An Application of Vague Relation to a Decision Making Problem

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Abstract**—**
In this present paper the Author is describing "An Application of Vague relation to a decision making problem" basic concepts of vague relation and their operations is recollected. Recently Gauand Buehrer reported in IEEE [2] the theory of vague sets. But vague sets and intuitionistic fuzzy sets are same concepts as clearly justified by Bustince and Burillo in [1]. Consequently in this paper the Intuitionistic fuzzy relations and Vague relations are same terminologies. and we will apply the technique of vague relations to a decision making problem.

Introduction:

The notion of fuzzy sets was introduced by Zadeh [11]In 1965. By ordinary sets we mean yes-or-no type rather than more-or-less type. In conventional dual logic, for instance, a statement can be true or false-and definitely nothing in between. Vagueness, imprecision and uncertainty have so far been modelled by classical set-theoretic approach. According to this approach, borderline elements can be either put into the set or should be kept outside it. Hence it becomes inadequate for applying to humanistic type of problems. When Zadeh introduced the notion of fuzzy set it was the revolutionary time for researchers to work over fuzzy sets. The fuzzy set theory turn out far reaching applications in human life. Also vague sets and vague relations can be modelled using this theory. Fuzzy set is a class of objects in which the transition forms membership to non-membership is gradual rather than abrupt. Such a class is characterized by a membership function which assigns to an element a grade or degree of membership between 0 and 1. For a beginner on fuzzy set theory, the work in [3], [4], [5], [6] etc. are good enough to start with If we look at the developmental history of mathematical systems or structures, we see that a mathematical system is, in general, suggested by situations which, while they are different, have some basic features in common so that the emergence of a mathematical system is essentially the result of a process of unification and abstraction. A mathematical system, thus, lays bare the structurally essential relations between otherwise distinct entities. So, it may be accepted that the results of the study of a mathematical system will be valid for each of those otherwise different situations which provided motivation and inspiration for the same. Such a study also provided an economy of effort and leads to a better and fuller understanding of the motivation situations. Fuzzy relations have a wide range of applications ([10], [8], [9], [7] [12]) in different areas in Computer Science specially in DBMS, in relational database of Codd's model (see [10], [12], [13]), in Management Science, in Medical Science, in Banking and Finance, in Social Sciences etc. In the present paper we introduce the notion of intuitionistic fuzzy relation, and study some operations on them, define compositions of intuitionistic fuzzy relation, and at the end we will show an application of intuitionistic fuzzy relation.

Definition 1.1 Intuitionistic Fuzzy Relations (IFR)

Let X and Y be two universes. An intuitionistic fuzzy relation (IFR) denoted by R(X? Y) of the universe X with the universe Y is an IFS of the Cartesian product X×Y.
The true membership value t_R (x,y) estimates the strength of the existence of the relation of R-type of the object x with the object y, whereas the false membership value f_R(x,y) estimates the strength of the non-existence of the relation of R-type of the object x with the object y.
The relation R(X? Y) could be in short denoted by the notation R, if there is no confusion.

Example : Consider two universes X = {a,b} and Y = {p,q,r}. Let R be an IFR of the universe X with the universe Y proposed by an intelligent agent as shown by the following table:

R(X? Y)	p	q	R
x	(.7,.2)	(.3,.5)	(.8,.2)
y	(.2,.4)	(.7,.3)	(.4,.4)

The proposed IFR reveals the strength of vague relation of every pair of X× Y; For example, it reveals that the object y of the universe X has R-relation with the element p of Y with the following estimation:
strength of existence of the relation = .7
strength of non-existence of the relation = .2
A relation E(X?Y) is called a Complete Relation from the universe X to the universe Y if $V_E(x,y) = [1,1]$ □ (x,y) □ X×Y.

1.2Compositions of IFRs

In this section we study different compositions of Intuitionistic fuzzy relations. We study the sup- min composition of Intuitionistic fuzzy relations. An IFS and an IFR, under a suitable composition, could yield a new IFR with an useful significance. Similarly two IFRs, under a suitable composition, could too yield a new IFR with an useful significance. Composition of relations is important for applications, because of the reason that if a relation of a universe X with another universe Y is known and if a relation of the universe Y with a third universe Z is known then the relation of X with Z could be computed.

Definition 1.2.1

Composition of an IFS and an IFR Let A be an IFS of the universe X and R be an IFR of the universe X with another universe Y. The composition of another universe Y. The composition of R with A, denoted by B = R ? A,Is an IFS in Y given by $V_{R?A}(y)$=[$isup_{x□X}$ {$t_A(x)$ □$t_R(x,y)$ },$isup_{x□X}$ {(1-f_A)(x) □ (1-f_R)(x,y)}] .

Definition 1.2.2

Composition of two IFRs

Let: R(X?Y) and S(Y?Z)be two IFRs. Then the composite relation B = R ? S, is an IFR of X with Zgiven by

$V_{R?S}(x,z)$=[$isup_{y□Y}$ { $t_R(x,y)$ $t_S(y,z)$ },$isup_{y□Y}$ {(1- f_R)(x,y) □ (1-f_S)(y,z)}] .

This composition yields an IF- valued link between the objects x (of X) and z (of Z) through the elements y (of Y).Clearly ?S ?S ? R.

Consider a IFR R(X? X) of a universe X with itself. This type of relation we will call as an 'IFR R on the universe X'.

Definition 1.2..3 An IFR on a universe X is said to be

(i) Reflexive :if □ x □ X,$V_R(x,x)$ = [1,1].
(ii) Symmetric :if □ x_1, x_2 □ X,$V_R(x_1,x_2)$ = $V_R(x_2,x_1)$.

Now we define inverse of an IFR R(X? Y).

Definition 1.2.4 Let R(X? Y)be an IFR relating X with Y. Then its inverse is denoted by R^1(Y? X) which is an IFR relating Y with X and is given by$V_{R-1}(y,x)$=$V_R(x,y)$ □ x □ X and □ y □ Y.It may be noticed that inverse and concepts.

Definition 1.2.5

An IFR on a universe X is said to be an Intuitionistic fuzzy tolerance relation (IFTR) or an Intuitionistic fuzzy proximity relation (IFPR) on X if it is both reflexive and symmetric.

Suppose that, in a biotechnology experiment four potentially new strains of bacteria B_1, B_2, B_3, and B_4 have been detected in the area around an anaerobic corrosion pit on a new aluminum- lithium alloy used in the fuel tanks of a new experimental aircraft. In order to propose methods to eliminate the bio-corrosion caused by these bacteria, the four strains must be categorized first of all. One way to categorize them is to compare them to one another. In a pair-wise comparison, one could develop an IFPR :-

R	B_1	B_2	B_3	B_4
B_1	[1,1]	[.3,.4]	[.8,.1]	[0,0]
B_2	[.3,.4]	[1,1]	[.6,.4]	[.2,.3]
B_3	[.8,.1]	[.6,.4]	[1,1]	[.8,.9]
B_4	[0,0]	[.2,.3]	[.8,.9]	[1,1]

The following proposition is straightforward.

Proposition 6.4.2

If R1 and R2 be two IFTRs on X, then

(i) R_1^{-1} is also a IFTR on X.

(ii) $R_1 I R_2$ is a IFTR on X.

(iii) $R_1 U R_2$ is a IFTR on X.

6.5An Application of IFRs in a Decision Making Problem

Now we present an application of Intuitionistic fuzzy relations in a critical decision making problem.

Consider the factories, industries, farms etc. in the city Calcutta where strikes, protests, demonstrations, forcing-demands, etc. are very common events specially at the labor-section every year or even sometimes very frequently in each year. Calcutta is a large democratic-city (in India) reigned by communist-government where the labor-section in the factories/industries enjoy full freedom of speech, human-rights, rights to protest, etc. Consider such a factory (hypothetical) of which the top- level management people do not prefer to face the labor-union when there is a strike or protest or demand for pay-hike, or demand for more perks and facilities etc. Rather, the immediate officers (of junior management level) in the factory face them, solve the issues with a lot of tension and strain, and try to satisfy the labor-union arriving at some compromise. The labor section constitutes about 90% of the total employees in the factory and the yearly output (consequently, the net yearly profit) is too much dependent on the hard work of these labors as the most of the jobs at this factory are done by them. The owners of the factory (the top level management people) always maintain an unofficial strategy, known as "Strategy-Knowledge" which they renew periodically, once in every five years, and then hand over this knowledge to the junior level managers of the factory as a highly-confidential document.
The "Strategy-Knowledge" is an Intuitionistic fuzzy relation R of a universe Dwight the universe S, where D=a universe of expected/possible demands(from the past history),S=a universe of suggested standard solutions(from the past history),and L= the class of labors in the factory.

The above set D does not guarantee that any type of demand d by a labor l □ L will be in D. Similarly the set S too does not guarantee that for any type of demand or for any combination of demands, there exist a standard solution s in S. These are specified from the past history and experience gathered by the top management people, and there is an unofficial instruction to each junior-manager that they must try to strict to this pre-specified S only. In fact that is what apparently called to be the “management quality” of a junior manager in the factory, how they can manage everything tactfully in compliance with the upper level constraints. The union puts various demands anytime, sometimes more than once in a year. For example, whenever there is a hike in petrol-price in the market, they will demand for more salary. Sometimes during winter they demand for more winter-clothes for their family- members, sometimes they demand for better medical treatment for their family members, sometimes they demand for one month salary bonus in addition to the existing bonus, sometimes they demand for higher rates of interest on their provident- fund deposits, etc. to list a few only. The demands vary from labor to labor of different departments of the factory. Suppose that, once upon a time, few labors go on strike in a factory. The different labors have different demands although most of the demands are common demands. How to solve this chaos means how to satisfy each of these labors individually. The problem is directly to be faced and solved by the junior managers by taking a very careful decision doing a lot of thinking, a lot of draft-computing on papers or in computers.

The decision- methodology will be based on the following stages mainly :-

(i) listen to the demands of each labor (individually), and the common demands from the union leaders

(ii) Make a link of these demands with the “Strategy Knowledge”

(iii) Compute a link of the labors with the solutions.

(iv) Let the set L be { l_1, l_2, l_3, ……., ln }. Let A be an IFS of the universe D, and R be an IFR of the universe D with the universe S.

Then the composition B = A ? R describes the state of the labour in terms of the solution as an IFS of S, with the IF value defined □ s □ S given by $V_B(s)$ = [$isup_{d□D}$ {$t_A(d)$ □$t_R(d,s)$ }, $isup_{d□D}$ {$(1-f_A)(d)$ □ $(1-f_R)(d,s)$}]………… (1)

If the state of a given labour l is described in terms of an IFS A of the universe D, then the labour l is assumed to be assigned solution in terms of IFS B of S through an IFR R of the “Strategy-Knowledge” from the universe D to the universe S, which is assumed to be given by an intelligent manager who can translate his own perception of the vagueness involved in IF-valued degree of association between demands and solutions.
Now consider many labours l_i□ L, R is an IFR from D to S and define an IFRQ from L to S. Then the equation (1) becomes V_T (l,s)= [$isup_{d□D}$ {$t_Q(l,d)$ □$t_R(d,s)$ },$isup_{d□D}$ {$(1- f_Q)(l,d)$ □ $(1-f_R)(d,s)$}]………..(2)
where T = R ? Q.
By R and Q, one can compute T. From the present knowledge of Q and T, the manager may compute the R for which t_R □ $(1- f_R)$ is greatest such that T = R ? Q holds, the most significant IFR translating the higher degree of associations of demand with solution, an approach to “Strategy-Knowledge”.
From computed R the manager can infer solution from the demand as an IF value. If the results are not satisfactory (not at all being acceptable to the concerned labours, the IFR R could be modified by the manager using a computer-based soft-computing system.

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