# Integration of certain generalized Gimel-function with respect to their parameters

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ABSTRACT
The object of the present paper is to establish four integrals associated with generalized multivariable Gimel-function defined here. The integration is perfomed with respect to a parameter. Such integrals are useful in the study of certain boundary value problems.

KEYWORDS : Generalized multivariable Gimel-function, multiple integral contours, integration with respect to a parameter.

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## 1. Introduction and preliminaries.

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental function of several complex variables.

$$\gimel(z_1,\cdots,z_r) = \gimel^{m_2,n_2;m_3,n_3;\cdots;m_r,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2};R_2;p_{i_3},q_{i_3},\tau_{i_3};R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)}}\left(\begin{array}{c|} \mathrm{z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{z}_r \end{array}\right.$$

$$[(\mathrm{a}_{2j};\alpha^{(1)}_{2j},\alpha^{(2)}_{2j};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha^{(1)}_{2ji_2},\alpha^{(2)}_{2ji_2};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha^{(1)}_{3j},\alpha^{(2)}_{3j},\alpha^{(3)}_{3j};A_{3j})]_{1,n_3},$$
$$[(\mathrm{b}_{2j};\beta^{(1)}_{2j},\beta^{(2)}_{2j};B_{2j})]_{1,m_2},[\tau_{i_2}(b_{2ji_2};\beta^{(1)}_{2ji_2},\beta^{(2)}_{2ji_2};B_{2ji_2})]_{m_2+1,q_{i_2}},[(b_{3j};\beta^{(1)}_{3j},\beta^{(2)}_{3j},\beta^{(3)}_{3j};B_{3j})]_{1,m_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha^{(1)}_{3ji_3},\alpha^{(2)}_{3ji_3},\alpha^{(3)}_{3ji_3};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{rj};\alpha^{(1)}_{rj},\cdots,\alpha^{(r)}_{rj};A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3};\beta^{(1)}_{3ji_3},\beta^{(2)}_{3ji_3},\beta^{(3)}_{3ji_3};B_{3ji_3})]_{m_3+1,q_{i_3}};\cdots;[(\mathrm{b}_{rj};\beta^{(1)}_{rj},\cdots,\beta^{(r)}_{rj};B_{rj})_{1,m_r}],$$

$$[\tau_{i_r}(a_{rji_r};\alpha^{(1)}_{rji_r},\cdots,\alpha^{(r)}_{rji_r};A_{rji_r})_{n_r+1,p_r}]:\ [(\mathrm{c}^{(1)}_j,\gamma^{(1)}_j;C^{(1)}_j)_{1,n^{(1)}}],[\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}},\gamma^{(1)}_{ji^{(1)}};C^{(1)}_{ji^{(1)}})_{n^{(1)}+1,p^{(1)}_i}]$$
$$[\tau_{i_r}(b_{rji_r};\beta^{(1)}_{rji_r},\cdots,\beta^{(r)}_{rji_r};B_{rji_r})_{m_r+1,q_r}]:[(\mathrm{d}^{(1)}_j),\delta^{(1)}_j;D^{(1)}_j)_{1,m^{(1)}}],[\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}},\delta^{(1)}_{ji^{(1)}};D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q^{(1)}_i}]$$

$$\left.\begin{array}{l};\cdots;[(c^{(r)}_j,\gamma^{(r)}_j;C^{(r)}_j)_{1,n^{(r)}}],[\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}},\gamma^{(r)}_{ji^{(r)}};C^{(r)}_j)_{n^{(r)}+1,p^{(r)}_i}] \\ ;\cdots;[(d^{(r)}_j,\delta^{(r)}_j;D^{(r)}_j)_{1,m^{(r)}}],[\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}},\delta^{(r)}_{ji^{(r)}};D^{(r)}_j)_{m^{(r)}+1,q^{(r)}_i}]\end{array}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\,\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1,\cdots,s_r) = \frac{\prod_{j=1}^{m_2}\Gamma^{B_{2j}}(b_{2j}-\sum_{k=1}^{2}\beta^{(k)}_{2j}s_k)\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha^{(k)}_{2j}s_k)}{\sum_{i_2=1}^{R_2}[\tau_{i_2}\prod_{j=n_2+1}^{p_{i_2}}\Gamma^{A_{2ji_2}}(a_{2ji_2}-\sum_{k=1}^{2}\alpha^{(k)}_{2ji_2}s_k)\prod_{j=m_2+1}^{q_{i_2}}\Gamma^{B_{2ji_2}}(1-b_{2ji2}+\sum_{k=1}^{2}\beta^{(k)}_{2ji2}s_k)]}$$

$$\frac{\prod_{j=1}^{m_3}\Gamma^{B_{3j}}(b_{3j}-\sum_{k=1}^{3}\beta_{3j}^{(k)}s_k)\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3}\prod_{j=n_3+1}^{p_{i_3}}\Gamma^{A_{3ji_3}}(a_{3ji_3}-\sum_{k=1}^{3}\alpha_{3ji_3}^{(k)}s_k)\prod_{j=m_3+1}^{q_{i_3}}\Gamma^{B_{3ji_3}}(1-b_{3ji3}+\sum_{k=1}^{3}\beta_{3ji3}^{(k)}s_k)]}$$

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$$\frac{\prod_{j=1}^{m_r}\Gamma^{B_{rj}}(b_{rj}-\sum_{k=1}^{r}\beta_{rj}^{(k)}s_k)\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r}\prod_{j=n_r+1}^{p_{i_r}}\Gamma^{A_{rji_r}}(a_{rji_r}-\sum_{k=1}^{r}\alpha_{rji_r}^{(k)}s_k)\prod_{j=m_r+1}^{q_{i_r}}\Gamma^{B_{rji_r}}(1-b_{rjir}+\sum_{k=1}^{r}\beta_{rjir}^{(k)}s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k)=\frac{\prod_{j=1}^{m^{(k)}}\Gamma^{D_j^{(k)}}(d_j^{(k)}-\delta_j^{(k)}s_k)\prod_{j=1}^{n^{(k)}}\Gamma^{C_j^{(k)}}(1-c_j^{(k)}+\gamma_j^{(k)}s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}}\prod_{j=m^{(k)}+1}^{q_{i^{(k)}}}\Gamma^{D_{ji^{(k)}}^{(k)}}(1-d_{ji^{(k)}}^{(k)}+\delta_{ji^{(k)}}^{(k)}s_k)\prod_{j=n^{(k)}+1}^{p_{i^{(k)}}}\Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)}-\gamma_{ji^{(k)}}^{(k)}s_k)]} \tag{1.3}$$

1) $[(c_j^{(1)};\gamma_j^{(1)})]_{1,n_1}$ stands for $(c_1^{(1)};\gamma_1^{(1)}),\cdots,(c_{n_1}^{(1)};\gamma_{n_1}^{(1)})$.

2) $m_2,n_2,\cdots,m_r,n_r,m^{(1)},n^{(1)},\cdots,m^{(r)},n^{(r)},p_{i_2},q_{i_2},R_2,\tau_{i_2},\cdots,p_{i_r},q_{i_r},R_r,\tau_{i_r},p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}},R^{(r)}\in\mathbb{N}$ and verify :

$0\leqslant m_2\leqslant q_{i_2},0\leqslant n_2\leqslant p_{i_2},\cdots,0\leqslant m_r\leqslant q_{i_r},0\leqslant n_r\leqslant p_{i_r},0\leqslant m^{(1)}\leqslant q_{i^{(1)}},\cdots,0\leqslant m^{(r)}\leqslant q_{i^{(r)}}$.

3) $\tau_{i_2}(i_2=1,\cdots,R_2)\in\mathbb{R}^+;\tau_{i_r}\in\mathbb{R}^+(i_r=1,\cdots,R_r);\tau_{i^{(k)}}\in\mathbb{R}^+(i=1,\cdots,R^{(k)}),(k=1,\cdots,r)$.

4) $\gamma_j^{(k)},C_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=1,\cdots,r);\delta_j^{(k)},D_j^{(k)}\in\mathbb{R}^+;(j=1,\cdots,m_k);(k=1,\cdots,r)$.

$\alpha_{kj}^{(l)},A_{kj}\in\mathbb{R}^+;(j=1,\cdots,n_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kj}^{(l)},B_{kj}\in\mathbb{R}^+;(j=1,\cdots,m_k);(k=2,\cdots,r);(l=1,\cdots,k)$.

$\alpha_{kji_k}^{(l)},A_{kji_k}\in\mathbb{R}^+;(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\beta_{kji_k}^{(l)},B_{kji_k}\in\mathbb{R}^+;(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r);(l=1,\cdots,k)$.

$\delta_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=m_k+1,\cdots,q_{i^{(k)}});(k=1,\cdots,r)$.

$\gamma_{ji^{(k)}}^{(k)}\in\mathbb{R}^+;(i=1,\cdots,R^{(k)});(j=n_k+1,\cdots,p_{i^{(k)}});(k=1,\cdots,r)$.

5) $c_j^{(k)}\in\mathbb{C};(j=1,\cdots,n_k);(k=1,\cdots,r);d_j^{(k)}\in\mathbb{C};(j=1,\cdots,m_k);(k=1,\cdots,r)$.

$a_{kji_k}\in\mathbb{C};(j=n_k+1,\cdots,p_{i_k});(k=2,\cdots,r)$.

$b_{kji_k}\in\mathbb{C};(j=m_k+1,\cdots,q_{i_k});(k=2,\cdots,r)$.

$d_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = m_k + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r).$

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = n_k + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r).$

The contour $L_k$ is in the $s_k(k = 1, \cdots, r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where $\sigma$ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1 - a_{2j} + \sum_{k=1}^{2} \alpha_{2j}^{(k)} s_k\right) (j = 1, \cdots, n_2), \Gamma^{A_{3j}}\left(1 - a_{3j} + \sum_{k=1}^{3} \alpha_{3j}^{(k)} s_k\right)$ $(j = 1, \cdots, n_3), \cdots, \Gamma^{A_{rj}}\left(1 - a_{rj} + \sum_{i=1}^{r} \alpha_{rj}^{(i)}\right) (j = 1, \cdots, n_r)$, $\Gamma^{C_j^{(k)}}\left(1 - c_j^{(k)} + \gamma_j^{(k)} s_k\right) (j = 1, \cdots, n^{(k)})(k = 1, \cdots, r)$to the right of the contour $L_k$ and the poles of $\Gamma^{B_{2j}}\left(b_{2j} - \sum_{k=1}^{2} \beta_{2j}^{(k)} s_k\right) (j = 1, \cdots, m_2), \Gamma^{B_{3j}}\left(b_{3j} - \sum_{k=1}^{3} \beta_{3j}^{(k)} s_k\right) (j = 1, \cdots, m_3)$ $, \cdots, \Gamma^{B_{rj}}\left(b_{rj} - \sum_{i=1}^{r} \beta_{rj}^{(i)}\right) (j = 1, \cdots, m_r)$, $\Gamma^{D_j^{(k)}}\left(d_j^{(k)} - \delta_j^{(k)} s_k\right) (j = 1, \cdots, m^{(k)})(k = 1, \cdots, r)$ lie to the left of the contour $L_k$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi$ where

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)} \gamma_j^{(k)} - \tau_{i^{(k)}} \left( \sum_{j=m^{(k)}+1}^{q_i^{(k)}} D_{ji^{(k)}}^{(k)} \delta_{ji^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_i^{(k)}} C_{ji^{(k)}}^{(k)} \gamma_{ji^{(k)}}^{(k)} \right) +$$

$$\sum_{j=1}^{n_2} A_{2j} \alpha_{2j}^{(k)} + \sum_{j=1}^{m_2} B_{2j} \beta_{2j}^{(k)} - \tau_{i_2} \left( \sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2} \alpha_{2ji_2}^{(k)} + \sum_{j=m_2+1}^{q_{i_2}} B_{2ji_2} \beta_{2ji_2}^{(k)} \right) + \cdots +$$

$$\sum_{j=1}^{n_r} A_{rj} \alpha_{rj}^{(k)} + \sum_{j=1}^{m_r} B_{rj} \beta_{rj}^{(k)} - \tau_{i_r} \left( \sum_{j=n_r+1}^{p_{i_r}} A_{rji_r} \alpha_{rji_r}^{(k)} + \sum_{j=m_r+1}^{q_{i_r}} B_{rji_r} \beta_{rji_r}^{(k)} \right) \tag{1.4}$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$\aleph(z_1, \cdots, z_r) = 0( |z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r} ), max( |z_1|, \cdots, |z_r| ) \rightarrow 0$

$\aleph(z_1, \cdots, z_r) = 0( |z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r} ), min( |z_1|, \cdots, |z_r| ) \rightarrow \infty$ where $i = 1, \cdots, r$ :

$\alpha_i = \min_{\substack{1 \leqslant j \leqslant m_i \\ 1 \leqslant j \leqslant m^{(i)}}} Re \left( \sum_{h=2}^{r} \sum_{h'=1}^{h} B_{hj} \frac{b_{hj}}{\beta_{hj}^{h'}} + D_j^{(i)} \frac{d_j^{(i)}}{\delta_j^{(i)}} \right)$ and $\beta_i = \max_{\substack{1 \leqslant j \leqslant n_i \\ 1 \leqslant j \leqslant n^{(i)}}} Re \left( \sum_{h=2}^{r} \sum_{h'=1}^{h} A_{hj} \frac{a_{hj} - 1}{\alpha_{hj}^{h'}} + C_j^{(i)} \frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right)$

**Remark 1.**
If $m_2 = n_2 = \cdots = m_{r-1} = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = B_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = B_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the generalized multivariable Gimel-function reduces in the generalized multivariable Aleph- function ( extension of multivariable Aleph-function defined by Ayant [1]).

**Remark 2.**
If $m_2 = n_2 = \cdots = m_r = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in a generalized multivariable I-function (extension of multivariable I-function defined by Prathima et al. [4]).

**Remark 3.**

If $A_{2j} = B_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = B_{rj} = A_{rji_r} = B_{rji_r} = 1$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in generalized of multivariable I-function (extension of multivariable I-function defined by Prasad [3]).

**Remark 4.**
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the generalized multivariable H-function (extension of multivariable H-function defined by Srivastava and Panda [5,6]).

In your investigation, we shall use the following notations.

$$\mathbb{A} = [(\mathrm{a}_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})]_{1,n_2},[\tau_{i_2}(a_{2ji_2};\alpha_{2ji_2}^{(1)},\alpha_{2ji_2}^{(2)};A_{2ji_2})]_{n_2+1,p_{i_2}},[(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3};\alpha_{3ji_3}^{(1)},\alpha_{3ji_3}^{(2)},\alpha_{3ji_3}^{(3)};A_{3ji_3})]_{n_3+1,p_{i_3}};\cdots;[(\mathrm{a}_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}};\alpha_{(r-1)ji_{r-1}}^{(1)},\cdots,\alpha_{(r-1)ji_{r-1}}^{(r-1)};A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.5}$$

$$\mathbf{A} = [(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,n_r}],[\tau_{i_r}(a_{rji_r};\alpha_{rji_r}^{(1)},\cdots,\alpha_{rji_r}^{(r)};A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.6}$$

$$A = [(\mathrm{c}_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,n^{(1)}}],[\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)},\gamma_{ji^{(1)}}^{(1)};C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}];\cdots;$$

$$[(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,n^{(r)}}],[\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)},\gamma_{ji^{(r)}}^{(r)};C_j^{(r)})_{n^{(r)}+1,p_i^{(r)}}] \tag{1.7}$$

$$\mathbb{B} = [(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})]_{1,m_2},[\tau_{i_2}(b_{2ji_2};\beta_{2ji_2}^{(1)},\beta_{2ji_2}^{(2)};B_{2ji_2})]_{m_2+1,q_{i_2}},[(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)};B_{3j})]_{1,m_3},$$

$$[\tau_{i_3}(b_{3ji_3};\beta_{3ji_3}^{(1)},\beta_{3ji_3}^{(2)},\beta_{3ji_3}^{(3)};B_{3ji_3})]_{m_3+1,q_{i_3}};\cdots;[(\mathrm{b}_{(r-1)j};\beta_{(r-1)j}^{(1)},\cdots,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})_{1,m_{r-1}}],$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}};\beta_{(r-1)ji_{r-1}}^{(1)},\cdots,\beta_{(r-1)ji_{r-1}}^{(r-1)};B_{(r-1)ji_{r-1}})_{m_{r-1}+1,q_{i_{r-1}}}] \tag{1.8}$$

$$\mathbf{B} = [(\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)};B_{rj})_{1,m_r}],[\tau_{i_r}(b_{rji_r};\beta_{rji_r}^{(1)},\cdots,\beta_{rji_r}^{(r)};B_{rji_r})_{m_r+1,q_{i_r}}] \tag{1.9}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,m^{(1)}}],[\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)},\delta_{ji^{(1)}}^{(1)};D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}];\cdots;$$

$$[(\mathrm{d}_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,m^{(r)}}],[\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)},\delta_{ji^{(r)}}^{(r)};D_j^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.10}$$

$$U = m_2,n_2;m_3,n_3;\cdots;m_{r-1},n_{r-1};V = m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)} \tag{1.11}$$

$$X = p_{i_2},q_{i_2},\tau_{i_2};R_2;\cdots;p_{i_{r-1}},q_{i_{r-1}},\tau_{i_{r-1}}:R_{r-1};Y = p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}};\tau_{i^{(r)}};R^{(r)} \tag{1.12}$$

## 2. Required integral.

In this paper, we require the following result. It's a hypergeometric function with unit argument given by Whittaker and Watson [7]

**Lemma.**

$$\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\frac{\Gamma(a+x)\Gamma(b-x)\Gamma(c-x)}{\Gamma(d-x)}e^{\pm\omega\pi x}\mathrm{d}x=\frac{\Gamma(a+b)\Gamma(a+c)\Gamma(d-a-b-c)}{\Gamma(d-b)\Gamma(d-c)}e^{\pm\omega\pi x} \tag{2.1}$$

provided $Re(d-a-b-c)>0$

## 3. Main integrals.

In this section, we evaluate four integrals with respect to their parameters involving the generalized multivariable Gimel-function.

**Theorem 1.**

$$\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\frac{\Gamma(b-x)\Gamma(c-x)}{\Gamma(d-x)}e^{\pm\omega\pi x}\,\gimel^{U;m_r,n_r+1:V}_{X;p_{i_r}+1,q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\mathrm{z}_1 & \mathbb{A};\ (\text{1-a-x;h}_1,\cdots,h_r;1),\mathbf{A}:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\ \mathbf{B}:\ \mathrm{B}\end{array}\right)\mathrm{d}x=$$

$$\frac{e^{\pm\omega\pi a}}{\Gamma(d-b)\Gamma(d-c)}\gimel^{U;m_r+1,n_r+2:V}_{X;p_{i_r}+2,q_{i_r}+1,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\mathrm{z}_1e^{\pm\omega\pi h_1} & \mathbb{A};\ (\text{1-a-b;h}_1,\cdots,h_r;1),(1-a-c;h_1,\cdots,h_r;1),\mathbf{A}:A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_re^{\pm\omega\pi h_r} & \mathbb{B};\ (\text{d-a-b-c;h}_1,\cdots,h_r;1),\mathbf{B}:B\end{array}\right) \tag{3.1}$$

provided

$$h_i>0(i=1,\cdots,r),\ Re(d-a-b-c-d)+\sum_{i=1}^{r}h_i\min_{\substack{1\leqslant j\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_j^{(i)}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>0.$$

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof
To prove the theorem 1, we replace the generalized multivariable Gimel-function by this multiple integrals contour with the help of (1.1), change the order of integrations which is justified under the conditions mentioned above. We get

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{k=1}^{r}\theta_k(s_k)z_k^{s_k}\left[\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\frac{\Gamma(b-x+\sum_{i=1}^{r}h_is_i-x)}{\Gamma(d-x)}e^{\pm\omega\pi x}\mathrm{d}x\right]\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{3.2}$$

Now, we evaluate the inner integral with the help of the result given by Whittaker and Watson [7] and interpret the resulting expression with the help of (1.1), we obtain the desired result.

**Theorem 2.**

$$\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\Gamma(b-x)\Gamma(a+x)\Gamma(c-x)e^{\pm\omega\pi x}\,\gimel^{U;m_r,n_r:V}_{X;p_{i_r}+1,q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\mathrm{z}_1 & \mathbb{A};\ \mathbf{A},(\text{1-a-c;h}_1,\cdots,h_r;1):A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\ \mathbf{B}:\ \mathrm{B}\end{array}\right)\mathrm{d}x=$$

$$\Gamma(a+b)\Gamma(a+c)e^{\pm\omega\pi a}\,\gimel^{U;m_r+1,n_r+2:V}_{X;p_{i_r}+2,q_{i_r}+1,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c}\mathrm{z}_1 & \mathbb{A};\ \mathbf{A},(\text{d-b;h}_1,\cdots,h_r;1),(d-c;h_1,\cdots,h_r;1):A\\ \cdot & \cdot\\ \cdot & \cdot\\ \mathrm{z}_r & \mathbb{B};\ (\text{d-a-b-c;h}_1,\cdots,h_r;1),\mathbf{B}:B\end{array}\right) \tag{3.3}$$

under the same existence conditions that (3.1).

**Theorem 3.**

$$\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\Gamma(b-x)\Gamma(a+x)\Gamma(c-x)e^{\pm\omega\pi x}$$

$$\gimel^{U;m_r,n_r+1:V}_{X;p_{i_r}+2,q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} z_1 & \mathbb{A};\ (1\text{-c+x};h_1,\cdots,h_r;1),\mathbf{A},(d-x;h_1,\cdots,h_r;1):A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B};\ \mathbf{B}:\ B \end{array}\right)dx=\Gamma(a+b)e^{\pm\omega\pi a}$$

$$\gimel^{U;m_r+1,n_r+1:V}_{X;p_{i_r}+3,q_{i_r}+1,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} z_1 & \mathbb{A};\ (1\text{-a-c};h_1,\cdots,h_r;1),\mathbf{A},(d-b;h_1,\cdots,h_r;1),(d-c;2h_1,\cdots,2h_r;1):A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B};\ (\text{d-a-b-c};2h_1,\cdots,2h_r;1),\mathbf{B}:B \end{array}\right) \tag{3.4}$$

provided

$$h_i>0(i=1,\cdots,r),\ Re(d-a-b-c-d)-2\sum_{i=1}^{r}h_i\min_{\substack{1\leqslant j\leqslant m_i\\1\leqslant j\leqslant m^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}B_{hj}\frac{b_{hj}}{\beta_{hj}^{h'}}+D_j^{(i)}\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>0.$$

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

**Theorem 4.**

$$\frac{1}{2\pi\omega}\int_{-\omega\infty}^{\omega\infty}\frac{\Gamma(b-x)\Gamma(a+x)}{\Gamma(c-x)}e^{\pm\omega\pi x}\gimel^{U;m_r,n_r+1:V}_{X;p_{i_r}+1,q_{i_r},\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} z_1 & \mathbb{A};\ (1\text{-d-x};h_1,\cdots,h_r;1),\mathbf{A}:A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B};\ \mathbf{B}:\ B \end{array}\right)dx=$$

$$\frac{e^{\pm\omega\pi a}\Gamma(a+b)}{\Gamma(c-b)}\gimel^{U;m_r+1,n_r+2:V}_{X;p_{i_r}+2,q_{i_r}+1,\tau_{i_r}:R_r:Y}\left(\begin{array}{c|c} z_1 & \mathbb{A};\ (1\text{-a-d};h_1,\cdots,h_r;1),\mathbf{A},(c-d;h_1,\cdots,h_r;1):A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B};\ (\text{c-a-b-d};h_1,\cdots,h_r;1),\mathbf{B}:B \end{array}\right) \tag{3.5}$$

provided

$$h_i>0(i=1,\cdots,r),\ Re(c-a-b-d)-\sum_{i=1}^{r}h_i\max_{\substack{1\leqslant j\leqslant n_i\\1\leqslant j\leqslant n^{(i)}}}Re\left(\sum_{h=2}^{r}\sum_{h'=1}^{h}A_{hj}\frac{a_{hj}-1}{\alpha_{hj}^{h'}}+C_j^{(i)}\frac{c_j^{(i)}-1}{\gamma_j^{(i)}}\right)$$

$|arg(z_k)|<\frac{1}{2}A_i^{(k)}\pi$ where $A_i^{(k)}$ is defined by (1.4).

To prove the formulae (3.3), (3.4) and (3.5), we use the similarly process.

## 4. Conclusion.

The generalized Gimel-function of several variables presented in this paper, are quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various known and (news) integrals with respect to parameters concerning the special functions of one variable and several variables.

## REFERENCES.

[1] F. Ayant, An integral associated with the Aleph-functions of several variables. International Journal of Mathematics Trends and Technology (IJMTT), 31(3) (2016), 142-154.

[2] B.L.J. Braaksma, Asymptotics expansions and analytic continuations for a class of Barnes-integrals, Compositio Math. 15 (1962-1964), 239-341.

[3] Y.N. Prasad, Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , 231-237.

[4] J. Prathima, V. Nambisan and S.K. Kurumujji, A Study of I-function of Several Complex Variables, International Journal of Engineering Mathematics Vol (2014), 1-12.

[5] H.M. Srivastava and R. Panda, Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24 (1975),119-137.

[6] H.M. Srivastava and R.Panda, Some expansion theorems and generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Paul. 25 (1976), 167-197.

[7] E.T. Whittaker end G.N. Watson, A course of modern analysis, Cambridge Univ. Press., Cambridge, (1952).