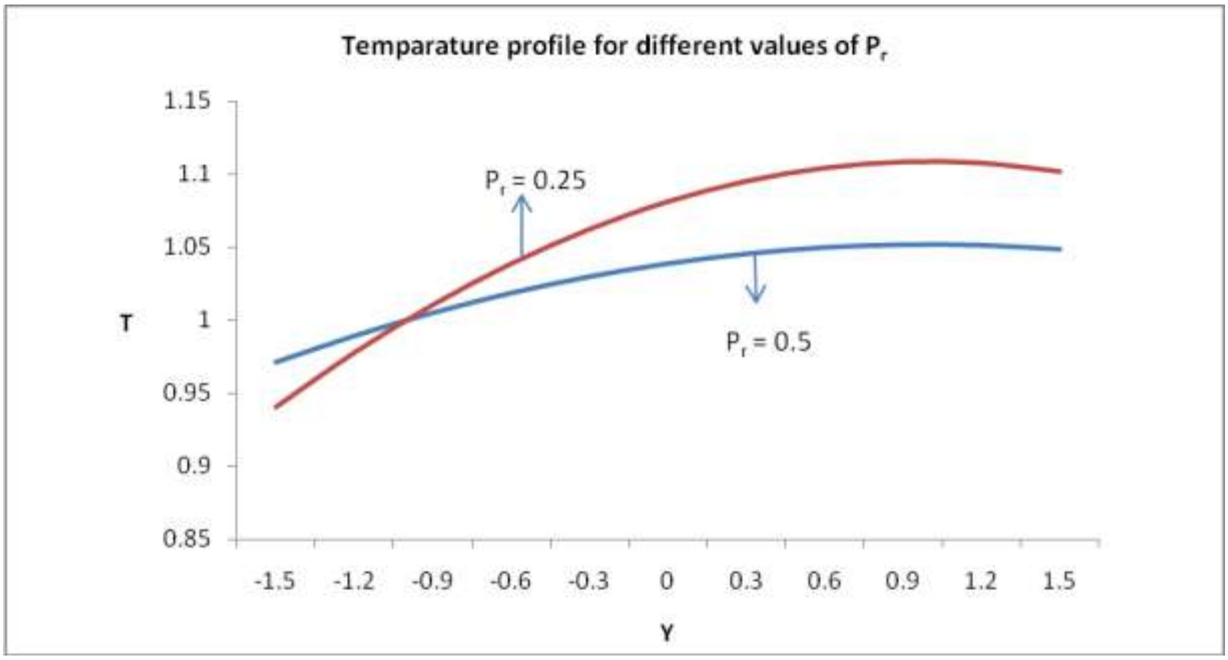

Fig.9 :- The temperature profile for different values of Prandtl number $p_r = 0.5, 0.25$ is plotted against the variable y when $\omega = 1.0$, t=1.0, S=0.1.

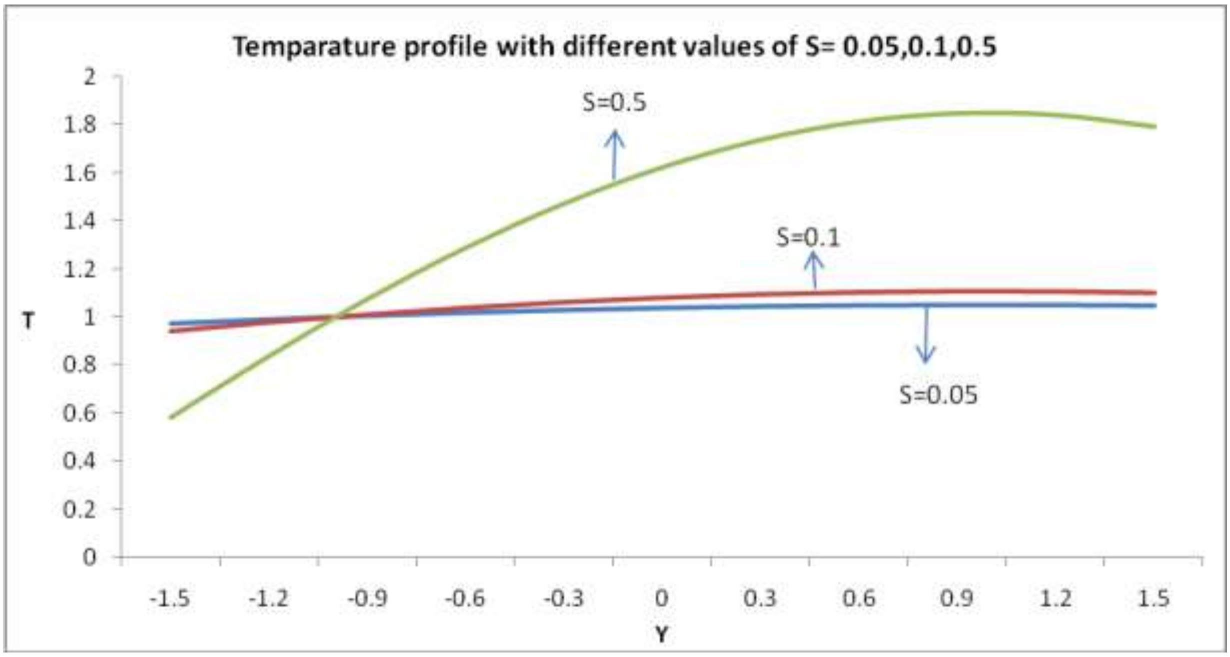

Fig.10:-The temperature distribution has been drawn against the variable y for various values of S=0.05, 0.1 and 0.5 when $p_r = 0.5, \omega = 1.0$ and t = 1.0.

RESULTS AND DISCUSSION

1. Velocity profile for different values of magnetic parameter **M.**

The velocity profile increases with the increase of Magnetic parameter M in Oscillatory motion where as the velocity decreases with increase of Magnetic Parameter M in the transilatory motion. The velocity profile is zero when y = 0 and For smaller values of M, the increase in velocity is small and for large values of M, The velocity profile is considerably high.

2. Velocity profile for different values of Prandtl number $\mathbf{p_r}$.

The velocity profile for different values of Prandtl number p_r is plotted against the Variable y. It is observed that the velocity of the fluid increases with the decrease of The Prandtl number in oscillatory motion as in the case of transilatory motion. It is almost zero at the plate y = 0 and it decreases considerably between the plates y = **-** 1.5 and y = 0 and it increases from the plate y = 0.

3. Velocity profile for different values of source or sink term **S.**

It is found that the velocity u increases with the increase in source or sink term in Oscillatory motion where as in transilatory motion the velocity decreases with increase in source or sink term S. The velocity decreases between the plates y = **-** 1.5 and y = 0 and is zero when y = 0 and velocity increases from zero as y increases from zero.

4. Velocity profile for different values of Elastic parameter $\mathbf{R_c}$.

It is shown that the velocity profile is zero at y = 0 and it decreases considerably between the plates y = **-** 1.5 and y = 0 and increases considerably from the plate y = 0. The velocity increases with the decrease in R_c in oscillatory motion while the velocity increases with the increase in R_c in transilatory motion.

5. Velocity profile for different values of Grashoff number $\mathbf{G_r}$.

The variation of velocity u is plotted against the variable y for different values of Grashoff number G_r. It is observed that the velocity decreases between the plates y = -1.5 and y = 0 and is zero at y = 0 and the increase in velocity is considerably high from y = 0. The velocity increases with the increase in Grashoff number in Oscillatory motion.

6. Velocity profile for different values of Froude's number $\mathbf{F_r}$.
It is found that the velocity decreases between the plates y = -1.5 and y = 0 and it is observed that the velocity is almost same for F_r = 0.1 and F_r = 0.5 in oscillatory motion where as in transilatory motion the velocity decreases with the decrease in Froude's number.

7. Velocity profile for different values of Reynold's number $\mathbf{R_e}$.

The velocity distribution has been obtained by plotting the graph against the variable y for various values of Reynold's number. The increase in velocity is considerably high from the plate y = 0. There is no change in velocity for R_e = 1 and R_e = 2 in oscillatory motion.

8. Velocity profile for different values of porous medium parameter Da= 2, 21.

The velocity distribution has been obtained by plotting the graph against the variable y for various values of porous medium parameter. Tha velocity profile increases with the increase in Da that is the velocity profile decreases with the increase in the parameter of porous media.

9. Temperature profile for different values of Prandtl number $\mathbf{p_r}$.

Temperature profile for different values of p_r is plotted against the variable y when ω =1.0, t = 1.0 and S =0.1. We notice that the temperature increases with the increase in Prandtl number in oscillatory motion as well as in transilatory motion.

10. Temperature profile for different values of **S**.

It is seen that the temperature increases with the increase in source or sink term S in oscillatory motion..The variation in temperature for S =0.1 and S = 0.05 is almost similar.

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