SOME PROPERTIES OF NANO γ-CLOSED SETS

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Abstract. In this paper we introduced and study a new class of sets called nano γ-closed sets and nano $\pi g\gamma^{\star}$-closed sets and discussed some its properties are investigated.

1. Introduction

The first introduced by Lellis Thivagar et al in 2013 [5] a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological

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space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

The first nano generlizing closed sets (resp. nano g-closed sets) was done by K.Bhuvaneswari et.al in 2014 [4] introduced and studied nano g-closed sets in nano topological spaces. In 2017, I. Rajasekaran et.al [9] studied the notion of generalized closed sets in nano topological spaces called nano πg-closed sets.

The main aim of this paper is to introduce properties of nano $\pi g\gamma^{\star}$-closed sets. The relationships of nano $\pi g\gamma^{\star}$-closed sets with various other sets are discussed.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [7] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

(2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

(3) *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Property 2.2. [5] *If (U, R) is an approximation space and $X, Y \subseteq U$; then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$;

(2) $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;

(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

(8) $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;

(9) $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

(10) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. [5] *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

(1) *U and $\phi \in \tau_R(X)$,*

(2) *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

(3) *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. [5] *If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5. [5] *If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.*

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. *A subset H of a nano topological space $(U, \tau_R(X))$ is called*

(1) *nano semi-open* [5] *if $H \subseteq Ncl(Nint(H))$.*
(2) *nano regular-open* [5] *if $H = Nint(Ncl(H))$.*
(3) *nano pre-open* [5] *if $H \subseteq Nint(Ncl(H))$.*
(4) *nano α-open* [6] *if $H \subseteq Nint(Ncl(Nint(H)))$.*
(5) *nano β-open* [8] *if $H \subseteq Ncl(Nint(Ncl(H)))$.*
(6) *nano π-open* [1] *if the finite union of nano regular-open sets.*

The complements of the above mentioned open sets is called their respective closed sets.

Definition 2.7. *A subset H of a nano topological space $(U, \tau_R(X))$ is called*

(1) *nano g-closed* [3] *if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*
(2) *nano $g\beta$-closed* [10] *if $N\beta cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.*

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(3) *nano gp-closed* [4] *if* $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano open.*

(4) *nano* πg*-closed* [9] *if* $Ncl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano* π*-open.*

(5) *nano* πgp*-closed* [11] *if* $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano* π*-open.*

(6) *nano* $\pi g\beta$*-closed* [12] *if* $N\beta cl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano* π*-open.*

(7) *nano* αg*-closed* [13] *if* $N\alpha cl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano open.*

(8) *nano* $g\alpha$*-closed* [13] *if* $N\alpha cl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano* α*-open.*

(9) *nano gp-closed* [4] *if* $Npcl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano open.*

(10) *nano gs-closed* [2] *if* $Nscl(H) \subseteq G$, *whenever* $H \subseteq G$ *and* G *is nano semi-open.*

The complements of the above mentioned closed sets is called their respective open sets.

3. On nano γ-closed and nano $\pi g\gamma^\star$-closed sets

Definition 3.1. *A subset* H *of nano topological space* $(U, \tau_R(X))$ *(briefly NT's) is called*

(1) *nano* γ*-closed if* $Nint(Ncl(H)) \cap Ncl(Nint(H)) \subseteq H$.

(2) *nano* $g\gamma$*-closed if* $N\gamma cl(H) \subseteq G$ *whenever* $H \subseteq G$ *and* G *is nano open.*

(3) *nano* $\pi g\gamma^\star$*-open set if* $Nint(N\gamma cl(H)) \subseteq G$ *whenever* $H \subseteq G$ *and* G *is nano* π*-open set.*

The complements of the above mentioned open sets is called their respective closed sets.

Example 3.2. *Let* $U = \{e_1, e_2, e_3, e_4\}$ *with* $U/R = \{\{e_1, e_2\}, \{e_3\}, \{e_4\}\}$ *and* $X = \{e_2, e_4\}$. *Then the nano topology* $\tau_R(X) = \{\phi, \{e_4\}, \{e_1, e_2\}, \{e_1, e_2, e_4\}, U\}$. *Here* $\{e_1\}$ *is nano* γ*-closed, nano* $g\gamma$*-closed and nano* $\pi g\gamma^\star$*-closed.*

Theorem 3.3. *In a NT's, every nano closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano closed set and G be a nano π-open set of U such that $H \subseteq G$. Then $H = N\gamma cl(H) \subseteq Ncl(H)$ and $G = Nint(N\gamma cl(H)) \subseteq Nint(H) \subseteq Nint(G)$. Therefore H is nano $\pi g\gamma^\star$-closed set.

Remark 3.4. *The converse of the Theorem 3.3 is not true in general the following example.*

Example 3.5. *In Example 3.2, then the subset of $H = \{e_1\}$ is nano $\pi g\gamma^\star$-closed but not closed.*

Theorem 3.6. *In a NT's, every nano semi-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano semi-closed set and G be a nano π-open set in U such that $H \subseteq G$. Then $H = N\gamma cl(H) \subseteq Nscl(H)$ and $G = Nint(N\gamma cl(H)) \subseteq Nint(H) \subseteq Nint(G)$. Therefore H is nano $\pi g\gamma^\star$-closed.

Remark 3.7. *The converse of the Theorem 3.6 is not true in general the following example.*

Example 3.8. *In Example 3.2, then the subset of $H = \{e_2\}$ is nano $\pi g\gamma^\star$-closed but not semi-closed.*

Theorem 3.9. *In a NT's, every nano pre-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano pre-closed set and G be a nano π-open set of U such that $H \subseteq G$. Then $H = N\gamma cl(H) \subseteq Npcl(H)$ and $G = Nint(N\gamma cl(H)) \subseteq Nint(H) \subseteq Nint(G)$. Thus H is a nano $\pi g\gamma^\star$-closed.

Remark 3.10. *The converse of the Theorem 3.9 is not true in general the following example.*

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Example 3.11. *In Example 3.2, then the subset of* $H = \{e_4\}$ *is nano* $\pi g\gamma^\star$*-closed but not nano pre-closed.*

Theorem 3.12. *In a NT's, every nano* α*-closed is nano* $\pi g\gamma^\star$*-closed.*

Proof. Let H be a nano α-closed set and G be a nano π-open in U such that $H \subseteq G$. Then $H = N\gamma cl(H) \subseteq N\alpha cl(H)$ and $G = Nint(N\gamma cl(H)) \subseteq Nint(H) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^\star$-closed.

Remark 3.13. *The converse of the Theorem 3.12 is not true in general the following example.*

Example 3.14. *In Example 3.2, then the subset of* $H = \{e_1\}$ *is nano* $\pi g\gamma^\star$*-closed but not nano* α*-closed.*

Theorem 3.15. *In a NT's, every nano* γ*-closed is nano* $\pi g\gamma^\star$*-closed.*

Proof. Let H be a nano γ-closed set and G be a nano π-open set in U such that $H \subseteq G$. Then $H = N\gamma cl(H)$ and $G = Nint(N\gamma cl(H)) = Nint(H) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^\star$-closed.

Remark 3.16. *The converse of the Theorem 3.15 is not true in general the following example.*

Example 3.17. *Then In Example 3.2, then the subset of* $H = \{e_1, e_2, e_4\}$ *is nano* $\pi g\gamma^\star$*-closed but not nano* γ*-closed.*

Theorem 3.18. *In a NT's, every nano* β*-closed is nano* $\pi g\gamma^\star$*-closed.*

Proof. Let H be a nano β-closed set and G be a nano π-open set in U such that $H \subseteq G$. Then $H = N\gamma cl(H) \subseteq N\beta cl(H)$ and $G = Nint(N\gamma cl(H)) = Nint(H) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^\star$-closed.

Remark 3.19. *The converse of the Theorem 3.18 is not true in general the following example.*

Example 3.20. *Then In Example 3.2, then the subset of* $H = \{e_1, e_2, e_4\}$ *is nano* $\pi g\gamma^\star$*-closed but not nano* β*-closed.*

Theorem 3.21. *In a NT's, every nano g-closed is nano* $\pi g\gamma^\star$*-closed.*

Proof. Let H be a nano g-closed set and G be a nano π-open in U such that $H \subseteq G$. Since every nano π-open is nano open, $Ncl(H) \subseteq G$. Now we have $N\gamma cl(H) \subseteq Ncl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Thus H is a nano $\pi g\gamma^\star$-closed.

Remark 3.22. *The converse of the Theorem 3.21 is not true in general the following example.*

Example 3.23. *In Example 3.2, then the subset of* $H = \{e_2\}$ *is nano* $\pi g\gamma^\star$*-closed but not nano g-closed.*

Theorem 3.24. *In a NT's, every nano gp-closed is nano* $\pi g\gamma^\star$*-closed.*

Proof. Let H be a nano gp-closed set and G be a π-open in U such that $H \subseteq G$. Since every nano π-open is nano open, $Npcl(H) \subseteq G$. Now we have $N\gamma cl(H) \subseteq Npcl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Thus H is a nano $\pi g\gamma^\star$-closed.

Remark 3.25. *The converse of the Theorem 3.24 is not true in general the following example.*

Example 3.26. *In Example 3.2, then the subset of* $H = \{e_4\}$ *is nano* $\pi g\gamma^\star$*-closed but not gp-closed.*

Theorem 3.27. *In a NT's, every nano* $g\beta$*-closed is nano* $\pi g\gamma^\star$*-closed.*

Proof. Let H be a nano $g\beta$-closed set and G be a nano π-open in U such that $H \subseteq G$. Since every nano π-open is nano open, $N\beta cl(H) \subseteq G$. Now we have $N\gamma cl(H) \subseteq N\beta cl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Therefore H is a nano $\pi g\gamma^\star$-closed.

Remark 3.28. *The converse of the Theorem 3.27 is not true in general the following example.*

Example 3.29. *In Example 3.2, then the subset of $H = \{e_1, e_2, e_4\}$ is nano $\pi g\gamma^\star$-closed but not $g\beta$-closed.*

Theorem 3.30. *In a NT's, every nano $g\alpha$-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano $g\alpha$-closed set and G is nano π-open in U such that $H \subseteq G$. Since every nano π-open is nano open, $N\alpha cl(H) \subseteq G$. Now we have $N\gamma cl(H) \subseteq N\alpha cl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^\star$-closed.

Remark 3.31. *The converse of the Theorem 3.30 is not true in general the following example.*

Example 3.32. *In Example 3.2, then the subset of $H = \{e_2\}$ is nano $\pi g\gamma^\star$-closed but not nano $g\alpha$-closed.*

Theorem 3.33. *In a NT's, every nano $g\gamma$-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano $g\gamma$-closed set and G be a nano π-open in U such that $H \subseteq G$. Since every nano π-open is nano open, $N\gamma cl(H) \subseteq G$. Thus $Nint(N\gamma cl(H)) \subseteq Nint(G) = G$. Hence H is nano $\pi g\gamma^\star$-closed.

Remark 3.34. *The converse of the Theorem 3.33 is not true in general the following example.*

Example 3.35. *In Example 3.2, then the subset of $H = \{e_1, e_2, e_4\}$ is nano $\pi g\gamma^\star$-closed but not nano $g\gamma$-closed.*

Theorem 3.36. *In a NT's, every nano gs-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano gs-closed set and G be a nano semi-open in U such that $H \subseteq G$. Since every nano semi-open is nano open, $Nscl(H) \subseteq G$. Now we have $N\gamma cl(H) \subseteq Nscl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Thus H is a nano $\pi g\gamma^\star$-closed.

Remark 3.37. *The converse of the Theorem 3.36 is not true in general the following example.*

Example 3.38. *In Example 3.2, then the subset of $H = \{e_1, e_4\}$ is nano $\pi g\gamma^\star$-closed but not nano gs-closed.*

Theorem 3.39. *In a NT's, every nano πg-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano πg-closed set and G be a nano π-open in U such that $H \subseteq G$. Hence $Ncl(H) \subseteq G$ and $N\gamma cl(H) \subseteq Ncl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^\star$-closed.

Remark 3.40. *The converse of the Theorem 3.39 is not true in general the following example.*

Example 3.41. *In Example 3.2, then the subset of $H = \{e_1, e_2\}$ is nano $\pi g\gamma^\star$-closed but not πg-closed.*

Theorem 3.42. *In a NT's, every nano αg-closed is nano $\pi g\gamma^\star$-closed.*

Proof. Let H be a nano αg-closed set and G be a nano open in U such that $H \subseteq G$. Since every nano α-open is nano open. Hence $N\alpha cl(H) \subseteq G$ and Now $N\gamma cl(H) \subseteq N\alpha cl(H) \subseteq G = Nint(N\gamma cl(H)) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^{\star}$-closed.

Remark 3.43. *The converse of the Theorem 3.42 is not true in general the following example.*

Example 3.44. *In Example 3.2, then the subset of $H = \{e_2, e_3, e_4\}$ is nano $\pi g\gamma^{\star}$-closed but not nano αg-closed.*

Theorem 3.45. *In a NT's, every nano πgp-closed is nano $\pi g\gamma^{\star}$-closed.*

Proof. Let H be a nano πgp-closed set and G is nano π-open in U such that $H \subseteq G$ and G be a nano π-open in U. Then $Npcl(H) \subseteq G$ and $N\gamma cl(H) \subseteq Npcl(H) \subseteq G$, $Nint(N\gamma cl(H)) \subseteq Nint(G) = G$. Hence H is nano $\pi g\gamma^{\star}$-closed.

Remark 3.46. *The converse of the Theorem 3.45 is not true in general the following example.*

Example 3.47. *In Example 3.2, then the subset of $H = \{e_4\}$ is nano $\pi g\gamma^{\star}$-closed but not πgp-closed.*

Theorem 3.48. *In a NT's, every nano $\pi g\beta$-closed is nano $\pi g\gamma^{\star}$-closed.*

Proof. Let H be a nano $\pi g\beta$-closed set and G be a nano π-open in U such that $H \subseteq G$. Since $N\beta cl(H) \subseteq G$ and $N\gamma cl(H) \subseteq N\beta cl(H) \subseteq G = Ncl(Nint(N\gamma cl(H))) \subseteq Nint(G)$. Thus H is nano $\pi g\gamma^{\star}$-closed.

Remark 3.49. *The converse of the Theorem 3.48 is not true in general the following example.*

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Example 3.50. *In Example 3.2, then the subset of* $H = \{e_1, e_2, e_4\}$ *is nano* $\pi g\gamma^\star$*-closed but not nano* $\pi g\beta$*-closed.*

Remark 3.51. *In a NT's the union of two nano* $\pi g\gamma^\star$*-closed set is nano* $\pi g\gamma^\star$*-closed the following example.*

Example 3.52. *Let* $U = \{n_1, n_2, n_3\}$ *with* $U/R = \{\{n_1, n_3\}, \{n_2\}\}$ *and* $X = \{n_3\}$*. Then the nano topology* $\tau_R(X) = \{\phi, \{n_1, n_3\}, U\}$*. Finally the subset* $P = \{n_1\}$ *and* $Q = \{n_2\}$ *then* $P \cup Q = \{n_1, n_2\}$ *is nano* $\pi g\gamma^\star$*-closed set.*

Remark 3.53. *In a NT's the intersection of two nano* $\pi g\gamma^\star$*-closed is nano* $\pi g\gamma^\star$*-closed the following example.*

Example 3.54. *In Example 3.52, finally the subset* $P = \{n_1, n_3\}$ *and* $Q = \{n_2, n_3\}$ *then* $P \cap Q = \{n_3\}$ *is nano* $\pi g\gamma^\star$*-closed set.*

Remark 3.55. *We obtain Definitions, Theorems, Remarks and Examples follows from the diagrams-I and II.*

Diagram-I

$$\begin{array}{ccc}
\textit{nano closed} & \textit{nano semi-closed} & \textit{nano pre-closed} \\
\searrow & \downarrow & \swarrow \\
\textit{nano } \alpha\textit{-closed} \rightarrow & \textit{nano } \pi g\gamma^\star\textit{-closed} & \leftarrow \textit{nano } \beta\textit{-closed} \\
\nearrow & \uparrow & \nwarrow \\
\textit{nano } g\textit{-closed} & \textit{nano } gp\textit{-closed} & \textit{nano } gs\textit{-closed}
\end{array}$$

Diagram-II

$$
\begin{array}{ccccc}
\textit{nano } \alpha g\textit{-closed} & & \textit{nano } \pi g\textit{-closed} & & \textit{nano } \pi gp\textit{-closed} \\
\searrow & & \downarrow & & \swarrow \\
\textit{nano } g\alpha\textit{-closed} & \rightarrow & \textit{nano } \pi g\gamma^{\star}\textit{-closed} & \leftarrow & \textit{nano } \gamma\textit{-closed} \\
\nearrow & & \uparrow & & \nwarrow \\
\textit{nano } \pi g\beta\textit{-closed} & & \textit{nano } g\beta\textit{-closed} & & \textit{nano } g\gamma\textit{-closed}
\end{array}
$$

Reverse of the above implications but not converse.

4. Further properties

Theorem 4.1. *Let U be a nano topological space if $H \subseteq U$ is nano $\pi g\gamma^{\star}$-closed then $Nint(N\gamma cl(H)) - H$ does not contain any non empty nano π-closed.*

Proof. Let H be a nano $\pi g\gamma^{\star}$-closed in U and $K \subseteq Nint(N\gamma cl(H)) - H$ such that K is nano π-closed in U. Then $(U - K)$ is nano π-open in U and $H \subseteq (U - K)$. Since H is nano $\pi g\gamma^{\star}$-closed, $Nint(N\gamma cl(H)) \subseteq (U - K) \Rightarrow K \subseteq (U - Nint(N\gamma cl(H)))$. Hence $K \subseteq (Nint(N\gamma cl(H)) - H) \cap (U - Nint(N\gamma cl(H))) \Rightarrow K = \phi$. Thus $Nint(N\gamma cl(H)) - H$ does not contain any non empty nano π-closed.

Theorem 4.2. *Let $K \subseteq H \subseteq U$ where H is nano $\pi g\gamma^{\star}$-closed and π-open in U, then K is nano $\pi g\gamma^{\star}$-closed relative to $H \iff K$ is nano $\pi g\gamma^{\star}$-closed in U.*

Proof. Let $K \subseteq H \subseteq H$ where H is a nano $\pi g\gamma^{\star}$-closed and nano π-open, then $Nint(N\gamma cl(H)) \subseteq H$. Since $K \subseteq H, Nint(N\gamma cl(H)) \subseteq Nint(N\gamma cl(H)) \subseteq H$. Let K be nano $\pi g\gamma^{\star}$-closed in H and let $K \subseteq G$ where G is nano π-open in U, then $K = K \cap H \subseteq G \cap H$, which is nano π-open in H. Thus $(Nint(N\gamma cl(K)))_H \subseteq G \cap H$. Also we have, $(Nint(N\gamma cl(K)))_H = (Nint(N\gamma cl(K)))_H = (Nint(N\gamma cl(K)))$. Therefore $(Nint(N\gamma cl(K))) \subseteq G \cap H \subseteq G$. Thus K is nano $\pi g\gamma^{\star}$-closed in U.

Conversely, let K be nano $\pi g\gamma^\star$-closed in U. Let $G \subseteq F$ where F is nano π-open in H. Then $F = G \cap H$ where G is nano π-open in U. Thus $K \subseteq F = G \cap H \subseteq G$. Since K is nano $\pi g\gamma^\star$-closed in U, $Nint(N\gamma cl(K)) \subseteq G$. Hence $(Nint(N\gamma cl(K)))_H = H \cap Nint(N\gamma cl(K)) \subseteq G \cap H = F$. Therefore K is nano $\pi g\gamma^\star$-closed relative to H.

Theorem 4.3. *If H is a nano $\pi g\gamma^\star$-closed and K is any set such that $H \subseteq K \subseteq Nint(N\gamma cl(H))$, then K is a nano $\pi g\gamma^\star$-closed.*

Proof. Let $K \subseteq G$ and G be a nano π-open. Since $H \subseteq K \subseteq G$ and H is nano $\pi g\gamma^\star$-closed, $Nint(N\gamma cl(H)) \subseteq G$. Now $Nint(N\gamma cl(K)) \subseteq Nint(N\gamma cl(H)) \subseteq G$. Thus K is a nano $\pi g\gamma^\star$-closed.

Theorem 4.4. *Let U be a nano topological space if $H \subseteq U$ is nowhere dense then H is nano $\pi g\gamma^\star$-closed.*

Proof. Let $H \subseteq G$ where G is nano π-open in U. Since H is nowhere dense, $Nint(Ncl(H)) = \phi$. We have $Nint(N\gamma cl(H)) \subseteq Nint(Ncl(H)) = \phi \subseteq G$. Thus H is nano $\pi g\gamma^\star$-closed.

Theorem 4.5. *In a NT's of U for each $a \in U$, $U - \{a\}$ is either nano $\pi g\gamma^\star$-closed or nano π-open in U.*

Proof. Suppose $U - \{a\}$ is not nano π-open then U is the only π-open containing $U - \{a\}$. Hence $Nint(N\gamma cl(U - \{a\})) \subseteq U \Rightarrow U - \{a\}$ is nano $\pi g\gamma^\star$-closed.

Theorem 4.6. *A subset $H \subseteq U$ is nano $\pi g\gamma^\star$-open $\iff$ $V \subseteq Ncl(N\gamma int(H))$ whenever V is nano π-closed and $V \subseteq H$.*

Proof. Assume that $H \subseteq U$ is nano $\pi g\gamma^\star$-open. Let V be nano π-closed such that $V \subseteq H$. Then $(U - H) \subseteq (U - V)$. Since $(U - H)$ is nano $\pi g\gamma^\star$-closed and $(U - V)$ is nano π-open, $Nint(N\gamma cl(U-H)) \subseteq (U-V) \Rightarrow (U - Ncl(N\gamma int(H))) \subseteq (U-V)$. Therefore $V \subseteq Ncl(N\gamma int(H))$.

Conversely, we assume V is nano π-closed and $V \subseteq H$ such that $V \subseteq Ncl(N\gamma int(H))$. Let $(U-H) \subseteq G$, where G is nano π-open. Then $(U-G) \subseteq H$ and since $(U-G)$ is nano π-closed, $(U-G) \subseteq Ncl(N\gamma int(H)) \Rightarrow Nint(N\gamma cl(U-H)) \subseteq G$. Thus $(U-H)$ is nano $\pi g\gamma^\star$-closed and H is nano $\pi g\gamma^\star$-open.

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