Analysis of Fuzzy Non Preemptive Priority Queue using Non-Linear Programming Approach

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Abstract:

The paper gives the membership function of the system characteristics in non preemptive priority queuing model with priority subscribers where the arrival rate and service rate are fuzzy. Here we transform a fuzzy priority queue into crisp queues by applying a $\alpha -$cut approach. A pair of non -linear programming programs is formulated. The membership function is derived from distinct values of α.To demonstrate the validity of the method, numerical example is given.

Key words: *Priority queues, Non preemptive priority, Parametric non –linear programming, Membership function.*

INTRODUCTION

The study of waiting lines is important aspect in queueing theory. We use different models and methods to analyse waiting time. Mathematical analysis of queues gives way to decrease the waiting time and waiting line. Queueing models have wide applications in several fields such as transportation engineering, service industry, production, communication systems, health care and information processing systems. The waiting discipline where customers are served with respect to their order of arrival is frequently found in queuing models, but then again many real queueing systems follow priority discipline model . Priority mechanisms are very useful scheduling method that allow messages of different classes to receive different quality of service (QOS). The priority queue has received great interest in the literature for this purpose. A priority mechanism is a useful method that allows different customer types of customers to receive different performance level. Priority queueing have many applications such as communication network, call centres and hospitals etc. Priority schemes are also known for their ease of implementation. Many queuing works is devoted to analyse priority queues. Here the queuing model considered is one where few customers are given a priority service over routine.

Two recognized priority disciplines are the preemptive and the non-preemptive disciplines. Under the first rule, a customer with high priority is permitted to go into service immediately even lower priority is already in service. The service of low customer is interrupted when higher class customer arrive, and will be restarted from the point of pause, when all the queues of higher priority have been emptied**.** In this situation the lower priority class customers are completely unseen and do not affect in any way the queues of the higher classes. In non-preemptive case there is no interruption. The non-preemptive priority queues are useful for performance evaluation of production, manufacturing, inventory controls and computer systems. Here the queuing model considered is one where customers are given a non–preemptive priority service over routine. Many authors have studied priority queues. An overview of basic priority queues in crisp environment have been published in [5],[6],[8],[11] and [14]. Given a model of this kind we require to find the performance measures.

The parameters in the non-preemptive priority may be fuzzy. The fuzzy queuing models are more realistic and practical than classical ones**.** Queuing models along with fuzzy increases their application. Dissimilar to the classic model that considers arrivals to follow a Poisson process and exponentially distributed service times, the arrival rate in numerous real conditions is more possibility than probabilistic and are not denoted by exact terms. The researchers like Li and Lee [12], Buckley [2], Negi and Lee [13], Kaufmann [10],Kao et al [9], Chen [3,4] has examined fuzzy queues by Zadeh's extension principle. Parametric linear programming approach to derive the membership functions of the system in fuzzy queues has been derived by Kao et al [9].Recent developments on fuzzy numbers by random variables can be used to analyse the queueing system. For example, Zadeh, L.A [15] has presented the idea of fuzzy probabilities. Here we study a fuzzy non-preemptive priority queue by non-linear programming approach.

MODEL DESCRIPTION

Fuzzy non-preemptive priority queue:

Suppose that customers of the k^{th} priority arrive according to poisson process and service distribution for the k^{th} priority be exponentially distributed with mean $\frac{1}{\mu_k}$ unit that begins service and completes its service before another item is admitted, regardless of priority.

We begin with $\rho_k = \frac{\lambda_k}{\mu_k}$ $(1 \le k \le r), \sigma_k = \sum_{i=1}^{i=k} \rho_i (\sigma_0 = 0, \sigma_r = \rho)$. The system is stationary for $\rho < 1$. Let $\eta_{\tilde{\lambda}}(x)$ and $\eta_{\tilde{\mu}}(y)$ denote the membership function of arrival rate and service rate. Without the loss of generality let us assume the performance measures for 3-priority queues. From the traditional queuing theory

$$W_{q,i} = \frac{\sum_{k=1}^{k=r} \frac{\rho_k}{\mu_k}}{(1-\sigma_{i-1})(1-\sigma_i)}$$

$$L_q = \sum_{i=1}^{r} L_q^{(i)} = \sum_{i=1}^{r} \lambda_i (W_{q,i})$$

Using the notion of $\alpha -$ cut the FM/FM/1 queue with 2-priority is reduced to M/M/1 queue with 2-priority customers with equal service rates. Hence $\mu_1 = \mu_2 = \mu$, $\rho_1 = \frac{\lambda_1}{\mu_1}$, $\rho_2 = \frac{\lambda_2}{\mu_2}, \rho_3 = \frac{\lambda_3}{\mu_3}$.

Since $\rho = \rho_1 + \rho_2, \rho = \frac{\lambda}{\mu}, \lambda = \lambda_1 + \lambda_2, \sigma_k = \sum_{i=1}^{i=k} \rho_i, \sigma_0 = 0$

$$W_q^{(1)} = \frac{\lambda}{\mu(\mu - \lambda_1)}$$

$$W_q^{(2)} = \frac{\lambda}{(\mu - \lambda_1)(\mu - \lambda)}$$

$$L_q^{(1)} = \frac{\lambda\lambda_1}{\mu(\mu - \lambda_1)}$$

$$L_q^{(2)} = \frac{\lambda\lambda_2}{(\mu - \lambda_1)(\mu - \lambda)}$$

Nonpreemptive priority queue with two priority queues:

To broaden the applicability of the standard queueing model with priority customer, we allow specification of the system parameters. Let the arrival rate of first priority customers , the arrival rate of second priority customers , service rate are approximately known and can be represented by the fuzzy numbers, $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}$ respectively. Let $\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}}(y)$ denote the membership function of $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}$ respectively. We then have the following fuzzy sets,

$$\tilde{\lambda}_1 = \left\{\left(x_1, \eta_{\tilde{\lambda}_1}(x_1)\right) | x_1 \in X_1\right\},$$

$$\tilde{\lambda}_2 = \left\{\left(x_2, \eta_{\tilde{\lambda}_2}(x_2)\right) | x_2 \in X_2\right\}$$

$$\tilde{\mu} = \left\{\left(y, \eta_{\tilde{\mu}}(y)\right) | y \in Y\right\}$$

Where X_1, X_2, Y are the crisp universal sets of the arrival rate of first priority, second priority, service rate respectively.

Let $f(x_1, x_2, y)$ denote the system characteristic of interest. Since $\tilde{\lambda}_1, \tilde{\lambda}_2, \mu$ are fuzzy numbers, $f(x_1, x_2, y)$ is also a fuzzy number. By Zadeh's extension principle the membership function of the system characteristic $f(x_1, x_2, y)$ is defined as

$$\eta_{f(x_1,x_2,y)}(z) = Sup_{\Omega}\ min\left\{\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}}(y) | z = f(x_1, x_2, y)\right\},$$

where the supremum is taken over the set

$$\Omega = \left\{x_1 \in X_1, x_2 \in X_2, y \in Y |\ 0 < \frac{x_1 + x_2}{y} < 1\right\}$$

Let the system characteristic of interest is the average waiting time of customers in queue. It follows that the average waiting time of customers in first priority queue is

$$f(x_1, x_2, y) = \frac{x_1+x_2}{y(y-x_1)}$$

The membership function for the average waiting of customers in first priority is

$$\eta_{\tilde{E}(W_q{}^1)}(z) = Sup_{\Omega}\ min\left\{\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}}(y) | z = \frac{x_1+x_2}{y(y-x_1)}\right\}$$

The average waiting time of customers in second priority is $f(x_1, x_2, y) = \frac{x_1+x_2}{(y-x_1)(y-(x_1+x_2))}$

The membership function for the average waiting of customers in second priority is

$$\eta_{E(W_q{}^1)}(z) = Sup_{\Omega}\ min\left\{\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}}(y) | z = \frac{x_1 + x_2}{(y - x_1)(y - (x_1 + x_2))}\right\}$$

These membership functions are not expressed in the usual forms, making it very difficult to imagine their shapes. Here we consider the representation problem using a mathematical programming technique. Parametric non linear programming is developed to find the α −cuts of (x_1, x_2, y) based on extension principle.

THE SOLUTION PROCEDURE

By constructing the membership function $\eta_{\tilde{E}(W_q{}^1)}$ of $\tilde{E}(W_q{}^1)$ is on the basis of deriving the α −cuts of $\tilde{E}(W_q{}^1)$. Denote the α −cuts of $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}$ as crisp intervals as follows.

$$\lambda_1(\alpha) = [x_{1\alpha}{}^L, x_{1\alpha}{}^U] = \left[\min_{x_1 \in X_1}\left\{x_1 | \eta_{\tilde{\lambda}_1}(x_1) \geq \alpha\right\}, \max_{x_1 \in X_1}\left\{x_1 | \eta_{\tilde{\lambda}_1}(x_1) \geq \alpha\right\}\right]$$

$$\lambda_2(\alpha) = [x_{2\alpha}{}^L, x_{2\alpha}{}^U] = \left[\min_{x_2 \in X_2}\left\{x_2 | \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha\right\}, \max_{x_2 \in X_2}\left\{x_1 | \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha\right\}\right]$$

$$\mu(\alpha) = [y_\alpha{}^L, y_\alpha{}^U] = \left[\min_{y \in Y}\left\{y | \eta_{\tilde{\mu}}(y) \geq \alpha\right\}, \max_{y \in Y}\left\{y | \eta_{\tilde{\mu}}(y) \geq \alpha\right\}\right]$$

The constant arrival rates and service rates are given as intervals when the membership functions are not less than a given possibility level for α. Hence the bounds of these intervals can be described as functions of α and can be obtained as $x_{1\alpha}{}^L = \min \eta_{\tilde{\lambda}_1}{}^{-1}(\alpha)$,$x_{1\alpha}{}^U = \max \eta_{\tilde{\lambda}_1}{}^{-1}(\alpha)$, $x_{2\alpha}{}^L = \min \eta_{\tilde{\lambda}_2}{}^{-1}(\alpha)$,

$$x_{2\alpha}{}^U = \max \eta_{\tilde{\lambda}_2}{}^{-1}(\alpha), y_\alpha{}^L = \min \eta_{\tilde{\mu}}{}^{-1}(\alpha) y_\alpha{}^L = \max \eta_{\tilde{\mu}}{}^{-1}(\alpha)$$

Hence we can use the α −cuts of $\tilde{E}(W_q{}^1)$ and $\tilde{E}(W_q{}^2)$ to construct its membership function .

Using Zadeh's extension principle,to derive the membership function $\eta_{\tilde{E}(W_q{}^1)}$ we need at least one of the following cases to hold such that $z = \frac{x_1+x_2}{y(y-x_1)}$ satisfies $\eta_{\tilde{E}(W_q{}^1)} = \alpha$

Case (i): $\left(\eta_{\tilde{\lambda}_1}(x_1) = \alpha, \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha, \eta_{\tilde{\mu}}(y) \geq \alpha\right)$

Case (ii): $\left(\eta_{\tilde{\lambda}_1}(x_1) \geq \alpha, \eta_{\tilde{\lambda}_2}(x_2) = \alpha, \eta_{\tilde{\mu}}(y) \geq \alpha\right)$

Case (iii): $\left(\eta_{\tilde{\lambda}_1}(x_1) \geq \alpha, \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha, \eta_{\tilde{\mu}}(y) = \alpha\right)$

This can be accomplished using parametric NLP techniques. The NLP to find the lower and upper bounds of the α −cuts of $\eta_{\tilde{E}(W_q{}^1)}$ for the Case (i) are

$$E\left(W_q{}^1\right)_\alpha^{L_1} = \min_{\Omega}\left(\frac{x_1 + x_2}{y(y - x_1)}\right)$$

$$E\left(W_q{}^1\right)_\alpha^{U_1} = \max_{\Omega}\left(\frac{x_1 + x_2}{y(y - x_1)}\right)$$

for the Case (ii) are $E\left(W_q{}^1\right)_\alpha^{L_2} = \min_{\Omega}\left(\frac{x_1+x_2}{y(y-x_1)}\right)$

$$E\left(W_q{}^1\right)_\alpha^{U_2} = \max_{\Omega}\left(\frac{x_1 + x_2}{y(y - x_1)}\right)$$

for the Case (ii) are $E\left(W_q{}^1\right)_\alpha^{L_3} = \min_{\Omega}\left(\frac{x_1+x_2}{y(y-x_1)}\right)$

$$E\left(W_q{}^1\right)_\alpha^{U_3} = \max_{\Omega}\left(\frac{x_1 + x_2}{y(y - x_1)}\right)$$

From the definitions $x_1 \in \lambda_1(\alpha), x_2 \in \lambda_2(\alpha), y \in \mu(\alpha)$ can be replaced by

$x_1 \in [x_{1\alpha}{}^L, x_{1\alpha}{}^U], x_2 \in [x_{2\alpha}{}^L, x_{2\alpha}{}^U], y \in [y_\alpha{}^L, y_\alpha{}^U]$ respectively.

Given $0 < \alpha_2 < \alpha_1 \leq 1$ we have $\left[x_{1\alpha_1}{}^L, x_{1\alpha_1}{}^U\right] \subseteq \left[x_{1\alpha_2}{}^L, x_{1\alpha_2}{}^U\right]$,

$\left[x_{2\alpha_1}{}^L, x_{2\alpha_1}{}^U\right] \subseteq \left[x_{2\alpha_2}{}^L, x_{2\alpha_2}{}^U\right], \left[y_{\alpha_1}{}^L, y_{\alpha_1}{}^U\right] \subseteq \left[y_{\alpha_2}{}^L, y_{\alpha_2}{}^U\right]$

Hence to find the membership function $\eta_{\tilde{E}(W_q{}^1)}$ it is sufficient to find the left and right shape functions of $\eta_{\tilde{E}(W_q{}^1)}$ which is equivalent in finding the lower bound

$$E\left(W_q{}^1\right)_\alpha^{L} = \min_{\Omega}\left(\frac{x_1 + x_2}{y(y - x_1)}\right)$$

such that $x_{1\alpha}{}^L \leq x_1 \leq x_{1\alpha}{}^U, x_{2\alpha}{}^L \leq x_2 \leq x_{2\alpha}{}^U, y_\alpha{}^L \leq y \leq y_\alpha{}^U$

$$E\left(W_q{}^1\right)_\alpha^{U} = \max_{\Omega}\left(\frac{x_1 + x_2}{y(y - x_1)}\right)$$

such that $x_{1\alpha}{}^L \leq x_1 \leq x_{1\alpha}{}^U, x_{2\alpha}{}^L \leq x_2 \leq x_{2\alpha}{}^U, y_\alpha{}^L \leq y \leq y_\alpha{}^U$

The crisp interval obtained above represents the α −cuts of $E\left(W_q{}^1\right)$. Again applying the results of Zimmermann and Kaufmann and convexity properties to $E\left(W_q{}^1\right)$, we have

$E\left(W_q{}^1\right)_{\alpha_1}^L \geq E\left(W_q{}^1\right)_{\alpha_1}^U, E\left(W_q{}^1\right)_{\alpha_1}^L \geq E\left(W_q{}^1\right)_{\alpha_1}^U$ where $0 < \alpha_2 < \alpha_1 \leq 1$

If both $E\left(W_q{}^1\right)_\alpha^L$ and $E\left(W_q{}^1\right)_\alpha^U$ are invertible with respect to α then a left shape function $L(z)=\left[E\left(W_q{}^1\right)_\alpha^L\right]^{-1}$ and a right shape function $R(z)=\left[E\left(W_q{}^1\right)_\alpha^U\right]^{-1}$ can be derived from which the membership function is constructed as

$$\eta_{\tilde{E}(W_q{}^1)} = \begin{cases} L(z), E\left(W_q{}^1\right)_{\alpha=0}^L \leq z \leq E\left(W_q{}^1\right)_{\alpha=1}^L \\ R(z), E\left(W_q{}^1\right)_{\alpha=1}^U \leq z \leq E\left(W_q{}^1\right)_{\alpha=0}^U \end{cases}$$

The membership functions of the average waiting time, expected number of customers in different priority queues can be derived in a similar manner.

NUMERICAL EXAMPLES

The Quality of Service (QoS) play a essential role in the analysis of a network traffic priority queueing systems with random switchover times is suggested . Here we consider a priority class queueing systems concerning switching to describe, model and analyse. The performance characteristics of priority queueing systems are used for estimating a Quality of Service . There will be no immediate interruptions of requests services under this discipline. Hence, on completion of service of each request , the device is ready to move to the non-empty queue with the highest priority level requests, if any are presented in the system and are waiting to be served. We consider a model of this type.

The fuzzy waiting time of first priority in the queue

Suppose the arrival, service rates are triangular fuzzy numbers given by $\tilde{\lambda}_1 = [5,6,7], \tilde{\lambda}_2 = [3,4,5], \tilde{\mu} = [14,15,16]$ per minute respectively.

It is easy to find that $[{x_{1\alpha}}^L, {x_{1\alpha}}^U] = [5+\alpha, 7-\alpha]$,

$[{x_{2\alpha}}^L, {x_{2\alpha}}^U] = [3+\alpha, 5-\alpha], [{y_\alpha}^L, {y_\alpha}^U] = [14+\alpha, 16-\alpha]$

The $\alpha-$ cuts of $({W_q}^1)$

$$({W_q}^1)_\alpha^L = \frac{8+2\alpha}{176-43\alpha+2\alpha^2}$$

$$({W_q}^1)_\alpha^U = \frac{12-2\alpha}{2\alpha^2+35\alpha+98}$$

With the help of MATLAB 7.0, the inverse functions of $({W_q}^1)_\alpha^L$ and $({W_q}^1)_\alpha^L$ exist, yield the membership function $\eta_{\tilde{E}({W_q}^1)}(z) = \begin{cases} L(z)\,; \frac{1}{22} \le z \le \frac{2}{27} \\ R(z); \frac{2}{27} \le z \le \frac{6}{49} \end{cases}$

$$where\ L(z) = \frac{(43x+2) \pm (441x^2+236x+4)^{\frac{1}{2}}}{4x}$$

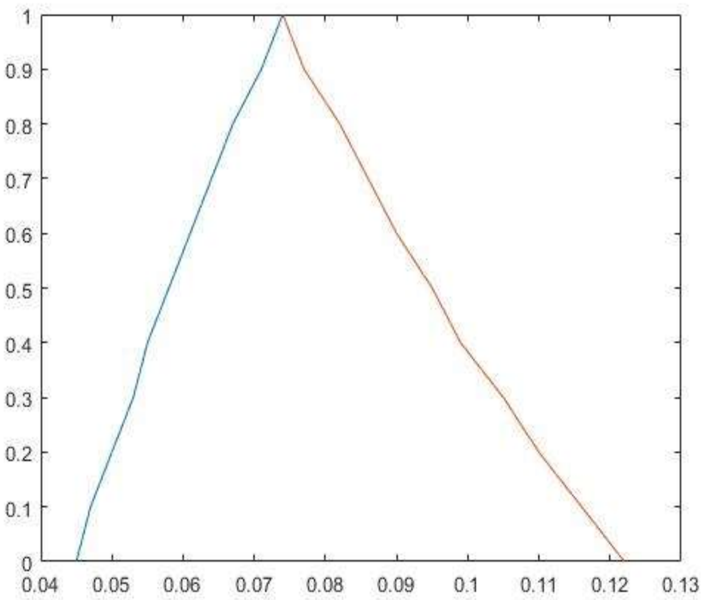
$$R(z) = \frac{-(35x+2) \pm (441x^2+236x+4)^{\frac{1}{2}}}{4x}$$

The $\alpha-$ cuts of arrival and service rates and fuzzy queue length of first priority

α	${x_{1\alpha}}^L$	${x_{1\alpha}}^U$	${x_{2\alpha}}^L$	${x_{2\alpha}}^U$	${y_\alpha}^L$	${y_\alpha}^U$	$E({W_q}^1)_\alpha^L$	$E({W_q}^1)_\alpha^U$
0.0	5	7	3	5	14	16	0.045	0.122
0.1	5.1	6.9	3.1	4.9	14.1	15.9	0.047	0.116
0.2	5.2	6.8	3.2	4.8	14.2	15.8	0.050	0.110
0.3	5.3	6.7	3.3	4.7	14.3	15.7	0.053	0.105
0.4	5.4	6.6	3.4	4.6	14.4	15.6	0.055	0.099
0.5	5.5	6.5	3.5	4.5	14.5	15.5	0.058	0.095
0.6	5.6	6.4	3.6	4.4	14.6	15.4	0.061	0.090
0.7	5.7	6.3	3.7	4.3	14.7	15.3	0.064	0.086
0.8	5.8	6.2	3.8	4.2	14.8	15.2	0.067	0.082
0.9	5.9	6.1	3.9	4.1	14.9	15.1	0.071	0.077
1.0	6.0	6.0	4.0	4.0	15.0	15.0	0.074	0.074

From the table we find that fuzzy waiting time of first priority in the queue

$E(W_q{}^1)$ at $\alpha = 1$ is 0.074, indicating that the waiting time of first priority in the queue is 0.074. Moreover the range of $E(W_q{}^1)$ never exceed 0.122 or fall below 0.045.



The membership function for waiting time of first priority in the queue

The fuzzy waiting time of second priority in the queue

Similarly the $\alpha -$ cuts of $(W_q{}^2)$ are

$$(W_q{}^2)_\alpha^L = \frac{8 + 2\alpha}{88 - 49\alpha + 6\alpha^2}$$

$$(W_q{}^2)_\alpha^U = \frac{12 - 2\alpha}{6\alpha^2 + 25\alpha + 14}$$

$$\eta_{\tilde{E}(W_q{}^1)}(z) = \begin{cases} L(z)\,;\ \frac{1}{11} \leq z \leq \frac{2}{9} \\ R(z);\ \frac{2}{9} \leq z \leq \frac{6}{7} \end{cases}$$

$$where\ L(z) = \frac{(49x + 2) \pm (289x^2 + 388x + 4)^{\frac{1}{2}}}{12x}$$

$$R(z) = \frac{-(25x + 12) \pm (289x^2 + 388x + 4)^{\frac{1}{2}}}{12x}$$

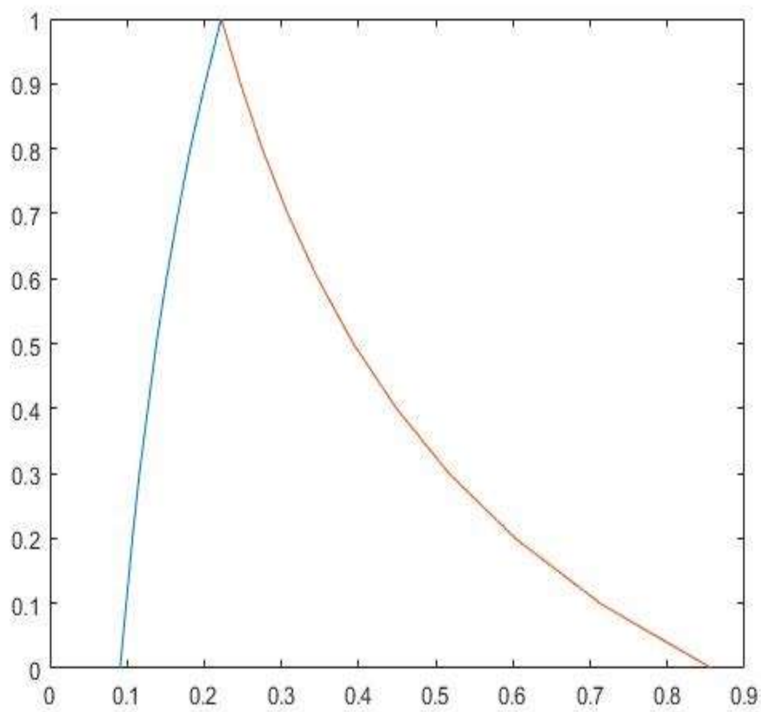
The $\alpha -$ cuts of arrival and service rates and fuzzy queue length of second priority

α	$x_{1\alpha}{}^L$	$x_{1\alpha}{}^U$	$x_{2\alpha}{}^L$	$x_{2\alpha}{}^U$	$y_\alpha{}^L$	$y_\alpha{}^U$	$E(W_q{}^2)_\alpha^L$	$E(W_q{}^2)_\alpha^U$
0.0	5	7	3	5	14	16	0.091	0.857

0.1	5.1	6.9	3.1	4.9	14.1	15.9	0.099	0.713
0.2	5.2	6.8	3.2	4.8	14.2	15.8	0.107	0.603
0.3	5.3	6.7	3.3	4.7	14.3	15.7	0.116	0.517
0.4	5.4	6.6	3.4	4.6	14.4	15.6	0.127	0.449
0.5	5.5	6.5	3.5	4.5	14.5	15.5	0.138	0.393
0.6	5.6	6.4	3.6	4.4	14.6	15.4	0.151	0.347
0.7	5.7	6.3	3.7	4.3	14.7	15.3	0.166	0.308
0.8	5.8	6.2	3.8	4.2	14.8	15.2	0.182	0.275
0.9	5.9	6.1	3.9	4.1	14.9	15.1	0.201	0.247
1.0	6.0	6.0	4.0	4.0	15.0	15.0	0.222	0.222

From the table we find that fuzzy waiting time of second priority in the queue

$E(W_q{}^2)$ at $\alpha = 1$ is 0.222, indicating that the waiting time of second priority in the queue is 0.222. Moreover the range of $E(W_q{}^2)$ never exceed 0.857or fall below 0.091.



The membership function for average waiting time of second priority in the queue

The fuzzy queue length of first priority

The $\alpha -$ cuts of $(L_q{}^1)$ are

$$(L_q{}^1)_\alpha^L = \frac{40 + 18\alpha + 2\alpha^2}{176 - 43\alpha + 2\alpha^2}$$

$$(L_q{}^1)_\alpha^U = \frac{84 - 26\alpha + 2\alpha^2}{98 + 35\alpha + 2\alpha^2}$$

$$\eta_{\tilde{E}(L_q{}^1)}(z) = \begin{cases} L(z)\,; \frac{5}{22} \le z \le \frac{4}{9} \\ R(z);\, \frac{4}{9} \le z \le \frac{6}{7} \end{cases}$$

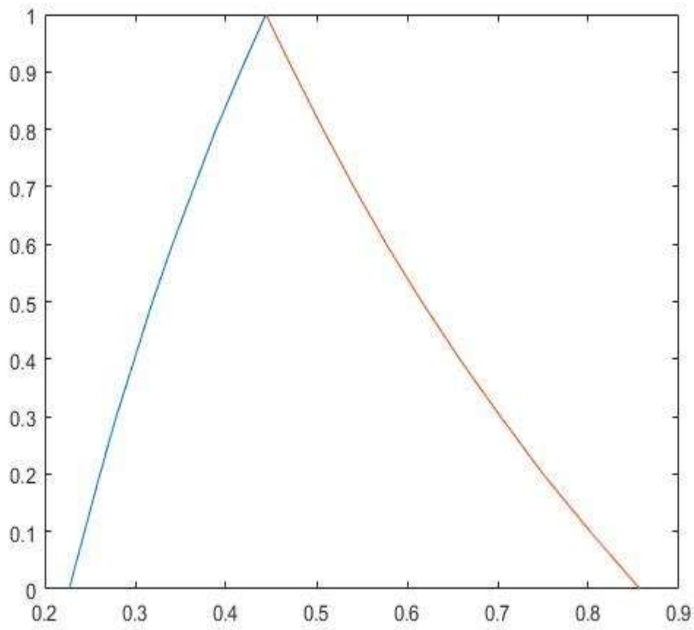
$$where\ L(z) = \frac{-(43x-18) \pm (441x^2+3276x+4)^{\frac{1}{2}}}{4x-4}$$

$$R(z) = \frac{-(35x+26) \pm (441x^2+3276x+4)^{\frac{1}{2}}}{4x-4}$$

α	${x_{1\alpha}}^L$	${x_{1\alpha}}^U$	${x_{2\alpha}}^L$	${x_{2\alpha}}^U$	${y_\alpha}^L$	${y_\alpha}^U$	$E\left({L_q}^1\right)_\alpha^L$	$E\left({L_q}^1\right)_\alpha^U$
0.0	5	7	3	5	14	16	0.227	0.857
0.1	5.1	6.9	3.1	4.9	14.1	15.9	0.244	0.802
0.2	5.2	6.8	3.2	4.8	14.2	15.8	0.261	0.750
0.3	5.3	6.7	3.3	4.7	14.3	15.7	0.279	0.703
0.4	5.4	6.6	3.4	4.6	14.4	15.6	0.299	0.658
0.5	5.5	6.5	3.5	4.5	14.5	15.5	0.319	0.616
0.6	5.6	6.4	3.6	4.4	14.6	15.4	0.341	0.577
0.7	5.7	6.3	3.7	4.3	14.7	15.3	0.365	0.541
0.8	5.8	6.2	3.8	4.2	14.8	15.2	0.389	0.507
0.9	5.9	6.1	3.9	4.1	14.9	15.1	0.416	0.475
1.0	6.0	6.0	4.0	4.0	15.0	15.0	0.444	0.444

From the table we find that fuzzy queue length of first priority in the queue

$E\left({L_q}^1\right)$ at $\alpha = 1$ is 0.444, indicating that the queue length of first priority in the queue is 0.444. Moreover the range of $E\left({L_q}^1\right)$ never exceed 0.857 or fall below 0.227.



The membership function for average queue length of first priority

The fuzzy queue length of second priority

The $\alpha -$ cuts of $\left(L_q{}^2\right)$ are

$$\left(L_q{}^2\right)_\alpha^L = \frac{24 + 14\alpha + 2\alpha^2}{88 - 49\alpha + 6\alpha^2}$$

$$\left(L_q{}^2\right)_\alpha^L = \frac{60 - 22\alpha + 2\alpha^2}{14 + 25\alpha + 6\alpha^2}$$

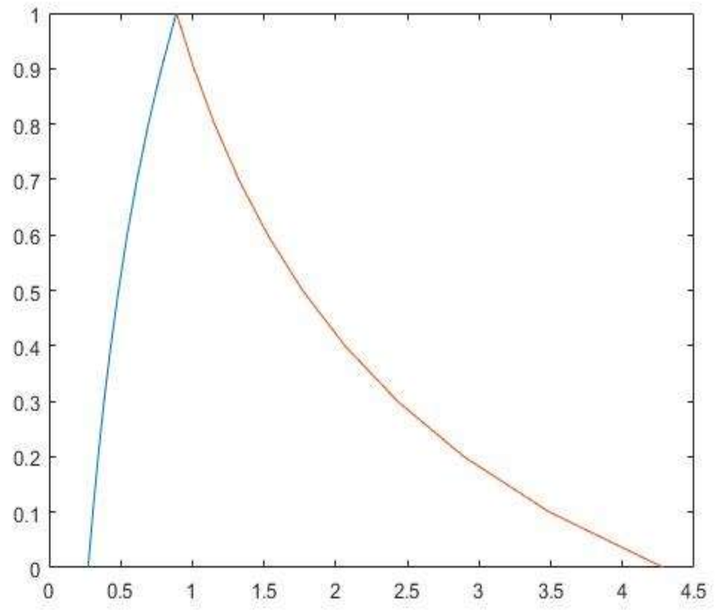
$$\eta_{\tilde{E}\left(L_q{}^2\right)}(z) = \begin{cases} L(z)\,; \ \frac{3}{11} \le z \le \frac{8}{9} \\ R(z); \ \frac{8}{9} \le z \le \frac{30}{7} \end{cases}$$

$$where\ L(z) = \frac{(49x + 14) \pm (289x^2 + 2652x + 4)^{\frac{1}{2}}}{12x - 4}$$

$$R(z) = \frac{-(25x + 22) \pm (289x^2 + 2652x + 4)^{\frac{1}{2}}}{12x - 4}$$

α	$x_{1\alpha}{}^L$	$x_{1\alpha}{}^U$	$x_{2\alpha}{}^L$	$x_{2\alpha}{}^U$	$y_\alpha{}^L$	$y_\alpha{}^U$	$E\left(L_q{}^2\right)_\alpha^L$	$E\left(L_q{}^2\right)_\alpha^U$
0.0	5	7	3	5	14	16	0.273	4.286
0.1	5.1	6.9	3.1	4.9	14.1	15.9	0.306	3.492
0.2	5.2	6.8	3.2	4.8	14.2	15.8	0.343	2.894
0.3	5.3	6.7	3.3	4.7	14.3	15.7	0.384	2.431
0.4	5.4	6.6	3.4	4.6	14.4	15.6	0.431	2.064
0.5	5.5	6.5	3.5	4.5	14.5	15.5	0.485	1.768
0.6	5.6	6.4	3.6	4.4	14.6	15.4	0.545	1.525
0.7	5.7	6.3	3.7	4.3	14.7	15.3	0.614	1.323
0.8	5.8	6.2	3.8	4.2	14.8	15.2	0.693	1.154
0.9	5.9	6.1	3.9	4.1	14.9	15.1	0.784	1.011
1.0	6.0	6.0	4.0	4.0	15.0	15.0	0.888	0.888

From the table we find that fuzzy queue length of second priority in the queue $E\left(L_q{}^2\right)$ at $\alpha = 1$ is 0.888, indicating that the queue length of second priority in the queue is 0.888. Moreover the range of $E\left(L_q{}^2\right)$ never exceed 4.286 or fall below 0.273



The membership function for average queue length of second priority

Conclusion:

The prioritization plays the important role in Qos. The mathematical models of priority queueing systems play an essential role in analysing and designing various networks such as Wireless Local Area Networks (WLAN)**.**Here $\alpha -$cut and Zadeh's extension principle is applied to non preemptive priority model and membership function of waiting time , average queue length for first and second priority is constructed using non-linear programming approach. The numerical example is also derived.

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