

Fine GS Closed Sets and Fine SG Closed Sets in Fine Topological Space

P.Deepika^{#1}, S.Chandrasekar^{*2}

^{#1} Assistant Professor, Department of Mathematics,
Valliammal College for women, Anna Nagar East,
Chennai, Tamilnadu, India.

^{*2} Assistant Professor, Department of Mathematics,
Arignar Anna Government Arts College,
Namakkal(DT), Tamil Nadu, India

Abstract

In 1990, S.P.Arya et al have introduced the concept of generalized semi closed sets to characterize the s-normality. While in 1987, P.Bhattacharyya et al have introduced the notion of semi generalized closed sets in topological spaces. Since then many works have been developed in the fields of generalized open and generalized closed sets. Powar P. L. and Rajak K. have introduced fine-topological space which is a special case of generalized topological space. Aim of this paper is we introduced in fine Fine gs closed sets and Fine sg closed sets in fine topological spaces

Keywords

Fine generalized semi closed set, Fine semi generalized closed set, F-gs interior, F-gs closure, F-sg interior, F-sg closure

AMS Mathematical Subject classification (2010): 54C08, 54C10

I. INTRODUCTION

In 1963 semi open sets and semi continuous functions were introduced and investigated by N. Levine [9]. In 1987, Bhattacharyya and Lahiri [2] used semi open sets to define and investigate the notion of semi generalized closed sets. Later, in 1990 S.P.Arya et al [1] have introduced the concepts of generalized semi closed sets using semi closure to characterize the s-normality axiom. Now, we found the various papers in the field of generalized open sets and generalized closed sets. Powar P. L. and Rajak K [12] have introduced fine-topological space which is a special case of generalized topological space. Aim of this paper is we introduced Fine gs closed sets and Fine sg closed sets in fine topological spaces and also study the notions like fine generalized semi closure and interior and fine semi generalized closure and interior operators in fine topological spaces

2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition: 2.1 [11,12]

Let (X, τ) be a topological space we define, $\tau(A_\alpha) = \tau_\alpha = \{G_\alpha (\neq X) : G_\alpha \cap A_\alpha \neq \emptyset, \text{ for } A_\alpha \in \tau \text{ and } A_\alpha \neq X, \emptyset \text{ for some } \alpha \in J, \text{ where } J \text{ is the index set}\}$. Now, define $\tau_f = \{\{\emptyset, X\} \cup \tau_\alpha\}$. The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine topological space X and generated by the topology τ on X .

The element of τ_f are called fine open sets in (X, τ, τ_f) and the complement of fine open set is called fine closed sets and it is denoted by τ_f^c

Example :2.2. [11,12]

Consider a topological space $X = \{p, q, r\}$ with the topology

$\tau = \{X, \emptyset, \{p\}\} \cong \{X, \emptyset, A_\alpha\}$ where $A_\alpha = \{p\}$. In view of Definition 2.1 we have,

$\tau_\alpha = \tau(A_\alpha) = \tau \{p\} = \{\{p\}, \{p, q\}, \{p, r\}\}$

then the fine collection is $\tau_f = \{\emptyset, X\} \cup \{\tau_\alpha\} = \{\emptyset, X, \{p\}, \{p, q\}, \{p, r\}\}$.

We quote some important properties of fine topological spaces.

Lemma: 2.3. [11,12]

Let (X, τ, τ_f) be a fine space then arbitrary union of fine open set in X is fine-open in X .

Lemma: 2.4 [11,12]

The intersection of two fine-open sets need not be a fine-open set as the following example shows.

Example:2.5 [11,12]

Let $X = \{p, q, r\}$ be a topological space with the topology

$\tau = \{X, \emptyset, \{p\}, \{q\}, \{p, q\}\}$, $\tau_f = \{X, \emptyset, \{p\}, \{q\}, \{p, q\}, \{q, r\}, \{p, r\}\}$. It is easy to see that, the above

collection τ_f is not a topology. Since, $\{p, r\} \cap \{q, r\} = \{r\} \notin \tau_f$. Hence, the collection of fine open sets in a fine space X does not form a topology on X , but it is a generalized topology on X .

Remark :2.6 [11,12]

In view of Definition 2.1 of generalized topological space and above Lemmas 2.3 and 2.4 it is apparent that (X, τ, τ_f) is a special case of generalized topological space. It may be noted specifically that the topological space plays a key role while defining the fine space as it is based on the topology of X but there is no topology in the back of generalized topological space.

Definition:2.7 [11,12]

A subset A of a Fine space (X, τ, τ_f) is called Fine semi-open if $A \subset \text{Fcl}(\text{Fint}(A))$.

The complement of Fine semi-open set is called Fine semi-closed.

The Fine semi-closure of a subset A of Fine space X , denoted by $\text{Fsc}(A)$, is defined to be the intersection of all Fine semi-closed sets containing A in Fine space X .

Definition :2.8

Let (X, τ, τ_f) be a Fine topological space. A subset A of a Fine space X is called Fsg-closed if $\text{Fsc}(A) \subset U$ whenever $A \subset U$ and U is Fine semi-open in Fine space (X, τ, τ_f) . The complement of Fsg-closed set is called Fsg-open.

Definition:2.9

Let (X, τ, τ_f) be a Fine topological space. A subset A of Fine space X is called Fine generalized semi closed (F-gs-closed) if $\text{F-scl}A \subseteq U$ whenever $A \subseteq U$ and U is Fine open. The complement of a F-gs-closed set is called a F-gs-open set.

Example :2.10

Let $X = \{1, 2, 3, 4\}$ be a topological space with the Fine topology

$\tau = \{X, \emptyset, \{1\}\}$, $\tau_f = \{X, \emptyset, \{1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$

Then $\text{F-GSO}(X) = \{\emptyset, X, \{1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$

Definition 2.12 [11]

Let A be a subset of a Fine space (X, τ, τ_f) is called Fine generalized closed (Fine g- closed) if $\text{cl}A \subseteq U$, whenever $A \subseteq U$ and U is Fine open. The complement of a Fine g-closed set is called the Fine g-open set.

Definition: 2.13 [11]

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called Fine g-closed if for each Fine closed set G of X , $f(G)$ is Fine g-closed set.

Definition:2.14

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called a Fine pre semi closed if image of each Fine semi closed set is Fine semi closed.

Lemma: 2.15

If a mapping $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is Fine presemi closed, then for each subset B of Fine space Y and each Fine semi open set V in Fine space X containing $f^{-1}(B)$, there exists a Fine semi open set U in Fine space X containing x such that $f^{-1}(U) \subset V$.

3. F-gs CLOSURE AND F-gs INTERIOR OF A SET

In this section, we introduce the notions of Fine generalized semi closure and Fine generalized semi interior operators of a subset of a Fine space X .

Definition:3.1

The generalized semi closure of a subset A of a Fine space X is the intersection of all Fine generalized semi closed sets containing A and is denoted by $\text{F-gs cl}(A)$.

Since every Fine g-closed set as well as Fine semiclosed set is F-gs-closed set and hence we have, $A \subset \text{F-gscl}(A) \subset \text{F-cl}A$ and $A \subset \text{F-gscl}A \subset \text{F-scl}A \subset \text{F-cl}A$.

Definition:3.3

A point x of a Fine space (X, τ, τ_f) is called a Fine generalized semi limit point (written as F-gs-limit point) of a subset A of Fine space X , if for each F-gs-open set U containing x , $A \cap (U - \{x\}) \neq \emptyset$.

The set of all F-gs-limit points of a set A will be denoted by $F\text{-gsd}(A)$ is called generalized semi-derived set of A .

One can easily prove the following.

Lemma:3.4 :

If A is a subset of a Fine space (X, τ, τ_f) , then $F\text{-gscl}A = A \cup F\text{-gsd}(A)$.

Note :

A point $x \in F\text{-gscl}(A)$ iff every F-gs-open set containing point x contains a point of A .

The following theorem can be easily proved.

Theorem:3.5

If A and B are subsets of a space (X, τ, τ_f) , then the following are true:

- (i) $F\text{-gsd}(A) \cup F\text{-gsd}(B) = F\text{-gsd}(A \cup B)$.
- (ii) $F\text{-gscl}(A \cup B) = F\text{-gscl}(A) \cup F\text{-gscl}(B)$.
- (iii) $F\text{-gscl}(F\text{-gscl}(A)) = F\text{-gscl}(A)$.

Next, we prove the following.

Theorem:3.6

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-closed map, then $F\text{-gscl}(f(A)) \subset f(\text{Fcl}(A))$ for every subset A of Fine space X .

Proof :

Let $A \subset X$. Since f is F-gs-closed map, $f(F\text{-cl}A)$ is F-gs-closed containing $f(A)$. Hence, $F\text{-gscl}(f(A)) \subset f(A) \subset f(F\text{-cl}A)$. Now, we define the following.

Definition:3.7

The Fine generalized semi-interior of a subset A of a Fine space X is the union of all F-gs-open sets contained in A and is denoted by $F\text{-gsint}(A)$. Since every Fine semiopen set is F-gs-open set and F-g-open set is F-gs-open and hence we have, $F\text{-sint}A \subset F\text{-gsint}A \subset F\text{-int}A$.

Easy proof of the following is omitted.

Lemma:3.8

For any subset A of a Fine space (X, τ, τ_f) then

- (i) $F\text{-gscl}(X-A) = (X - F\text{-gsint}A)$.
- (ii) $F\text{-gsint}(X-A) = (X - F\text{-gscl}A)$.

Recall the following.

Definition: 3.9

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called F-gs-open if for each Fine open set U of Fine space X , $f(U)$ is F-gs-open set in Fine space Y .

Now, we characterize the F-gs-open mappings in the following.

Theorem: 3.10

For a map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$,

- (i) f is F-gs-open.
- (ii) $f(F\text{-int}A) \subset F\text{-gsint}(f(A))$ for each subset A of Fine space X .
- (iii) For each $x \in X$ and for each Fine open set U containing x , there is a F-gs-open set V containing $f(x)$ such that $V \subset f(U)$.
- (iv) For each subset B of Fine space X , $f^{-1}(F\text{-gscl}(A)) \subset F\text{-cl}(f^{-1}(B))$.

Then (1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4).

Proof :

We prove (iii) \Rightarrow (iv) only :

Let $x \in f^{-1}(F\text{-gscl}(B))$. If $x \notin F\text{-cl}(f^{-1}(B))$, then $x \in U = X - F\text{-cl}(f^{-1}(B))$. Then from (3), there is a F-gs-open set V such that $f(x) \in V \subset f(U)$. Now, $V \subset f(U) \subset f(X - f^{-1}(B)) \subset Y - B$, which shows that $B \subset Y - V$. Since $Y - V$ is F-gs-closed, $F\text{-gscl}(B) \subset Y - V$. Now, $f(x) \in F\text{-gscl}(B)$. Hence, $f(x) \notin V$, which is contradiction. Hence, (iii) \Rightarrow (iv). Next, we prove the following.

Theorem :3.11

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-open, $B \subset Y$ and G is a Fine closed set containing $f^{-1}(B)$, then there is a F-gs-closed set V such that $B \subset V$ and $f^{-1}(V) \subset G$.

Easy and routine proof of the theorem is omitted.

Recall the following.

Definition: 3.12

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called a F-gs-continuous if $f^{-1}(V)$ is F-gs-closed set in Fine space X for every Fine closed set V of Fine space Y.

Equivalently, a map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-continuous if and only if the inverse image of each Fine open set is F-gs-open set.

Clearly, every Fine space continuous map is Fg-continuous, every Fg-continuous map is F-gs-continuous and every Fine semi continuous map is F-gs-continuous map. We, characterize the F-gs-continuous mappings in the following.

Theorem: 3.13

Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a map.

- (i) f is F-gs-continuous.
- (ii) For each $x \in X$ and for each Fine open set V containing $f(x)$, there is a F-gs-open set U containing x such that $f(U) \subset V$.
- (iii) $f(\text{F-gscl}(A)) \subset \text{F-cl}(f(A))$ for each subset A of Fine space X .
- (iv) $\text{F-gscl}(f^{-1}(B)) \subset f^{-1}(\text{cl}(B))$ for each subset B of Fine space Y .

Then (i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (iv).

Straight forward proof of the theorem is omitted.

Next, we recall the following.

Theorem :3.14

If a mapping $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is irresolute and Fine pre semiclosed, then for each F-gs-closed set A of Fine space Y , $f^{-1}(A)$ is F-gs-closed.

However, we have the following.

Lemma :3.15

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is Fine closed and F-gs-continuous and B is a F-gs-closed set of Y , then $f^{-1}(B)$ is F-gs-closed set in Fine space X .

Proof :

Let B be a F-gs-closed subset of Fine space Y . Let $f^{-1}(B) \subset U$, where U is an Fine open set of Fine space X . Since f is Fine closed, there is an Fine open set V such that $B \subset V$ and $f^{-1}(V) \subset U$. Since B is F-gs-closed set, $\text{F-scl}(B) \subset U$. Hence $f^{-1}(\text{F-scl}(B)) \subset U$. Since $\text{F-scl}(B) \subset \text{F-cl}(B)$, Fine closed set and f is F-gs-continuous, $f^{-1}(\text{F-cl}(B))$ is F-gs-closed set in Fine space X . Hence, $\text{F-scl}(f^{-1}(\text{F-cl}(B))) \subset U$ which implies that $\text{F-scl}(f^{-1}(B)) \subset U$ and hence $f^{-1}(B)$ is F-gs-closed set in Fine space X . Hence the theorem.

Clearly, note that the compositions of two F-gs-continuous maps need not be F-gs-continuous. But, we have the following decompositions.

Theorem: 3.16

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-continuous and $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$ is continuous then $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$ is F-gs-continuous.

Proof :

Easy.

Theorem: 3.17

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-gs-continuous and Fine closed and $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$ is F-gs-continuous then $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$ is F-gs-continuous. In view of the above lemma - 3.16, we can prove the above theorem.

4. F-sg CLOSURE AND F-sg INTERIOR OF A SET

In this section, we introduce the Fine semi generalized closure and Fine semi generalized interior operators of a set A of a Fine space X .

Definition : 4.1

Fine semi generalized closure of a subset A of a Fine space X is the intersection of all F-sg-Fine closed sets containing A and is denoted by $\text{F-sgcl}(A)$.

Since every Fine semi closed set is F-sg closed and every F-sg closed set is F-gs-closed set and hence, $\text{F-sgcl}(A) \subset \text{F-scl}(A) \subset \text{F-cl}(A)$ and $\text{F-gscl}(A) \subset \text{F-sgcl}(A) \subset \text{F-scl}(A) \subset \text{F-cl}(A)$.

Definition:4.2

A point x of a Fine space (X, τ, τ_f) is called a Fine semi generalized limit point (written as F-sg-limit point) of a subset A of Fine space X , if for each F-sg-open set U containing x , $A \cap (U - \{x\}) \neq \emptyset$.

The set of all F-sg-limit points of A will be denoted by $\text{F-sgd}(A)$ and is called Fine semi generalized derived set of A .

One can easily prove the following.

Lemma:4.3

If A is a subset of a Fine space (X, τ, τ_f) . then $F\text{-sgcl}A = A \cup F\text{-sgd}(A)$.

Note :

A point $x \in F\text{-sgcl}(A)$ iff every $F\text{-sg-open}$ set containing point x contains the point of A . Now, the following theorem can be easily proved.

Theorem:4.4

If A and B are subsets of a Fine space (X, τ, τ_f) . Then the following are true:

- (i) $F\text{-sgd}(A) \cup F\text{-sgd}(B) = F\text{-sgd}(A \cup B)$.
- (ii) $F\text{-sgcl}(A) \cup F\text{-sgcl}(B) = F\text{-sgcl}(A \cup B)$.
- (iii) $F\text{-sgcl}(F\text{-sgcl}(A)) = F\text{-sgcl}(A)$.

Recall the following.

Definition:4.5

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called a Fine semi generalized closed (i.e., $F\text{-sg-closed}$) map if for each Fine closed set G of Fine space X , $f(G)$ is $F\text{-sg-closed}$ set in Fine space Y .

Next, we give the following.

Theorem:4.6

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is $F\text{-sg-closed}$ then $F\text{-sgcl}(f(A)) \subset f(F\text{-cl}A)$ for each subset A of Fine space X .

Proof is obvious.

We, define the following.

Definition:4.7

The Fine semi generalized interior of a subset A of a Fine space X is the union of all $F\text{-sg-open}$ sets contained in A and is denoted by $F\text{-sgint}(A)$.

Since every Fine semiopen set is $F\text{-sg-open}$ set and hence, $F\text{-sint}(A) \subset F\text{-sgint}(A) \subset F\text{-int}(A)$. And, every $F\text{-sg-open}$ set is $F\text{-gs-open}$ set and hence, $F\text{-sint}(A) \subset F\text{-sgint}(A) \subset F\text{-gsint}(A) \subset F\text{-int}(A)$.

One can easily prove the following.

Lemma :4.8

For any subset A of a Fine space (X, τ, τ_f) , the following are true:

- (i) $F\text{-sgcl}(X-A) = X - F\text{-sgint}(A)$.
- (ii) $F\text{-sgint}(X-A) = X - F\text{-sgcl}(A)$.

Recall the following.

Definition:4.9

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called $F\text{-sg-open}$ if for each Fine open set U of Fine space X , $f(U)$ is $F\text{-sg-open}$ set in Fine space Y . Now, we characterize the $F\text{-sg-open}$ mappings in the following.

Theorem:4.10

For a mapping $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$:

- (i) f is $F\text{-sg-open}$ map.
- (ii) $f(F\text{-int}A) \subset F\text{-sgint}(f(A))$ for each subset A of Fine space X .
- (iii) For each $x \in X$ and for each open set U containing x , there is a $F\text{-sg-open}$ set V containing $f(x)$ such that $V \subset f(U)$.
- (iv) For each subset $B \subset Y$, $f^{-1}(F\text{-sgcl}(B)) \subset F\text{-cl}(f^{-1}(B))$.

Then $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4)$.

Proof is similar to Theorem-4.9 above. Next, we give the following.

Theorem:4.11

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is $F\text{-sg-open}$ map, $B \subset Y$ and G is Fine closed set containing $f^{-1}(B)$ then there is a $F\text{-sg-closed}$ set V such that $B \subset V$ and $f^{-1}(V) \subset G$.

Proof : Easy.

Recall the following.

Definition:4.12

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called a $F\text{-sg-continuous}$ if $f^{-1}(V)$ is $F\text{-sg-closed}$ set in Fine space X for every Fine closed set V of Y .

Equivalently, a map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is $F\text{-sg-continuous}$ if and only if the inverse image of each Fine open set is $F\text{-sg-open}$ set. Clearly, every $F\text{-sg-continuous}$ map is $F\text{-gs-continuous}$ and every Fine semi continuous map is $F\text{-sg-continuous}$ map.

We, characterize the $F\text{-sg-continuous}$ mappings in the following.

Theorem:4.13

Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a map.

- (i) f is $F\text{-sg-continuous}$.

- (ii) For each $x \in X$ and for each Fine open set V containing $f(x)$, there is a F-sg-open set U containing x such that $f(U) \subset V$.
- (iii) $f(\text{F-sgcl}(A)) \subset \text{F-cl}(f(A))$ for each subset A of Fine space X .
- (vi) $\text{F-sgcl}(f^{-1}(B)) \subset f^{-1}(\text{F-cl}(B))$ for each subset B of Fine space Y .

Then $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4)$.

Straightforward proof of the theorem is omitted.

Next, we recall the following.

Definition:4.14

A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called a F-sg- irresolute if $f^{-1}(A)$ is F-sg- closed for each F-sg closed set A of Fine space Y .

Clearly, the compositions of two F-sg-continuous functions need not be F-sg-continuous. But, we have the following decompositions of F-sg-continuous mappings.

Lemma:4.15

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-sg-irresolute and $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$ is F-sg-continuous, then $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$ is F-sg-continuous.

Theorem: 4.16

If $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is F-sg-continuous and $g : (Y, \sigma, \sigma_f) \rightarrow (Z, \rho, \rho_f)$ is continuous then $g \circ f : (X, \tau, \tau_f) \rightarrow (Z, \rho, \rho_f)$ is F-sg-continuous.

Proof:

Easy.

CONCLUSIONS

Many different forms of generalized closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we introduced in fine Fine gs closed sets and Fine sg closed sets in fine topological spaces and investigate some of the basic properties. This shall be extended in the future Research with some applications

ACKNOWLEDGMENT

I wish to acknowledge friends of our institution and others those who extended their help to make this paper as successful one. I acknowledge the Editor in chief and other friends of this publication for providing the timing help to publish this paper

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