Edge Odd Graceful Labeling of Some Special Graphs

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Abstract

The concept of Edge Odd Graceful Labeling was studied from Edge –Odd Graceful Labeling of the Complete Bipartite Graph. It is a special type of labeling of a graph G that is if there is a bijection f from the edge of G to the set $\{1,3,5,\dots,(2q-1)\}$ so that the induced mapping $f^*: V \to \{0,1,2,\dots,2q\}$ given by $f^*(u) = \sum\{f(uv)/uv \epsilon E\} \,(mod 2q)$ with p vertices. This paper deals with the edge odd graceful labeling of some special graphs like Dutch Windmill Graph D_5^m , D_4^m and Circular Ladder Graph CL_n

Keywords

Circular Ladder Graph, Dutch Windmill Graph, Edge, Odd.

1. Introduction

Graph labeling[1] plays an important role in research now a days. Graph Labeling is an acting tool which contribute a lot in various field and makes very easy to handle many areas such as communication network, circuit Layout etc. A graph labeling is a map which carries graph elements (vertices or edges) to the integers subject to certain conditions .In 1967 Rosa introduced the concept of graph labeling methods. Then Lo introduced edge graceful graphs, Dr.A.Soliraju and K.Chithra introduced Edge-Odd graceful labeling (EOGL). In this paper the Dutch Wind mill graph D_5^m , D_4^m and Circular Ladder Graph CL_n are shown as EOGL graph.

2. Definition

2.1 Dutch Windmill Graph

The Dutch Windmill graph D_n^m [10] is the graph consisting m copies of complete graph K_n with a vertex in common .

2.2 Circular ladder graph

Circular ladder graph CL_n [2] is the Cartesian product of a cycle of length $n \geq 3$ and an edge. It is denoted as $C_n \times P_1$.

2.3 Edge Odd Graceful Labeling (EOGL)

A graph Labeling is said to be a EOGL [5,6,7,8,9,10] if there exists a bijection $f: E \to \{1,3,5,\dots,(2q-1)\}$ so that the induced mapping $f^*: V \to \{0,1,2,\dots,2q\}$ given by $f^*(u) = \sum\{f(uv) \setminus uv \epsilon E\} \,(mod 2q)$ with p vertices.

3. Theorem

Every D_5^m (m is odd) is an Edge Odd graceful Labeling Graph.

Proof

Let the vertex set of D_5^m be $V = V_1 \cup V_2$ where $V_1 = \{u\}$ common vertex and other vertices be $V_2 = \{u_i / i = 1 \text{ to } 4m\}$.

Edge set $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6$, where

$E_1 = \{uu_i / i = 1to4m\}$, $E_2 = \{u_i u_{i+1} \ / \ i = 1,3,\dots.to(4m-1)\}$, $E_3 = \{u_{4i-3}u_{4i} / i = 1 \text{ to } m\}$,
$E_4 = \{u_{4i-3}u_{4i-1} / i = 1 \text{ to } m\}$, $E_5 = \{u_{4i-2}u_{4i} / i = 1 \text{ to } m\}$, $E_6 = \{u_{4i-2}u_{4i-1} / i = 1 \text{ to } m\}$

First the edges are labeled as follows

$f(uu_{4i}) = 20i - 17$ for $i = 1 \text{ to } m$
$f(uu_{4i-1}) = 20i - 1$ for $i = 1 \text{ to } m$

$f(uu_{4i-2}) = 20i - 3$ for $i = 1\ to\ m$
$f(uu_{4i-3}) = 20i - 19$ for $i = 1\ to\ m$
$f(u_{4i-3}u_{4i}) = 20i - 15$ for $i = 1\ to\ m$
$f(u_{4i-3}u_{4i-1}) = 20i - 5$ for $i = 1\ to\ m$
$f(u_{4i-3}u_{4i-2}) = 20i - 13$ for $i = 1\ to\ m$
$f(u_{4i-2}u_{4i}) = 20i - 7$ for $i = 1\ to\ m$
$f(u_{4i-2}u_{4i-1}) = 20i - 11$ for $i = 1\ to\ m$
$f(u_{4i-1}u_{4i}) = 20i - 9$ for $i = 1\ to\ m$

Now, the vertex are labeled by induced mapping as follows ,
$f^*(u) = 0$
$f^*(u_{4i-3}) = [f(uu_{4i-3}) + f(u_{4i-3}u_{4i-2}) + f(u_{4i-3}u_{4i-1}) + f(u_{4i-3}u_{4i})](mod 20m), i = 1,2, \dots, m$
$f^*(u_{4i-2}) = [f(uu_{4i-2}) + f(u_{4i-2}u_{4i-3}) + f(u_{4i-2}u_{4i-1}) + f(u_{4i-2}u_{4i})](mod 20m), i = 1,2, \dots, m$
$f^*(u_{4i-1}) = [f(uu_{4i-1}) + f(u_{4i-1}u_{4i-3}) + f(u_{4i-1}u_{4i-2}) + f(u_{4i-1}u_{4i})](mod 20m), i = 1,2, \dots, m$
$f^*(u_{4i}) \quad = [f(uu_{4i}) + f(u_{4i}u_{4i-3}) + f(u_{4i}u_{4i-2}) + f(u_{4i}u_{4i-1})](mod 20m), i = 1,2, \dots, m$

Now the edge labels and vertex labels are distinct in D_5^m (when m is odd)
So D_5^m (when m is odd) is an Edge Odd Graceful Labeling Graph.

3.1 Example

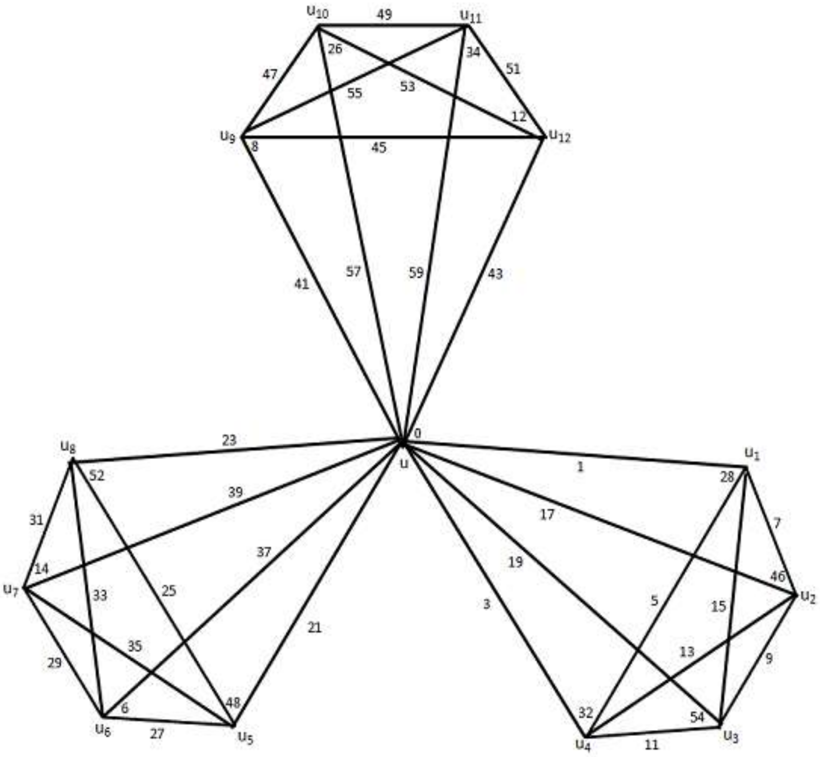

Fig 1 - Dutch Windmill Graph D_5^3

4. Theorem

The Dutch Windmill Graph D_4^m (m even) is edge odd Graceful Labeling Graph.

Proof

Let the vertex set of D_4^m be $V = V_1 \cup V_2$ where $V_1 = \{u\}$ common vertex and other vertices be $V_2 = \{u_i / i = 1\ to\ 3m\}$.

Edge set $E = E_1 \cup E_2 \cup E_3 \cup E_4$, where
$E_1 = \{uu_i / i = 1\ to\ 3m\}$, $E_2 = \{u_{3i-2}u_{3i-1} / i = 1,2,3, \dots, m\}$, $E_3 = \{u_{3i-1}u_{3i} / i = 1,2,3, \dots, m\}$ and $E_4 = \{u_{3i-2}u_{3i} / i = 1,2,3, \dots, m\}$.
First the edges are labeled as follows
$f(uu_{3i-2}) = 4i - 3\ \ for\ i = 1\ to\ m$

$f(u_{3i-2}u_{3i-1}) = 4m + 4i - 3 \ \ for\ i = 1\ to\ m$
$f(u_{3i-1}u_{3i}) = 4i - 1 \ \ for\ i = 1\ to\ m$
$f(uu_{3i}) = 4m + 4i - 1\ for\ i = 1\ to\ m$
$f(uu_{6i-4}) = 8m + 4i - 3 \ \ for\ \ i = 1\ to\ \frac{m}{2}$
$f(uu_{6i-1}) = 10m + 4i - 1 \ \ for\ \ i = 1\ to\ \frac{m}{2}$
$f(u_{6i-5}u_{6i-3}) = 10m + 4i - 3 \ \ for\ \ i = 1\ to\ \frac{m}{2}$
$f(u_{6i-2}u_{6i}) = 8m + 4i - 1 \ \ for\ \ i = 1\ to\ \frac{m}{2}$

Now the induced vertex labels are
$f^*(u) = 0$
$f^*(u_{6i-5}) = [f(uu_{6i-5}) + f(u_{6i-5}u_{6i-4}) + f(u_{6i-5}u_{6i-3})](mod12m) for\ i = 1\ to\ \frac{m}{2}$
$f^*(u_{6i-4}) = [f(uu_{6i-4}) + f(u_{6i-5}u_{6i-4}) + f(u_{6i-3}u_{6i-4})](mod12m) for\ i = 1\ to\ \frac{m}{2}$
$f^*(u_{6i-3}) = [f(uu_{6i-3}) + f(u_{6i-5}u_{6i-3}) + f(u_{6i-4}u_{6i-3})](mod12m) for\ i = 1\ to\ \frac{m}{2}$
$f^*(u_{6i-2}) = [f(uu_{6i-2}) + f(u_{6i-2}u_{6i-1}) + f(u_{6i-2}u_{6i})](mod12m) for\ i = 1\ to\ \frac{m}{2}$
$f^*(u_{6i-1}) = [f(uu_{6i-1}) + f(u_{6i-2}u_{6i-1}) + f(u_{6i-1}u_{6i})](mod12m) for\ i = 1\ to\ \frac{m}{2}$
$f^*(u_{6i}) = [f(uu_{6i}) + f(u_{6i-1}u_{6i}) + f(u_{6i-2}u_{6i})](mod12m) for\ i = 1\ to\ \frac{m}{2}$
Now the edge labels and vertex labels are distinct in D_4^m (when m is even)
So the Dutch Windmill Graph D_4^m (m even) is edge odd Graceful Labeling Graph.

4.1 Example

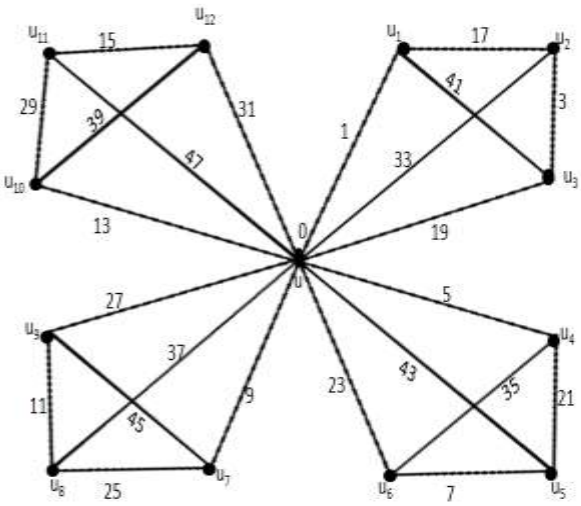

Fig 2 Dutch Windmill Graph D_4^4

5 Theorem

Every Circular ladder graph CL_n $(n \geq 3)$ is edge odd graceful labeling graph.

Proof

Let the vertices of CL_n be $\{u_1, u_2, u_3, \ldots, u_n, v_1, v_2, v_3, \ldots, v_n\}$.
Edge set be $E = E_1 \cup E_2 \cup E_3 \cup E_4$, Where $E_1 = \{u_iu_{i+1}/i = 1,2, \ldots, (n-1)\}$, $E_2 = \{u_1u_n , v_1v_n\}$, $E_3 = \{v_iv_{i+1}/i = 1,2, \ldots, (n-1)\}$, $E_4 = \{u_iv_i /i = 1,2, \ldots, n\}$.

First the edges are labeled as follows
$f(u_iu_{i+1}) = 2i - 1 \ \ for\ \ i = 1,2, \ldots, (n-1),$
$f(u_1u_n) = 2n - 1$
$f(v_iv_{i+1}) = 4n + 2i - 1 \ \ for\ \ i = 1,2, \ldots, (n-1)$
$f(v_1v_n) = 6n - 1$
$f\ (u_iv_i) = 4n - 2i + 1 \ \ for\ \ i = 1,2, \ldots, n$

Now the induced vertex labels are

$f^*(u_1) = 6n - 1$
$f^*(u_{i+1}) = 4n + 2i - 1 \ for \ i = 1,2, \dots, (n-1)$
$f^*(v_1) = 14n - 1(mod\ 6n)$
$f^*(v_{i+1}) = 12n + 2i - 1(mod\ 6n) for\ i = 1,2, \dots, (n-1)$
Now the edge labels and vertex labels are distinct in CL_n
So Every CL_n $(n \geq 3)$ is an Edge Odd Graceful Labeling Graph.

5.1 Example

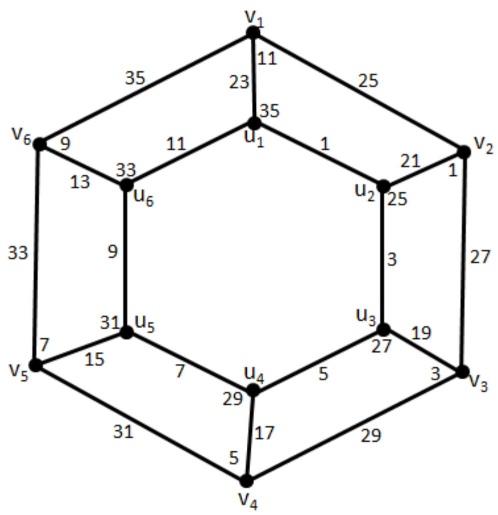

Fig 3 - Circular Ladder Graph CL_6

6 Conclusion

In this Paper Dutch Windmill Graph D_5^m when m is odd and D_4^m when m is even are proved as Edge - Odd Graceful Labeling Graphs are proved .Finally we Proved Every Circular ladder graph CL_n $(n \geq 3)$ is edge odd graceful labeling graph.

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