Support Independence in Graphs

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Abstract

In any graph G, the support of a vertex is defined as the sum of degrees of its neighbours. A graph G is said to be balanced, if every vertex of G has same support. G is called highly unbalanced when no two vertices of G have same support. In this paper, we introduce the concept of support independence in graphs. A subset S of a vertex set is said to be support independent, if no two vertices in S are having same support. The support independence number of G is the cardinality of maximum support independent set in G. We obtain the support independence number of some standard graphs and derived graphs.

Keywords: *Splitting graphs, cosplitting graphs, Support of a vertex, Support independence.*

AMS Subject Classification Code (2010): 05C (Primary)

1 Introduction

Only finite, simple, undirected graphs are considered in this paper. We refer [7] for further notations and terminology.The degree of a vertex v is denoted by $d(v)$. A *full vertex* of G is a vertex which is adjacent to every other vertices of G.A graph G is said to be r – *regular*, if every vertex of G has degree r. $D(G)$denote the set of degrees of all vertices in G.

In a graph $G(V, E)$, for any vertex $v \in V$, the *open neighbourhood* of v is the set of all vertices adjacent to v. That is, $N(v) = \{u \in V(G) / uv \in E(G)\}$. The *closed neighbourhood* of v is defined by $N[v] = N(v) \cup \{v\}$. Clearly, if $N[u] = N[v]$, then u and v are adjacent and $d(u) = d(v)$.

The concept of support of a vertex has been introduced and studied by Selvam Avadayappan and G. Mahadevan [6]. The *supports(v)* of a vertex v is the sum of degrees of its neighbours. That is, $s(v) = \sum_{u \in N(v)} d(u)$. Note that the support of any vertex in an r – *regular* graph is r^2.

A graph G is said to be a *balanced graph*, if any two vertices in G have the same support. It is easy to observe that the complete bipartite graphs $K_{m,n}$ and any regular graphs are balanced graphs. A graph G is said to be *highly unbalanced*, if distinct vertices of G have distinct supports. For example, a highly unbalanced graph is shown in Figure 1.

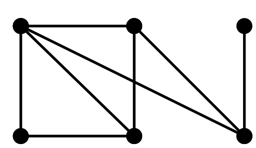


Figure 1

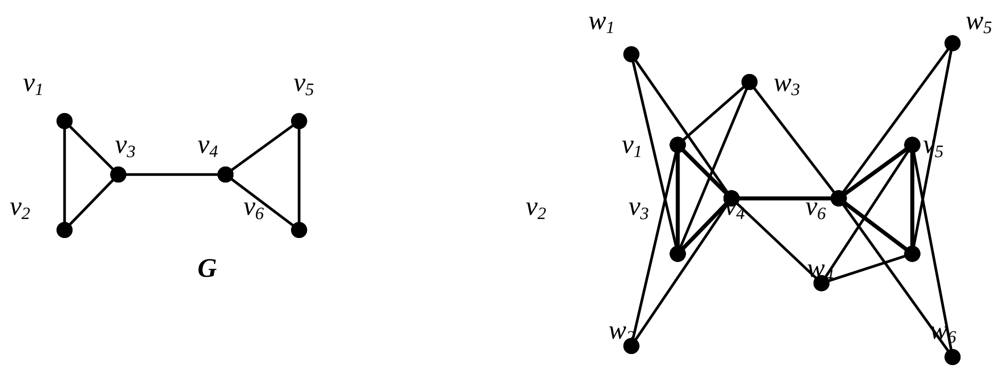
The following results have been proved in [6]:

Result A $\sum_{v\in V} s(v) = \sum_{v\in V} d(v)^2$.

Result B For any balanced graph G, $\delta(G) = 1$ if and only if $G \cong K_{1,n}$, $n \geq 1$.

The study on this parameter has its own importance as any two vertices of same degree need not be of same importance in any graph unless they are isomorphic images of each other. The degrees of its neighbours contribute much in determining the weightage of a vertex in a graph. Hence it becomes essential to study about the support of the vertices also.

The concept of splitting graph of a graph was introduced by Sampath Kumar and Walikar [8]. For a graph G, the graph $S(G)$, obtained from G, by adding a new vertex w for every vertex $v \in V$ and joining w to all vertices of G adjacent to v, is called the *splitting graph* of G. For example, a graph G and its splitting graph $S(G)$ are shown in Figure 2.



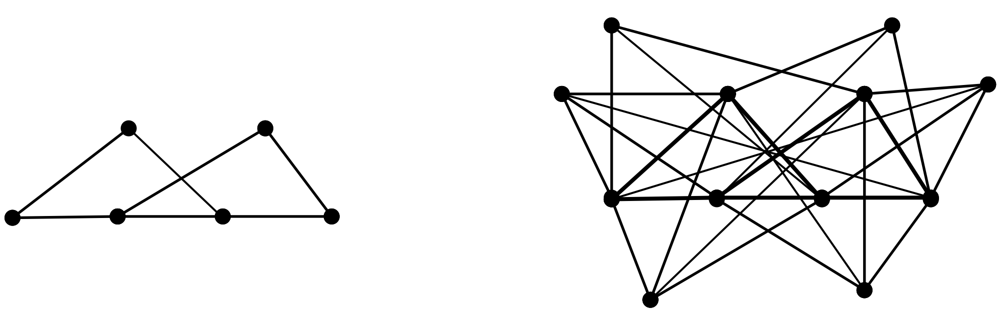
G **S(G)**

Figure 2

A necessary and sufficient condition for a graph to be a splitting graph is given in [8].

Result D[8] A graph G is a splitting graph if and only if $V(G)$ can be partitioned into two sets $V_1$and V_2 such that there exists a bijective mapping f from V_1 to V_2 and $N(f(v)) = N(v) \cap V_1$, for any $v \in V_1$.

Let G be a graph with vertex set $\{v_1, v_2, \ldots, v_n\}$. The *cosplitting graphCS(G)* is the graph obtained from G, by adding a new vertex $w_i$for each vertex v_i and joining w_i to all vertices which are not adjacent to v_i in G. For example, a graph G and its cosplitting graph $CS(G)$ are shown in Figure 3.



G **CS(G)**

Figure 3

For further details on various other types of splitting graphs, one can refer [1], [2], [3] and [4].

In a graph G, deleting an edge uv and introducing a new vertex w and the new edges uw and vw is called the *subdivision of the edge uv.* The *subdivision graph*, denoted by $S_1(G)$, is obtained from the graph G by subdividing every edge of Gonce. For example, $S_1(K_{1,5})$ is shown in Figure 4.

Figure 4

In literature, a lot of work has been done based on support of vertices. One such is a kind of classification of irregular graphs, namely support neighbourly irregular graphs which has been introduced in [5]. A connected graph is said to be *support neighbourly irregular* (or simply *SNI*), if no two vertices having same support are adjacent.

A graph proving the existence of SNI graphs is shown in Figure 5. Of course, all highly unbalanced graphs are SNI.

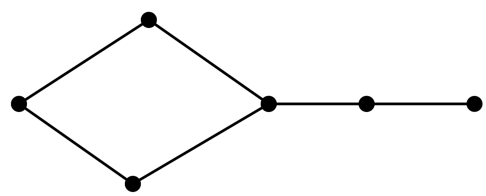


Figure 5

In [5], the following result has been proved which is needed for further discussion of our paper.

Result D A graph G is balanced if and only if G is a regular graph or a biregular bipartite graph with each partition having vertices of same degree.

In this paper, we introduce the concept of support independence in graphs and obtain some interesting results.

2 Support independence in graphs

In this section, we first introduce the concept of support independence number of a graph and compute the same for some standard graphs and the complement of a graph.

Recall that a subset S of V is said to be independent if no two vertices in S are adjacent. Independence number β of a graph G is the cardinality of a maximum independent set in G. Here we define a similar type of independence depending on the support of vertices.

A subset S of V is said to be *support independent* if no two vertices in Shave same support. A support independent set is said to be *maximal* if there exists no support independent set S' such that $S \subset S'$.

In particular, S is said to be *maximum* if there exists no such S' with$|S| < |S'|$. The *Support independence number*β_S is the cardinality of a maximum support independent set. Any support independent set with $|\beta_S|$ vertices is simply called β_S – set.

For example, $\beta_S = 3$ for the graph G shown in Figure 4 with a β_S – set$S = \{u,v,w\}$.

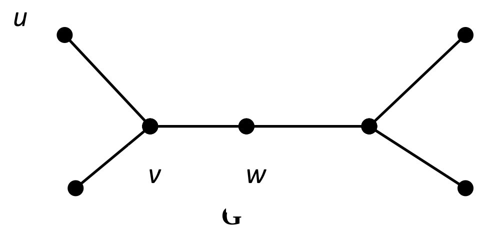


Figure 4

It is clear that ifS(G) denotes the support set of a graph G, that is, a set containing supports of all vertices in G, then $\beta_S$is nothing but | S(G) |.

The following facts can be easily verified for support independent sets:

Fact 2.1 $\beta_S(K_n) = 1$*, for* $n \geq 1$.

Fact 2.2 $\beta_S(K_{m,n}) = 1$ *for* $m, n \geq 1$.

Fact 2.3$\beta_S(C_n) = 1$*, for* $n \geq 3$.

Fact 2.4 $\beta_S(W_n) = 2$*, for* $n \geq 4$*,* where W_n is a wheel graph which is nothing but $C_1 \vee K_n$.

Fact 2.5 $\beta_S(K_n^c) = 1$*, for* $n \geq 1$.

Fact 2.6$\beta_S(P_n) = 1, n \leq 3$; $\beta_S(P_4) = 2$; $\beta_S(P_n) = 3, n \geq 5$.

Fact 2.7 For any balanced graph G, $\beta_S(G) = 1$.

Proposition 2.8 $\beta_S(G) = 1$ if and only if G is a regular graph or a biregular bipartite graph with each partition having vertices of same degree.

Proof For any balanced graph G, $\beta_S(G) = 1$.The result follows from Result D. ■

Fact2.9 For ahighly unbalanced graph G,$\beta_S(G) = n$.

Proposition 2.10 For any graph G of order n, $1 \leq \beta_S(G) \leq n$.

From Fact 2.7 and Fact 2.9, we note that the above inequality is sharp.When we compare the parameters β and β_S of a graph G, we find that they are independent of each other. K_n proves the existence of graphs with $\beta = \beta_S$ whereas, $P_n, n \geq 5$ stands as an example for graphs with $\beta > \beta_S$. Finally highly unbalanced graphs are graphs with $\beta < \beta_S$. It is interesting to note that in K_n, $\beta + \beta_S = 2$ and in the graph $K_n \times K_2$, $\beta + \beta_S = 3$.

Theorem 2.11 For any two given integers k and n such that $2 \leq k \leq n$, there exists a graph of order n with$\beta + \beta_S = k$.

ProofWhen $k = 2$, K_n is the required graph of order n.And for $k = 3$, $K_{n-2} \times K_2$serves as an example for the family of a graph of order n and $\beta + \beta_S = 3$. Assume that $4 \leq k \leq n$.Construct a graph G with the vertex set $V(G) = \{u_1, u_2, \ldots, u_{k-3}, v_1, v_2, \ldots, v_{n-k+3}\}$ and the edge set $E(G) = \{u_j v_1, v_s v_r \mid 1 \leq j \leq k-3; 1 \leq r,s \leq n-k+3; r \neq s\}$. Clearly G is of order n. Also we note that $\{v_2, u_j \mid 1 \leq j \leq k-3\}$ is a maximum β - set of G. Hence $\beta(G) = k-2$. In addition, $\{u_1, v_2\}$ is a maximum β_S – set in Gfixing $\beta_S(G) = 2$. All these facts prove the existence of a graph with the required properties. ■

For $k = 5$ and $n = 7$, the constructed graph with $\beta + \beta_S = 5$ is shown in Figure 5. Here $\{u,v,w\}$ is a maximum independent set whereas $\{u,w\}$ is a maximum support independent set.

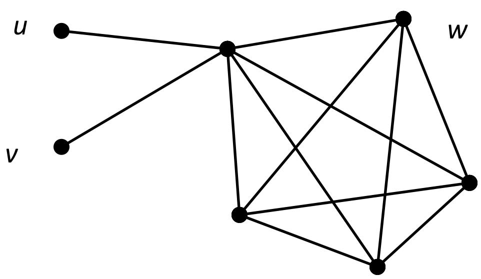


Figure 5

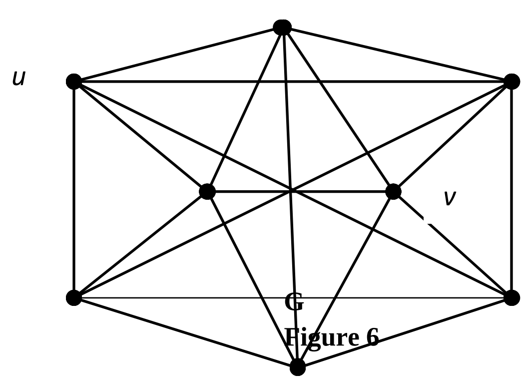
It is trivial that $\beta + \beta_S = 2$ if and only if $G \cong K_n$. In the next theorem, we characterise the graphs for which $\beta + \beta_S = 3$.

Theorem 2.12 Let G be any connected graph. Then$\beta + \beta_S = 3$ if and only if G is balanced of independence number two.

Proof It is obvious that both $\beta$and $\beta_S$are positive integers. For both parameters having a total of three, there are only two possibilities $\beta = 1$ and $\beta_S = 2$ or$\beta = 2$ and $\beta_S = 1$. Discussing the first case, $\beta = 1$ is possible only when G is complete, for which $\beta_S = 1$, leading to a contradiction. Now considering the second case, $\beta = 2$ and$\beta_S = 1$ weget G to bebalanced with independence number two.

■

For example, a balanced graph with independence number two for which $\beta + \beta_S = 3$ isshown in Figure 6. Suppose $\{u,v\}$ is a β- set of G. Then every other vertex in $V(G) \setminus \{u,v\}$ must be adjacent either to u or to v. Therefore $N[u] \cup N[v] = V(G)$ for two non-adjacent vertices u and v in G.



G

Figure 6

Note that if $\beta(G) = 1$, then G is complete for which $\beta_S(G) = 1$. But the converse need not be true.

Theorem 2.13 For any given integer $k \geq 2$, there exists a graph G with $\beta_S(G) = 1$ and $\beta(G) = k$.

Proof Consider $K_{2,k}$. Since it is a balanced graph, $\beta_S(G) = 1$ and also $\beta(G) = k$. ■

Theorem 2.14 There exists a graph G in which $\beta_S(G) = \beta(G) = \chi(G) = k$ for any $k \geq 2$.

Proof Construct the complete *k – partite* graph $G = K_{1,2,\dots,k}$ with vertex set $V(G) = \{v_{ij} / 1 \leq i \leq k;\ 1 \leq j \leq i\}$ and the edge set $E(G) = \{v_{ij}v_{rs} / 1 \leq i, r \leq k;\ 1 \leq j \leq i; 1 \leq r \leq s$ and $i \neq r\}$. Here we note that $\{v_{i1} / 1 \leq i \leq k\}$ is a β_S - *set* of G whereas $\{v_{kj} / 1 \leq j \leq k\}$ is a β - *set* of G. Therefore $\beta_S(G) = \beta(G) = k$. Also since G is *k – partite*, we have $\chi(G) = k$. Hence the result. ■

As an illustration, for $k = 4$, the constructed graph with $\beta_S(G) = \beta(G) = \chi(G) = 4$ is shown in Figure 7.

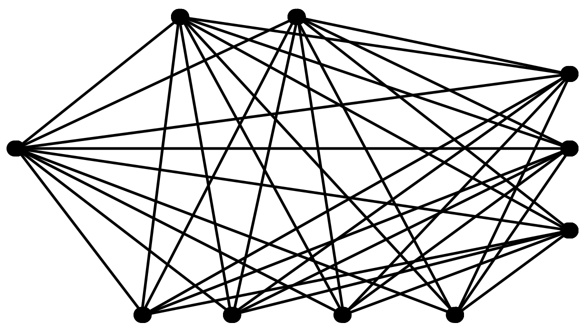


Figure 7

Next we concentrate on the support of a vertex in the complement of a graph.

Lemma 2.15 Support of any vertex v in $\bar{G}$, $s_{\bar{G}}(v) = (n-1)^2 - 2\mathrm{m} + \mathrm{s_G}(v) - d_G(v)(n-2)$, where m is the size of the graph G.

Proof Let us calculate support of a vertex in $\bar{G}$. The support of any vertex v in $\bar{G}$, $s_{\bar{G}}(v) = \sum_{u \in N_{\bar{G}}(v)} d_{\bar{G}}(u) = \sum_{u \in N_{\bar{G}}(v)} (n-1-d_G(u)) = \sum_{u \in N_G^c[v]} (n-1-d_G(u))$. Also we know that $|N_G^c[v]| = n - 1 - \mathrm{d_G}(v)$. Therefore $\sum_{u \in N_G^c[v]} (n-1-d_G(u)) = \big(n-1-\mathrm{d_G}(\mathrm{v})\big)(\mathrm{n}-1) - \sum_{u \in N_G^c[v]} d_G(u) = \big(n-1-\mathrm{d_G}(\mathrm{v})\big)(\mathrm{n}-1) - 2\mathrm{m} + \mathrm{s_G}(v) + d_G(v)$, since we know that $\sum_{u \in N_G^c[v]} d_G(u) + \sum_{u \in N_G(v)} d_G(u) + \mathrm{d_G}(\mathrm{v}) = 2\mathrm{m}$. On simplification, we get the lemma. ■

Proposition 2.16 The complement of a balanced graph is balanced if and only if it is regular.

Proof Let G be a balanced graph. Then by Proposition 2.8, we have, G is a regular graph or a biregular bipartite graph with each partition having vertices of same degree. The complement of a regular graph is also regular and hence balanced. Suppose G is (r,k) - biregular bipartite with vertices of same degree in same partition. Then $\bar{G}$ is a $(n - 1 - r,\ n - 1 - k)$ – biregular graph in which vertices of degree $n - 1 - k$ are having support $(n - 1 - k)^2$ and the remaining vertices are of degree $(n - 1 - r)^2$. Since $r \neq k$, $\bar{G}$ cannot be balanced.

■

Theorem 2.17 For any highly unbalanced graph G, $\bar{G}$ is also highly unbalanced if there do not exist vertices u and v in G, with $d_G(u) \neq d_G(v)$ and $s_G(u) \neq s_G(v)$ such that $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$.

Proof Let G be a highly unbalanced graph and $\beta_S(G)$ be its support independence number. Then $\beta_S(G) = |V(G)|$. By previous lemma, we have $s_{\bar{G}}(v) = \big(n-1-\mathrm{d_G}(\mathrm{v})\big)(\mathrm{n}-1) - 2\mathrm{m} + \mathrm{s_G}(v) + d_G(v)$. Suppose in $\bar{G}$, there exist two vertices u and v of same support. That is, $s_{\bar{G}}(u) = s_{\bar{G}}(v)$.

Then either the vertices *u* and *v* have the same degree and the same support in *G* or $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$. The first case is impossible, since *G* is highly unbalanced. Hence the theorem.

■

Now we introduce a relation ρ on the vertex set of *G* such that for any two vertices *u* and *v* in *G*, $u \rho v$ if and only if $d(u) = d(v)$ and $s(u) = s(v)$. One can easily verify that ρ is an equivalence relation defined on *V(G)*. Therefore *V(G)* can be partitioned into equivalence classes induced by ρ.

Theorem 2.18 For any graph *G* of order n, $\beta_S(\bar{G}) \le \eta$ (and also $\beta_S(G) \le \eta$) where η is the number of equivalence classes induced by ρ defined as above in *V(G)*. And the equality holds if and only if *G* has no two vertices *u* and *v* such that $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$.

Proof Let *G* be any graph of order *n*. It is obvious that $\beta_S(G) \le \eta$. We know that the vertices of same degree and same support in *G* will be of same degree and same support in $\bar{G}$ too. Hence each equivalence class induced by ρ contributes atmost one to any β_S – *set* of $\bar{G}$. That is, $\beta_S(\bar{G}) \le \eta$.

Also as in the discussion of Theorem 2.17, two vertices *u* and *v* in $\bar{G}$ have same support, if and only if they belong to same equivalence class induced by ρ in *V(G)* or $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$. If no two such vertices exist in *G*, then β_S – *set* of $\bar{G}$ contains exactly one vertex from each equivalence class induced. Therefore $\beta_S(\bar{G}) = \eta$.

■

For example, a graph *G* with its complement $\bar{G}$, for which $\beta_S(\bar{G}) < \eta$ is shown in Figure 8. Here in *G*, no two vertices have the same degree and the same support and hence $\eta = 7$. But the pendant vertex *u* and the vertex *v* of degree 2 satisfies $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$. They have same support in $\bar{G}$. We note that $\beta_S(\bar{G}) = 6$.

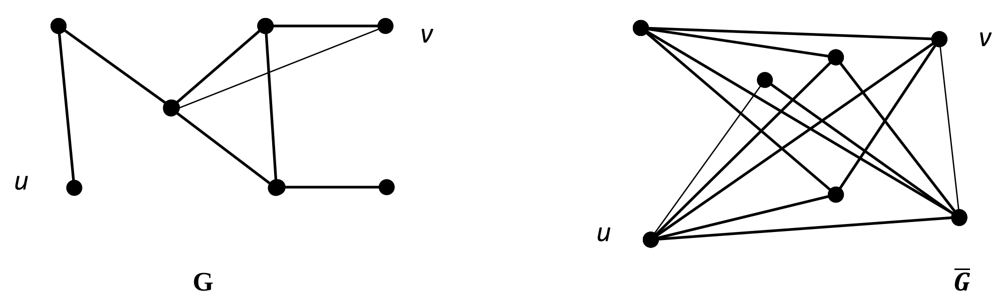


Figure 8

Theorem 2.18 can be restated as follows:

Theorem 2.19 For any graph *G*, $\beta_S(G) = \beta_S(\bar{G}) = \eta$ if and only if *G* has no two vertices *u* and *v* of different degrees with same support or such that $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$. ■

As an illustration, one can verify the following graph *G* for which $\eta = 3$ is given by Figure 9. It contains no two vertices *u* and *v* such that $d_G(u) - d_G(v) = \frac{s_G(u) - s_G(v)}{n-2}$. The graph $\bar{G}$ with $\beta_S(\bar{G}) = \eta = 3$ is also given.

Figure 9

Now we examine the relation between the support independence number of a graph and that of its splitting graph.

Theorem 2.20 For any graph G, $\beta_S(S(G)) = 2\beta_S(G)$, where $S(G)$ is its splitting graph.

Proof Let G be a graph and $S(G)$ be its splitting graph with the vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and $V(S(G)) = V(G) \cup \{w_1, w_2, ..., w_n\}$ such that $w_i$is the corresponding newly added vertex with respect to v_i. It is easy to note that $d_{S(G)}(v_i) = 2d_G(v_i)$ and $d_{S(G)}(w_i) = d_G(v_i)$

Now let us first calculate the support of any vertex v_i in $S(G)$. $s_{S(G)}(v_i) = \sum_{u \in N_{S(G)}(v_i)} d_{S(G)}(u) = \sum_{v_j \in N_G(v_i)} d_{S(G)}(v_j) + \sum_{v_j \in N_G(v_i)} d_G(w_j) = 2s_G(v_i) + s_G(v_i)$, since $d_{S(G)}(v_i) = 2d_G(v_i)$ and $d_{S(G)}(w_i) = d_G(v_i)$. Hence we conclude that $s_{S(G)}(v_i) = 3s_G(v_i)$.

For any newly added vertex w_i, $s_{S(G)}(w_i) = \sum_{u \in N_G(v_i)} d_{S(G)}(u) = 2\sum_{u \in N_G(v_i)} d_G(u) = 2s_G(v_i)$. Therefore every support independent set in G is a support independent set in $S(G)$ also. In addition, any set of newly added vertices corresponding to vertices in a support independent set is also a support independent set in $S(G)$.

Of them one can easily note that the set of vertices in a maximum support independent set of G along with their corresponding newly added vertices is the maximum support independent set in $S(G)$. In other words, $\beta_S(S(G)) = 2\beta_S(G)$. ■

For example, a graph G with $\beta_S(G) = 3$ and its splitting graph with$\beta_S(S(G)) = 6$ are shown in Figure 10.

Figure 10

Lemma 2.21 Let G be a graph of order n and size m. If v is a vertex in G, then the support of v in a cosplitting graph $CS(G)$ of$G$ is $nd(v)$. Also if w is the newly added vertex in $CS(G)$ corresponding to a vertex v in G, then the support of w in $CS(G)$ is$n^2 + s(v) - 2m$.

Proof Let G be a graph and $CS(G)$ be its cosplitting graph with the vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and $V(CS(G)) = V(G) \cup \{u_1, u_2, ..., u_n\}$ such that u_i is the corresponding newly added vertex with respect to v_i. Now support of the vertex u_i, for any i, $1 \leq i \leq n$ is $nd(u_i)$ and for every vertex v_i, $s(v_i) = \sum_{u \in N_{CS(G)}(v_i)} d_{CS(G)}(u) = \sum_{v_j \in N_G(v_i)} d_{CS(G)}(v_j) + \sum_{v_j \in N_G^c(v_i)} d(u_j)$.

The degree of v_i in $CS(G)$ is n for any i and the degree of w_j is $n - d(v_j)$ for any j. Hence, $s_{CS(G)}(v_i) = nd_G(v_i) + \sum_{v_j \in N_G^c(v_i)} (n - d(v_j)) = nd_G(v_i) + (n - d_G(v_i))n - \sum_{v_j \in V(G)} d_G(v_j) + s_G(v_i)$. That is, we get $s_{CS(G)}(v_i) = n^2 + s(v) - 2m$.

■

Theorem 2.22 For any graph G, $\beta_S(CS(G)) \leq \beta_S(G) + |D(G)|$, where $CS(G)$ is the cosplitting graph of G.

Proof Let G be a graph and $CS(G)$ be its cosplitting graph. By above lemma, it is quite obvious that any two vertices which are of same support in G will be of same support in $CS(G)$. Hence a maximum support independent set of G is a support independent set of $CS(G)$ also.

In addition, it follows from above lemma that the newly added vertices which correspond to the vertices of same degree in G have same support in $CS(G)$. Every set consisting of newly added vertices that correspond to vertices of distinct degrees in G forms a support independent set in $CS(G)$. If $CS(G)$ contains no two vertices v_i and w_j with same support, it becomes obvious that $\beta_S(CS(G)) = \beta_S(G) + |D(G)|$. Otherwise $\beta_S(CS(G)) < \beta_S(G) + |D(G)|$. ■

Note that for the graph G shown in Figure 11, $\beta_S(CS(G)) < \beta_S(G) + |D(G)|$. Here we note that $\beta_S(G) = 5$ and $|D(G)| = 4$. But $\beta_S(CS(G)) = 8$.

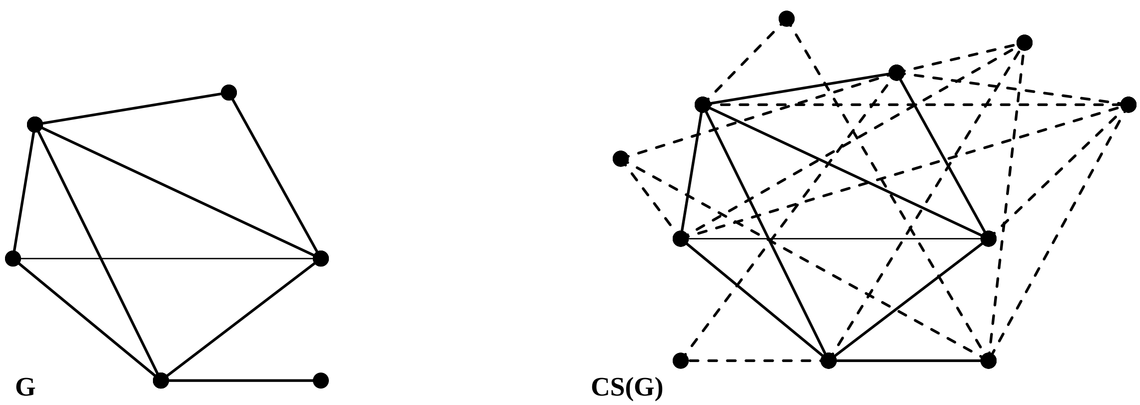


Figure 11

Also a graph G and its cosplitting graph $CS(G)$ with $\beta_S(CS(G)) = \beta_S(G) + |D(G)|$ are shown in Figure 12.

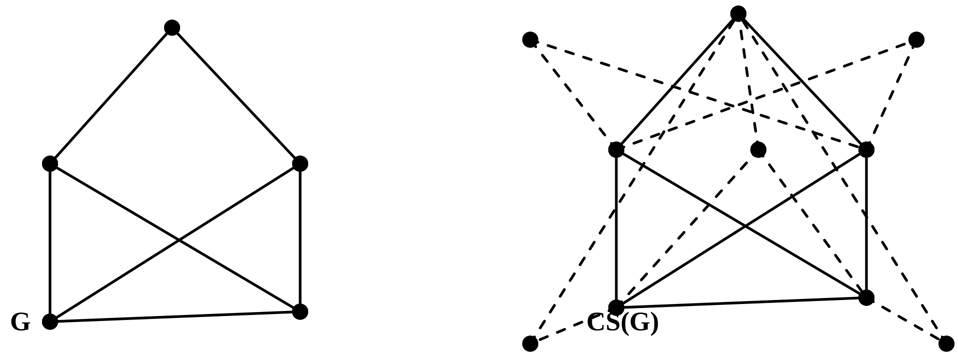


Figure 12

Let G be any graph and $S_1(G)$ be its subdivision graph. Now we compute the support of any vertex v in $S_1(G)$. If v is already a vertex of$G$, then it is obvious that $s(v) = 2d(v)$. On the other hand, if v is the newly added vertex in $S_1(G)$ when subdividing the edge uw of G, then $s(v) = d(u) + d(w)$.

Proposition 2.23 The support independence number of a subdivision graph $\beta_S(S_1(G)) \geq |D(G)|$.

Proof Let G be a graph whose subdivision graph is $S_1(G)$. From the above discussion, we have thevertices of distinct degrees in G have distinct supports in $S_1(G)$. Hence any set of vertices of distinct degrees in$G$ forms a support independent set of $S_1(G)$. Therefore $\beta_S(S_1(G)) \geq |D(G)|$. ■

Theorem 2.24 For any graph of order n and size m, if G contains no two vertices u and v such that $s_G(v) - s_G(u) = p(d_G(u) - d_G(v))$, then$\beta_S(G \vee K_p^c) = n$or $n + 1$, for any $p \geq 1$.

Proof Let us first find the support of a vertex v in the graph $H = G \vee K_p^c$. Now the degree of every vertex v of G gets increased by p in H. In addition, each vertex v in G has p new neighbours of degree n in H. Hence $s_H(v) = s_G(v) + pd_G(v) + np$. Suppose $w_1, w_2, \ldots, w_p$ are newly added vertices in H, then $s_H(w_i) = 2m + np$for any i, $1 \leq i \leq p$.We can note that for any two vertices u and $v$in G,$s_H(v) = s_H(u)$ only when$s_G(v) - s_G(u) = p(d_G(u) - d_G(v))$ which is impossible.In addition,if G contains a vertex v such that $s_G(v) = 2m - pd_G(v)$, then $s_H(w_i) = s_H(v)$ for any i, $1 \leq i \leq p$ and in this case $\beta_S(G \vee K_p^c) = n$.Otherwise,$\beta_S(G \vee K_p^c) = n+1$. ■

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