Ion Acoustic Waves in a Unidirected Dusty Plasma

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Abstract: *In the new investigation of ion acoustic (IA) waves with mobile dust particles, usual ion and electron components, the Korteweg-de Vries (KdV) equation is derived to establish both compressive and rarefactive IA solitons.*

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1. Introduction

Investigation of nonlinear phenomena in various media is the thrust area of research in science and technology. Many non-linear fascinating structures like solitary waves, double layers etc. are abundant in the complex space laboratory. Ion acoustic waves (IAW) with two and multi-component plasmas are being studied continuously for the last four decades. The emergence of relativistic and quantum effects in plasmas opens a new dimension in the studies of solitary waves. Further the occurrence of charged dust particles in space plasmas has created immense interest in the minds of plasma workers. Besides, insertion of dust particles (charged) into the laboratory plasma is found by the experimentalists to drastically change the properties of IA waves. A lot of investigation has already been completed in the last few decades to the studies of various properties of IA waves in plasmas without dust particles.

Dust grains of micrometer and sub- micrometer size, because of field emission, plasma current are found to change the properties of plasma waves in space. In many astrophysical environments such as in inter stellar medium, in asteroid zones, in cometery tails, in planetary rings, in the earth's magnetosphere, in the neighbourhood of stars [1], [2] and in radio frequency discharges, dusty plasmas are abundantly found. It has been found that the presence of static charged dust grains in plasma can generate extremely low frequency dust acoustic wave [3] in absence of magnetic field. The fluctuations of dust charge are found to act as damping source to dust acoustic waves. Many results (with minor corrections) of negative ion plasmas can be adapted to dusty plasma for its low frequency behaviour when the wave length and the inter-particle distance are much larger than the grain size.

Rao et al. [4] have studied dust acoustic (DA) waves propagating linearly as a normal mode and non-linearly as supersonic solitons of either positive or negative electrostatic potentials in a dusty plasma with inertial charged dusts and Boltzmann distributed electrons and ions. In the study of non-linear dust acoustic waves [5] in unmagnetized dusty plasma with the effects of vortex-like and non thermal ion distributions, large amplitude rarefactive as well as compressive solitons are reported to exist. In a similar approach with sufficient non thermality in the ions following Cairn's distribution [6] , and sufficient negative charges on the dust along with electrons, Verheest and Pillay [7] have investigated the existence of both positive and negative solitary structures in some parameter space. Besides taking some non thermal parameter in the thermal distribution of ions as variable, DA solitary waves [8] are studied in dusty plasmas with the help of pseudo-potential method. The existence of rarefactive solitons in magnetized non thermal dusty electronegative plasma is also reported [9] in which width is shown to decrease at the increase of $\overline{B_0}$, whereas amplitude is shown to remain unaffected.

Ion acoustic waves in dusty plasma have also been observed experimentally. Phase velocity of the wave called dust ion acoustic wave (DIAW) is found to increase due to decrease in electron's density [3] and waves are shown at high frequency affected by heavy mass [3] of negatively charged immobile dust grains. The non-linear DIA waves leading to shock waves are also experimentally observed [10]. The non-linear DIA waves and dust ion-acoustic shock waves implicitly related to KdV-Burger equation due to non adiabatic charge variation of dust particles have also been studied [11].

DA and DIA solitons in space plasma with stationary or mobile dusts have been investigated to conclude solitons of positive or negative potentials on the basis of implicitly occurring dust charges Z_d in some forms. But relativistic effects to the small particles electrons and ions in the space regions, like Van Allen Radiation belt, laser plasma interaction, earth's magnetosphere, outer boundary of radiation belt etc are not considered when dust charges may be present. DIA solitons in an unmagnetized plasma with relativistic ions, Boltzmann electrons (where relativistic effects is not explicit) and stationary dusts are discussed only in [12], [13].

In this paper, we have considered the plasma model consisting of mobile dust particles, unidirected warm ions and electrons, to derive Korteweg de-Vries (KdV) equation for solitary wave structures.

2. Equations governing the dynamics of motion

In order to investigate the propagation of solitary waves in a homogeneous unmagnetized plasma, we start with a set of interpenetrating fluid characterized by the equations of continuity and motion of the negatively charged dust particles, ions and electrons with the Poisson's equation. The governing equations are as follows:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0 \quad (1)$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = Z_d \frac{\partial \varphi}{\partial x} \quad (2)$$

for dust,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \quad (3)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{1}{Q}\left(\frac{\partial \varphi}{\partial x} + \frac{\alpha}{n_i}\frac{\partial n_i}{\partial x}\right) \quad (4)$$

for ions,

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0 \quad (5)$$

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = \frac{1}{Q'}\left(\frac{\partial \varphi}{\partial x} - \frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \quad (6)$$

for electrons,

and

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e + Z_d n_d - n_i \quad (7)$$

where the suffixes d, i and e stand for dust particle, positive ion and electron respectively; $Q=m_i/m_d$, $Q'=m_e/m_d$ (electron to dust mass ratio) and $\alpha=T_i/T_e$ is the ratio of the ion temperature to electron temperature. The physical quantities appearing in these equations are normalised as follows- the densities n_d, n_i and n_e by unperturbed ion density n_{io}, time t by the inverse of the characteristic dust plasma frequency $\omega_{pd}^{-1} = (m_d/4\pi n_{io}e^2)^{1/2}$, distance x by the Debye length $\lambda_{De}=(k_bT_e/4\pi n_{io}e^2)^{1/2}$, velocities by the dust-acoustic speed $C_d=(k_bT_e/m_d)^{1/2}$ and electric potential φ by $(k_bT_e)/e$; k_b is the Boltzmann constant.

3. Derivation of Korteweg-de Vries equation and its solution

To derive the KdV equation from the basic equations (1)-(7) for the description of the propagation of dust ion acoustic waves, we expand the flow variables asymptotically about the equilibrium state in terms of the smallness parameter ε as follows:

$$n_d = n_{d0} + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \cdots$$

$$n_i = 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \cdots$$

$$n_e = n_{e0} + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \cdots$$

$$v_d = \varepsilon v_{d1} + \varepsilon^2 v_{d2} + \cdots \tag{8}$$

$$v_i = \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \cdots$$

$$v_e = \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \cdots$$

$$\varphi = \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \cdots$$

We use the stretched variables

$$\eta = \varepsilon^{\frac{1}{2}}(x - Vt)\,, \qquad \tau = \varepsilon^{3/2} Vt \tag{9}$$

for the derivation of the KdV equation. Using (8) and (9) in equations (1)-(7) and equating the coefficients of the lowest order perturbation in ε with the use of the boundary conditions $n_{d1} = n_{i1} = n_{e1} = 0$, $v_{d1} = v_{i1} = v_{e1} = 0$, $\varphi_1 = 0$ at $|\eta| \to \infty$, we get

$$n_{d1} = -\frac{Z_d\, n_{d0} \varphi_1}{n_{i0} V^2}$$

$$v_{d1} = -\frac{Z_d\, \varphi_1}{V}$$

$$n_{i1} = \frac{\varphi_1}{QV^2 - \alpha}$$

$$v_{i1} = \frac{V\varphi_1}{QV^2 - \alpha} \tag{10}$$

$$n_{e1} = -\frac{n_{e0}\varphi_1}{n_{i0}(Q'V^2 - 1)}$$

$$v_{e1} = -\frac{V\varphi_1}{Q'V^2 - 1}$$

$$n_{e0} + Z_d n_{d0} - 1 = 0$$

$$n_{e1} + Z_d n_{d1} - n_{i1} = 0$$

From the last equation of (10), the expression for dust ion acoustic speed V can be written as

$$\frac{r}{(Q'V^2 - 1)} + \frac{{Z_d}^2(1-r)}{V^2} + \frac{1}{QV^2 - \alpha} = 0 \tag{11}$$

Equation (11) is a quadratic equation in V^2, and consequently it represents two types of dust ion acoustic modes propagating with different phase velocities, namely

$${V_1}^2 = \frac{b + \sqrt{b^2 - 4ac}}{2a} \tag{12}$$

for the fast dust ion acoustic mode, and

$${V_2}^2 = \frac{b - \sqrt{b^2 - 4ac}}{2a} \tag{13}$$

for the slow dust ion acoustic mode, where,

$$a = Qr + fQQ' + Q'$$

$$b = \alpha r + \alpha f Q' + fQ + 1$$

$$c = f\alpha \tag{14}$$

$$r = \frac{n_{e0}}{n_{i0}}$$

and

$$f = (1-r)Z_d$$

Again, equating the coefficients of the next order perturbation, we can get the following equations:

$$\frac{\partial n_{d1}}{\partial \tau} - V\frac{\partial n_{d2}}{\partial \eta} + \frac{\partial}{\partial \eta}(n_{d0}v_{d2}) + \frac{\partial}{\partial \eta}(n_{d1}v_{d1}) = 0 \tag{15}$$

$$\frac{\partial v_{d1}}{\partial \tau} - V\frac{\partial v_{d2}}{\partial \eta} + v_{d1}\frac{\partial v_{d1}}{\partial \eta} - Z_d\frac{\partial \varphi_2}{\partial \eta} = 0 \tag{16}$$

$$\frac{\partial n_{i1}}{\partial \tau} - V\frac{\partial n_{i2}}{\partial \eta} + \frac{\partial}{\partial \eta}(v_{i2}) + \frac{\partial}{\partial \eta}(n_{i1}v_{i1}) = 0 \tag{17}$$

$$\frac{\partial v_{i1}}{\partial \tau} - Vn_{i1}\frac{\partial v_{i1}}{\partial \eta} - V\frac{\partial v_{i2}}{\partial \eta} + v_{i1}\frac{\partial v_{i1}}{\partial \eta} + \frac{1}{Q}\left(n_{i1}\frac{\partial \varphi_1}{\partial \eta} + \frac{\partial \varphi_2}{\partial \eta} + \alpha\frac{\partial n_{i2}}{\partial \eta}\right) = 0 \tag{18}$$

$$\frac{\partial n_{e1}}{\partial \tau} - V\frac{\partial n_{e2}}{\partial \eta} + \frac{\partial}{\partial \eta}(n_{e0}v_{e2}) + \frac{\partial}{\partial \eta}(n_{e1}v_{e1}) = 0 \tag{19}$$

$$n_{e0}\frac{\partial v_{e1}}{\partial \tau} - Vn_{e1}\frac{\partial v_{e1}}{\partial \eta} - Vn_{e0}\frac{\partial v_{e2}}{\partial \eta} + n_{e0}v_{e1}\frac{\partial v_{e1}}{\partial \eta} - \frac{1}{Q'}\left(n_{e1}\frac{\partial \varphi_1}{\partial \eta} + n_{e0}\frac{\partial \varphi_2}{\partial \eta} + \frac{\partial n_{e2}}{\partial \eta}\right) = 0 \tag{20}$$

$$n_{e2} + Z_d n_{d2} - n_{i2} - \frac{\partial^2 \varphi_2}{\partial \eta^2} = 0 \tag{21}$$

Eliminating $\frac{\partial v_{d2}}{\partial \eta}$ from the equations (15) and (16), $\frac{\partial v_{i2}}{\partial \eta}$ from the equations (17) and (18), $\frac{\partial v_{e2}}{\partial \eta}$ from the equations (19) and (20) we get respectively

$$\frac{\partial n_{d2}}{\partial \eta} + \frac{(1-r)Z_d}{V^2}\left(\frac{2}{V}\frac{\partial \varphi_1}{\partial \tau} + \frac{\partial \varphi_2}{\partial \eta} - \frac{3Z_d}{V^2}\varphi_1\frac{\partial \varphi_1}{\partial \eta}\right) = 0 \tag{22}$$

$$\frac{\partial n_{i2}}{\partial \eta} - \frac{1}{(QV^2-\alpha)}\left\{\frac{2QV}{(QV^2-\alpha)}\frac{\partial \varphi_1}{\partial \tau} + \frac{\partial \varphi_2}{\partial \eta} + \frac{(3QV^2-\alpha)}{(QV^2-\alpha)^2}\varphi_1\frac{\partial \varphi_1}{\partial \eta}\right\} = 0 \tag{23}$$

$$\frac{\partial n_{e2}}{\partial \eta} + \frac{r}{(Q'V^2-1)}\left\{\frac{2Q'V}{(Q'V^2-1)}\frac{\partial \varphi_1}{\partial \tau} + \frac{\partial \varphi_2}{\partial \eta} - \frac{(3Q'V^2-1)}{(Q'V^2-1)^2}\varphi_1\frac{\partial \varphi_1}{\partial \eta}\right\} = 0 \tag{24}$$

The values of n_{d1}, v_{d1}, n_{i1}, v_{i1}, n_{e1}, v_{e1} are substituted from (10) in deducing these equations.

Adding (22), (23), (24) and making use of the relations (21) and (11), the KdV equation can be obtained as

$$\frac{\partial \varphi_1}{\partial \tau} + p\varphi_1\frac{\partial \varphi_1}{\partial \eta} + q\frac{\partial^3 \varphi_1}{\partial \eta^3} = 0 \tag{25}$$

where $p=AB$, $q=A/2$

$$A = \frac{V^3(QV^2-\alpha)^2(Q'V^2-1)^2}{QV^4(Q'V^2-1)^2 + Q'V^4(QV^2-\alpha)^2 r + (QV^2-\alpha)^2(Q'V^2-1)^2 {Z_d}^2(1-r)} \tag{26}$$

$$B = \frac{1}{2}\left\{\frac{(3QV^2-\alpha)}{(QV^2-\alpha)^3} - \frac{3{Z_d}^3(1-r)}{V^4} - \frac{(3Q'V^2-1)r}{(Q'V^2-1)^3}\right\} \tag{27}$$

Introducing the variable $\xi = \eta - c\tau$ and using the boundary conditions

$$\varphi_1(\xi) = \frac{d\varphi_1}{d\xi} = \frac{d^2\varphi_1}{d\xi^2} = 0$$

as $|\xi| \to \infty$, we get the solution of the KdV equation (25) as

$$\varphi_1 = \varphi_0 \operatorname{sech}^2\left(\frac{\xi}{\Delta}\right) \tag{28}$$

where $\varphi_0 = 3c/p$ is the amplitude of the dust ion acoustic soliton, $\Delta = \left(\frac{4q}{c}\right)^{1/2}$ is the width of the dust ion acoustic soliton.

4. Discussion

In this model of dusty plasma, compressive as well as rarefactive DIA solitons are seen to exist based primarily on dust charges Z_d and ion to dust mass ratio Q.

The amplitude of compressive DIA solitons starting with sharp fall initially continuously decreases with the dust charge Z_d [Fig-1] corresponding to the faster mode-1 for fixed values of Q (= $\frac{m_i}{m_d}$)=0.1, □ =0.1 and r=0.05. The smaller amount of negative dust charge Z_d in the plasma causes to diminish the amplitudes of the compressive DIA solitons rapidly for massive density concentration of ions in r (=0.05). But as the amount of dust charge Z_d increases, the soliton's amplitudes steadily decreases with Z_d for the same set of parameters. To the contrary, the growth of DIA rarefactive solitons of very small amplitude appears to remain constant (dotted line) throughout the range of Z_d corresponding to the slower mode-2 [Fig-1].

The significantly small amplitude rarefactive DIA solitons increases with temperature □ (= $\frac{T_i}{T_e}$) for all the values of the mass ratio Q= 0.2, 0.15, 0.1, 0.05 corresponding to the slower mode-2 [Fig-2]. But the amplitudes of the rarefactive DIA solitons perceptibly decrease as Q increases. Otherwise, increase of ion mass (or decrease of dust atomic mass) forces to diminish the amplitudes of the rarefactive DIA solitons.

The existence of solitary wave profiles is nicely depicted in Fig-3 for three values of mass ratio Q (= $\frac{m_i}{m_d}$) = 0.5, 0.3 and 0.1 The small amplitudes of compressive solitary waves are found to be smaller and smaller corresponding to smaller values of the mass ratio Q. For small concentration of ions corresponding to fixed mass ratio of the dust in Q are seen to generate very small amplitude compressive solitons for □ =0.1, r=0.05 and Z_d=100. Otherwise, higher concentration of ions in this dusty plasma is observed to yield relatively higher amplitude compressive solitons.

The amplitudes of the compressive (rarefactive) DIA solitons steadily increase (remain constant) corresponding to mode-1 (mode-2) with mass ratio Q=$\frac{m_i}{m_d}$ for both the values of Z_d=75, 100 at □ =0.2, r=0.05 [Fig-4]. The small amplitudes of compressive solitons are relatively much higher than the corresponding counter-parts of rarefactive solitons which remain constant. Small Z_d is found to produce higher amplitude compressive solitons [Fig-4, mode-1] in this dusty plasma.

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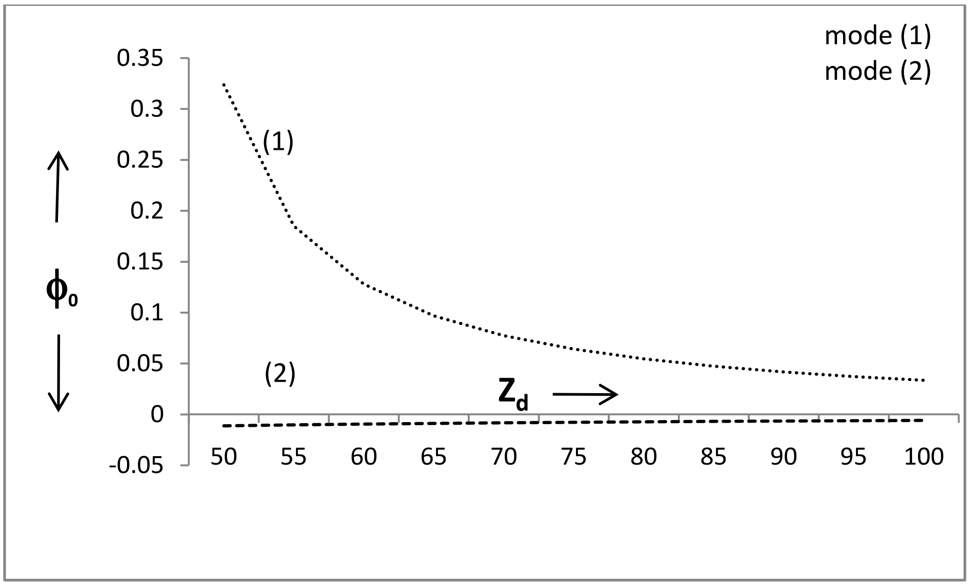

Fig 1: Amplitudes of Dust Ion Acoustic compressive solitons versus dust charge Z_d corresponding to faster mode (1) but rarefactive solitons for slower mode (2) for fixed values of Q=0.1, □=0.1, r=0.05

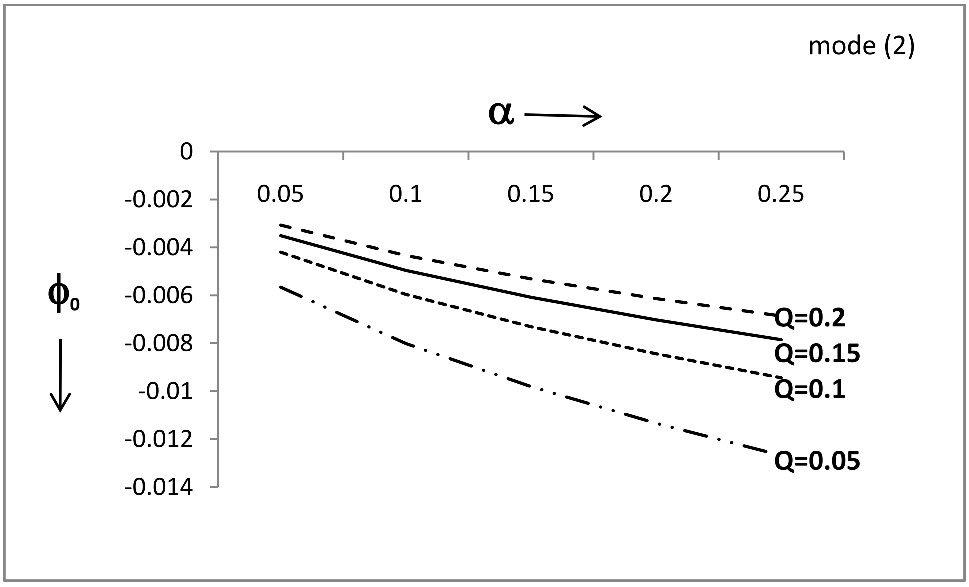

Fig 2: Amplitudes of Dust Ion Acoustic rarefactive solitons versus temperature ratio □ corresponding to Q=0.05, 0.1, 0.15, 0.2 for slower mode (2) for fixed values of r=0.05, Z_d=100 as shown in the figure.

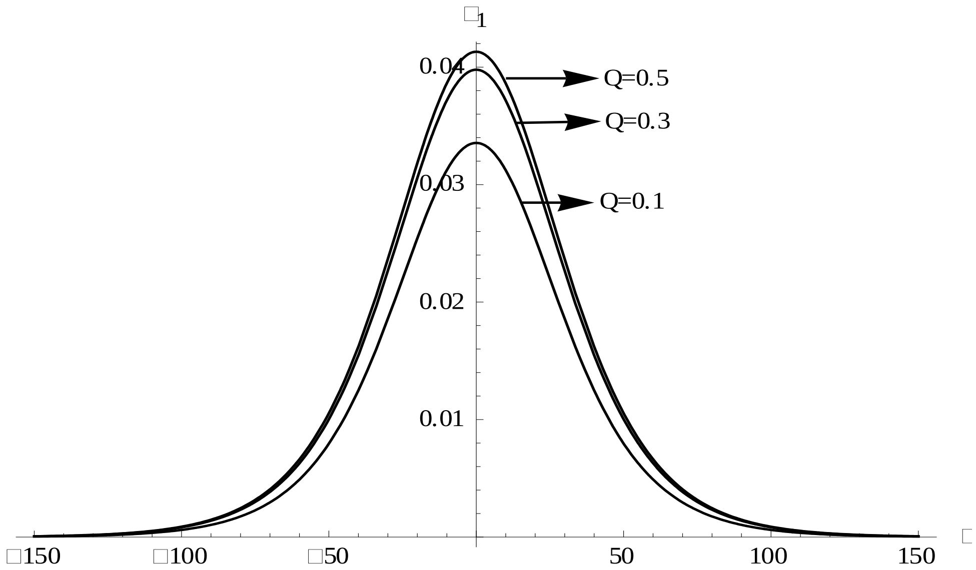

Fig 3: Compressive soliton profiles with respect to space□ □ □ corresponding to Q= $\frac{m_i}{m_d}$)=0.5, 0.3, 0.1 for fixed values of □ =0.1, r=0.05 and Z_d=100.

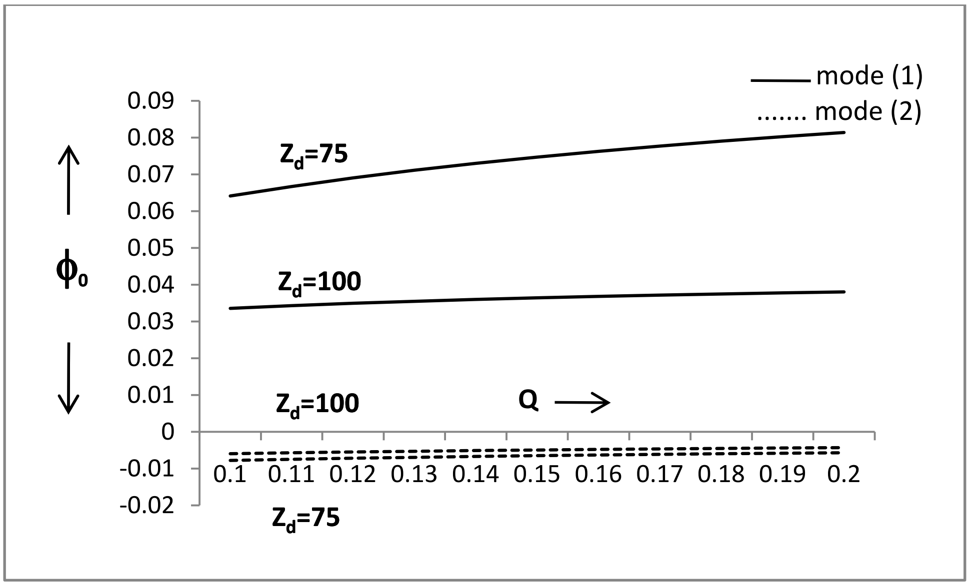

Fig 4: Amplitudes of Dust Ion Acoustic compressive solitons versus ion-dust mass ratio Q= $\frac{m_i}{m_d}$ for Z_d=75, 100 corresponding to faster mode (1) but rarefactive solitons for slower mode (2) for fixed values of □ =0.1, r=0.05