An Approach for Solving Fuzzy Sequencing Problems with Octogonal Fuzzy Numbers using Robust Ranking Techniques

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ABSTRACT

In this paper, we consider the Fuzzy sequencing problem where processing time taken as octagonal fuzzy numbers.It can be solved using Robust Ranking method and fuzzy sequencing problem can be converted into a crisp valued sequence problem and solved by Johnson's Algorithm which is illustrated through a numerical example.

Keywords:*Octogonal fuzzy numbers,Robust Ranking Technique,Fuzzy sequencingproblem.*

1 INTRODUCTION

A job sequencing problems arise when we are concerned with situations where there is a choice in which a number of tasks can be performed .Job sequencing problem has become the major problem in the computer field.Sequencing problem is considered to be one of the classic and important application of OR.The main role of the classical sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the totalamount of time required to complete the process of all the jobs.One of the renowned work in the field of considered till date is by Johnson's who gave the algorithm in 1954 for production scheduling in which he had minimized the total idle time of machines and the total production times of the job's, Later in 1967 Smith and Dudek developed a general algorithm for the solution of the 'n' job on 'm' machine sequencing problem of the flow shop when number passing is allowed.In recent years ,theory of fuzzy sets has been applied to model the imprecision in optimization ,in particular in sequencing problem in papers(4,5,11)some single machine sequencing problem with fuzzy processing times,fuzzy due dates and fuzzy precedence constraints have been discussed.Also in(3) an optimality evaluation of sequences under fuzzy parameters has been investigated .

In this paper we wish to propose a new approach to sequencing problem with fuzzy parameters.

2.PRELIMINARIES

2.1 Definition (Fuzzy Set) [12]

If X is a collection of objects denoted generically by x, then the fuzzy set Ấ in X is defined to be a set of ordered pairs. Where $\mu_{\tilde{A}}$(x) is called the membership function for the fuzzy set. The membership function maps each element of x to a value between (0, 1).

2.2 Definition(Fuzzy Number)[9]

A Fuzzy number Ấ in the real line R is a fuzzy set $\mu_{\tilde{A}}$(x): R→ (0, 1) that satisfies the following

properties.

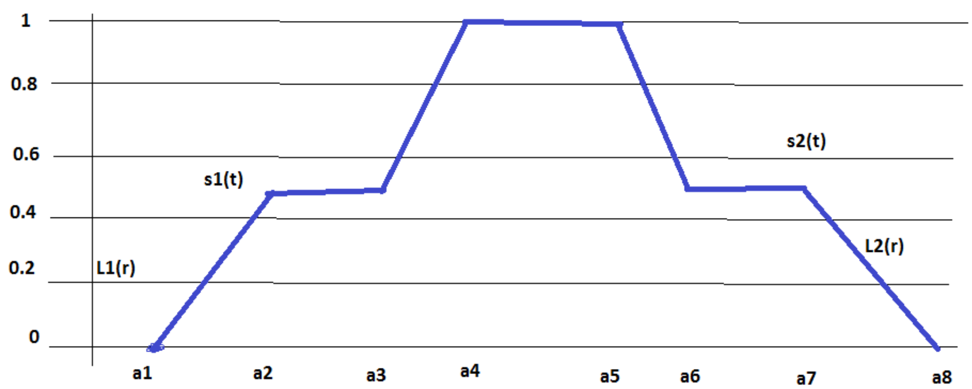
i) There exists an at least one x∈R with $\mu_{\tilde{A}}$(x) =1
ii) $\mu_{\tilde{A}}$(x) is piece wise continuous

3 OCTAGONAL FUZZY NUMBER

A fuzzy number is the normal Octagonal fuzzy number is denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$

where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ k\left[\frac{x-a_1}{a_2-a_1}\right] & for\ a_1 \le x \le a_2 \\ k & for\ a_2 \le x \le a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ 1 & for\ a_4 \le x \le a_5 \\ k+(1-k)\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ k & for\ a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ 0 & for\ x \ge a_8 \end{cases}$$



4. ROBUST RANKING TECHNIQUE

To provide results which are consistent with human intuition, robust ranking technique is used and its satisfies compensation, linearity and additive properties. If $\tilde{a}$ is a convex fuzzy number, the robust ranking index is defined by

$$R(\tilde{a}) = \int_0^1 (0.5) \quad (a_\alpha^L\ ,\ a_\alpha^U)\ d\alpha$$

Where $(a_\alpha^L\ ,\ a_\alpha^U) = [\{(b-a)\alpha + a, d-(d-c)\alpha\}, \{(f-e)\alpha + e, h-(h-g)\alpha\}]$

is the α- level cut of a fuzzy number $\tilde{a}$. Here this method is proposed for ranking the objective values. The representative value of fuzzy number $\tilde{a}$ is given by Robust ranking index $R(\tilde{a})$.

5. PROCESSING OF ‘n’ JOBS THROUGH ‘m’ MACHINES [7]

Let there are ‘n’ jobs say A_1 ,A_2,…. A_n be processed through ‘m’ machines say M_1 ,M_2,…. M_n in the order M_1 M_2…. M_n. Let t_{ij} be the fuzzy processing time taken by i^{th}job to be completed by j^{th} machine. The well-known Johnson Method (   ) can be extended to this problem, forming two fictitious machines G and H(say), If either (or) both of the following conditions are satisfied.

$$\min_i(\ t_{i1}\ ) \ge \max_i(t_{ij}\ )\ ,\ j = 2,3, \dots\ m-1$$

$\min(\ _i\, t_{im}\ ) \geq \max_i(t_{ij}\ )\ ,\ i = 1,2,3,..n$

Let G and H are given by $G_i = \sum_{j=1}^{m-1} t_{ij}$ and $H_i = \sum_{j=2}^{m} t_{ij}$, i=1,2,…n

6. PROCEDURE FOR SOLVING FUZZY SEQUENCING PROBLEMS

Step (1): Using Robust Ranking Technique , the fuzzy sequencing problem can be Converted into crisp sequencing problem.
Step (2): The Optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.
Step (3): After finding the optimal sequence, Determine the total elapsed fuzzy time and also the fuzzy idle time on machines.

7.NUMERICAL EXAMPLES

Shahi Export House has to process 5 items through 3 stages of production viz. Cutting, Sewing & Pressing. Processing times are given in the following table.

Items	Cutting M_1	Sewing M_2	Pressing M_3
A_1	(-2,-1,0,1,4,5,6,7)	(-3,-2,-1,0,1,2,3,4)	(7,8,10,11,14,15,17,18)
A_2	(1,2,3,4,5,6,9,10)	(-3,-2,-1,0,4,5,6,7)	(6,8,9,10,12,13,14,16)
A_3	(1,2,4,5,8,10,12,14)	(-4,-3,0,1,2,3,4,5)	(-1,0,1,2,4,5,6,7)
A_4	(2,3,6,7,8,10,11,13)	(-2,-1,0,1,2,3,4,5)	(5,6,7,8,12,13,14,15)
A_5	(3,5,6,8,10,12,13,15)	(-1,0,1,2,3,4,5,6)	(1,2,3,4,5,6,7,8)

Determine an order in which these items should be processed so as to minimize the total processing time.

Step (1): Using Robust Ranking Technique for Octagonal fuzzy number, the fuzzy times can be converted in to crisp times

$$R(\tilde{a}) = \int_0^1 (0.5)\quad (a_\alpha^L\ ,\ a_\alpha^U)\ d\alpha$$

R(-2,-1,0,1,4,5,6,7)= 5

R(1,2,3,4,5,6,9,10)= 10

R(1,2,4,5,8,10,12,14)= 14

R(2,3,6,7,8,10,11,13)= 15

R(3,5,6,8,10,12,13,15)= 18

R(-3,-2,-1,0,1,2,3,4)= 1

R(-3,-2,-1,0,4,5,6,7) = 4

R(-4,-3,0,1,2,3,4,5) = 2

R(-2,-1,0,1,2,3,4,5) = 3

R(-1,0,1,2,3,4,5,6) = 5

R(7,8,10,11,14,15,17,18) = 25

R(6,8,9,10,12,13,14,16) = 17

R(-1,0,1,2,4,5,6,7) = 6

R(5,6,7,8,12,13,14,15) = 20

R(1,2,3,4,5,6,7,8) = 9

Items	Cutting M_1	Sewing M_2	Pressing M_3
A_1	5	1	25
A_2	10	4	17
A_3	14	2	6
A_4	15	3	20
A_5	18	5	9

Step (2): Three Machine problem can be converted into two machine problem

Items	Machine M_1	Machine M_2
A_1	6	26
A_2	14	21
A_3	16	8
A_4	18	23
A_5	23	14

Optimum sequence

A_1	A_2	A_4	A_5	A_3

Step (3): Total elapsed time and Idle time

Items	Machine M_1		Machine M_2		Machine M_3	
	Time in	Time out	Time in	Time out	Time in	Time out
J_3	0	5	5	6	6	31
J_1	5	15	15	19	31	48
J_4	15	30	30	33	48	68
J_5	30	48	48	53	68	77
J_2	48	**62**	62	**64**	**77**	**83**

Total Elapsed time = 83Hrs

Idle time on Machine M_1 = 21 Hrs

Idle time on Machine M_2 = 49Hrs

Idle time on Machine M_2 = 6 Hrs

8. CONCLUSION

In this paper ,a simple method of solving fuzzy sequencing problem were introduced by using ranking of octagonal fuzzy numbers. The fuzzy sequencing problem of Octogonal fuzzy numbers has been transformed into crisp sequencing problem using Robust ranking technique .By this method we obtained the optimal solution in fuzzy nature and the optimal total cost in crisp nature.

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