Best Proximity Point For New Symmetric Rational Cyclic Contraction in Metric Spaces

Vinitha Dewangan[#1] and Amitabh Banerjee[*2]
[#1]Govt. J.Y. Chhattishgarh College Raipur, Chhattishgarh, India
[*2]Govt. S.N. College Nagari Distt.-Dhamtari, Chhattishgarh, India

Abstract — *In this paper we obtain new type of best proximity point theorems for new symmetric rational cyclic contraction in metric space (X, d). These results generalize and improve some main results in Yadav et al. (Best proximity point theorems for MT-K and MT-C rational cyclic contractions in metric spaces.)*

Keywords — *fixed point, cyclic map, best proximity point, rational cyclic contraction, MT-function.*

1. Introduction

Let (X, d) be a metric space and let A and B be nonempty subsets of X. A mapping T on $A \cup B$ is called a cyclic mapping if $T(A) \subseteq B$ and $T(B) \subseteq A$. $x \in A \cup B$ is called a best proximity point if $d(x, Tx) = d(A, B)$ is satisfied, where $d(A, B) = \inf d(x, y): x \in A, y \in B$. In 2003 Kirk et al. [8] gave the first result in cyclic contraction. After that several fixed point and best proximity point results have been proved in cyclic contraction. Some of these works may be noted in [2,4,5,7]. Eldred, Kirk and Veeramani [1] proved the existence of a best proximity point for relatively non-expansive mappings using the notion of proximal normal structure.

In 2006, Eldred and Veeramani [3] introduced the concept of cyclic contraction as follows.

Definition 1.1. [3] Let A and B be nonempty subsets of a metric space (X, d). A map $T: A \cup B \to A \cup B$ is called a cyclic contraction if the follow conditions hold:

(1) $T(A) \subseteq B$ and $T(B) \subseteq A$.
(2) There exists $k \in (0,1)$ such that $d(Tx, Ty) \leq kd(x, y) + (1 - k)dist(A, B)$ for all $x \in A$ and $y \in B$.

Eldred and Veeramani [3] first established the following interesting best proximity point theorem.

Theorem 1.2. (*[3], Proposition 3.2*). Let A and B be nonempty closed subsets of a complete metric space X. Let $T: A \cup B \to A \cup B$ be a cyclic contraction map, let $x_0 \in A$ and define $x_{n+1} = Tx_n, \quad n \in \mathbb{N}$. Suppose $\{x_{2n}\}$ has a convergent subsequence in A. Then there exists $x \in A$ such that d(x, T x) = dist(A, B).

2. Preliminaries

We define MT-function which will be used throughout the paper to get the best proximity point theorems.

Definition 2.1 (See [9]) A function $\varphi: [0, \infty) \to [0,1)$ is said to be an MT-function if $\lim_{s \to t^+} \varphi(s) < 1$ for all $t \in [0, \infty)$ (Mizoguchi-Takahashi's condition [11]).

It is obvious that if $\varphi: [0, \infty) \to [0,1)$ is a non-decreasing function or a non-increasing function, then φ is an MT-function. So, the set of MT-functions is a rich class, but it is worth to mention here that there exist functions which are not MT-functions.

Example 2.2. (See [11]) $\varphi: [0, \infty) \to [0,1)$ be defined by

$$\varphi(t) = \begin{cases} \dfrac{\sin(t)}{t}, & t \in (0, \pi/2] \\ 0 & , otherwise. \end{cases}$$

Since $\lim_{s \to t^+} \varphi(s) = 1, \varphi$ is not an MT-function. Very recently, Du. [11] first proved some characterizations of MT-functions.

Theorem 2.3. $\varphi: [0,\infty) \to [0,1)$ be a function. Then the following statements are equivalent.

(a) φ is an MT-function,

(b) For each $\in [0,\infty)$, there exists $r_t^{(1)}$ and $\varepsilon_t^{(1)} > 0$ such that $\varphi(s) < r_t^{(1)}$ for all $s \in (t, t+\varepsilon_t^{(1)})$.

(c) For each $t \in [0,\infty)$, there exists $r_t^{(2)}$ and $\varepsilon_t^{(2)} > 0$ such that $\varphi(s) < r_t^{(2)}$ for all $s \in (t, t+\varepsilon_t^{(2)})$.

(d) For each $t \in [0,\infty)$, there exists $r_t^{(3)}$ and $\varepsilon_t^{(3)} > 0$ such that $\varphi(s) < r_t^{(3)}$ for all $s \in (t, t+\varepsilon_t^{(3)})$.

(e) For each $t \in [0,\infty)$, there exists $r_t^{(4)}$ and $\varepsilon_t^{(4)} > 0$ such that $\varphi(s) < r_t^{(4)}$ for all $s \in (t, t+\varepsilon_t^{(4)})$.

(f) For any non-increasing sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[0,\infty)$ we have $0 \le \sup_{n\in\mathbb{N}} \varphi(x_n) < 1$.

(g) φ is a function of contractive factor [10]; that is, for any strictly decreasing sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[0,\infty)$, we have $0 \le \sup_{n\in\mathbb{N}} \varphi(x_n) < 1$.

In this paper, we will establish some best proximity point theorems for MT-K rational cyclic and MT-C rational cyclic contractions.

3. Best proximity point theorems

In this section, we first establish a convergence theorem which is one of the main results in this paper.

Theorem 3.1. Let A, B be nonempty subsets of a metric space (X,d) and $T: A \to B$ and $S: B \to A$ be maps. If there exists a non-decreasing function $\mu: [0,\infty) \to [0,\infty)$ and an MT-function $\varphi: [0,\infty) \to [0,1)$ such that

$$d(Tx,Sy) \le \varphi\left(\mu\big(d(x,y)\big)\right)\left[\frac{d(x,Tx)d(x,Sy)+d(y,Sy)d(y,Tx)}{d(x,Tx)+d(y,Sy)}\right] + \left[1-\varphi\left(\mu\big(d(x,y)\big)\right)\right]d(A,B), \tag{1}$$

for all $x \in A$ and $y \in B$, then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A,B).$$

Proof. Let $x_0 \in A$ be given. Define $x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$ for each $n \in \mathbb{N} \cup \{0\}$. Then $x_{2n} \in A$ and $x_{2n+1} \in B$, for each $n \in \mathbb{N} \cup \{0\}$. By (1), we have

$$\begin{aligned} d(x_1,x_2) &= d(Tx_0,Sx_1) \\ &\le \varphi\left(\mu\big(d(x_0,x_1)\big)\right)\left[\frac{d(x_0,Tx_0)d(x_0,Sx_1)+d(x_1,Sx_1)d(x_1,Tx_0)}{d(x_0,Tx_0)+d(x_1,Sx_1)}\right] \\ &+ \left[1-\varphi\left(\mu\big(d(x_0,x_1)\big)\right)\right]d(A,B) \end{aligned}$$

$$\leq \varphi\left(\mu\left(d(x_0,x_1)\right)\right)\left[\frac{d(x_0,x_1)d(x_0,x_2)+d(x_1,x_2)d(x_1,x_1)}{d(x_0,x_1)+d(x_1,x_2)}\right]$$
$$+[1-\varphi(\mu(d(x_0,x_1)))]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_0,x_1)\right)\right)\left[\frac{d(x_0,x_1)d(x_0,x_2)+0}{d(x_0,x_2)}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_0,x_1)\right)\right)\right]d(A,B)$$

$$\leq \varphi\left(\mu\left(d(x_0,x_1)\right)\right)d(x_0,x_1)+\left[1-\varphi\left(\mu\left(d(x_0,x_1)\right)\right)\right]d(A,B).$$

Since φ is an MT-function, then from Theorem (2.3),

$$0 \leq \sup_{n\in\mathbb{N}} \varphi\left(d(x_n,x_{n+1})\right) < 1.$$

So that we have $0 \leq \sup_{n\in\mathbb{N}} \varphi\left(\mu\left(d(x_0,x_1)\right)\right) < 1.$

Suppose $k = \sup_{n\in\mathbb{N}} \varphi\left(\mu\left(d(x_0,x_1)\right)\right) < 1.$

Therefore $0 \leq k <$, since $\varphi\left(\mu\left(d(x_0,x_1)\right)\right) \leq$ we get

$$d(x_1,x_2) \leq kd(x_0,x_1)+(1-k)d(A,B). \tag{2}$$

Similarly, from inequality (1), we have

$$d(x_2,x_3) = d(Sx_1,Tx_2)$$
$$\leq \varphi\left(\mu\left(d(x_1,x_2)\right)\right)\left[\frac{d(x_1,Sx_1)d(x_1,Tx_2)+d(x_2,Tx_2)d(x_2,Sx_1)}{d(x_1,Sx_1)+d(x_2,Tx_2)}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_1,x_2)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_1,x_2)\right)\right)\left[\frac{d(x_1,x_2)d(x_1,x_3)+d(x_2,x_3)d(x_2,x_2)}{d(x_1,x_2)+d(x_2,x_3)}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_1,x_2)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_1,x_2)\right)\right)\left[\frac{d(x_1,x_2)d(x_1,x_3)+0}{d(x_1,x_3)}\right]+\left[1-\varphi\left(\mu\left(d(x_1,x_2)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_1,x_2)\right)\right)d(x_1,x_2)+\left[1-\varphi\left(\mu\left(d(x_1,x_2)\right)\right)\right]d(A,B),$$

which implies that,

$$d(x_2,x_3) \leq kd(x_1,x_2)+(1-k)d(A,B). \tag{3}$$

From (2) and (3), it follows that

$$d(x_2,x_3) \leq kd(x_1,x_2)+(1-k)d(A,B)$$
$$\leq k[kd(x_0,x_1)+(1-k)d(A,B)]+(1-k)d(A,B)$$
$$\leq k^2d(x_0,x_1)+(k-k^2)d(A,B)+(1-k)b(A,B)$$
$$\leq k^2d(x_0,x_1)+(1-k^2)d(A,B).$$

Hence inductively, we have

$$d(x_n,x_{n+1}) \leq kd(x_n,x_{n-1})+(1-k)d(A,B)$$
$$\leq k^2d(x_{n-1},x_{n-2})+(1-k^2)d(A,B)$$

$$\leq \ldots$$
$$\leq k^n d(x_0, x_1) + (1 - k^n) d(A, B).$$

Since $0 \leq k < 1$, we obtain $\lim_{n \to \infty} k^n = 0$, that is

$\lim_{n \to \infty} d\,(x_n, x_{n+1}) = d(A, B).$

The following result is immediate from Theorem(3.1).

Corollary 3.2. Let A, B be nonempty subsets of a metric space (X, d) and $T: A \to B$ and $S: B \to A$. If the pair of maps T and S satisfy MT-K condition, then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \to \infty} d\,(x_n, x_{n+1}) = d(A, B).$$

Corollary 3.3. Let A, B be nonempty subsets of a metric space. Suppose that the mapping $T: A \to B$ and $S: B \to A$ satisfies K-rational cyclic condition. For a fixed element x_0 in A, let $x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$. Then $d(x_n, x_{n+1}) \to$ d(A, B).

Theorem 3.4. Let A and B be a nonempty closed subsets of a metric space (X, d) and $T: A \to B$ and $S: B \to A$ be maps. Suppose that there exists a non-decreasing function $\mu: [0, \infty) \to [0, \infty)$ and an MT-function $\varphi: [0, \infty) \to [0,1)$ such that

$$d(Tx, Sy) \leq \varphi\left(\mu\big(d(x,y)\big)\right)\left[\frac{d(x,Tx)d(x,Sy) + d(y,Sy)d(y,Tx)}{d(x,Tx) + d(y,Sy)}\right] + [1 - \varphi\left(\mu\big(d(x,y)\big)\right)]d(A,B), \tag{4}$$

for all $x \in A$ and $y \in B$. If $x_0 \in A$, define $x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$, $n \in \mathbb{N} \cup \{0\}$, then the sequence $\{x_n\}$ is bounded.

Proof. It follows from Theorem (3.1), that $\{d(x_{2n-1}, \ x_{2n})\}$ is convergent and hence it is bounded. Suppose $\{x_{2n_k}\}$ be a subsequence of x_{2n} which converges to some element x in A. Since the maps T and S satisfies MT-K rational cyclic condition, we get

$$d(x_{2n_k}, Tx_0) = d(Sx_{2n_k-1}, Tx_0)$$
$$\leq \varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\left[\frac{d(x_{2n_k-1}, Tx_0)d(x_{2n_k-1}, Sx_{2n_k-1}) + d(x_0, Sx_{2n_k-1})d(x_0, Tx_0)}{d(x_0, Tx_0) + d(x_{2n_k-1}, Sx_{2n_k-1})}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\left[\frac{d(x_{2n_k-1}, x_{2n_k})d(x_{2n_k-1}, Tx_0) + d(x_0, Tx_0)d(x_0, x_{2n_k})}{d(x_{2n_k-1}, x_{2n_k}) + d(x_0, Tx_0)}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\left[\frac{d(x_{2n_k-1}, x_{2n_k})d(x_{2n_k-1}, Tx_0)}{d(x_{2n_k-1}, x_{2n_k}) + d(x_0, Tx_0)}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\left[\frac{d(x_{2n_k-1}, x_{2n_k})d(x_{2n_k-1}, Tx_0)}{d(x_{2n_k-1}, Tx_0) + d(x_0, x_{2n_k})}\right]$$
$$+\left[1-\varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\right]d(A,B)$$
$$\leq \varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)d(x_{2n_k-1}, x_{2n_k}) + \left[1-\varphi\left(\mu\left(d(x_{2n_k-1}, x_0)\right)\right)\right]d(A,B).$$

Hence from Theorem (3.1), $\{d(x_{2n-1}, x_{2n})\}$ converges to d(A, B). So the last inequality implies that $d(x_{2n_k}, Tx_0) \leq d(A,B)$. Therefore the sequence $\{x_{2n}\}$ is bounded. Hence the sequence $\{x_n\}$ is also bounded.

The following result is a special case of Theorem (3.4).

Corollary 3.5. Let A and B be a nonempty closed subsets of a metric space (X, d) and $T: A \to B$ and $S: A \to B$ be maps. If the pair of maps T and S satisfies MT-K rational cyclic condition. If $x_0 \in A$, define $x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$, $n \in \mathbb{N}$ then the sequence $\{x_n\}$ is bounded.

Corollary 3.6. Let A and B be a nonempty closed subsets of a metric space. Suppose that the mappings $T: A \to B$ and $S: A \to B$ form a K-rational cyclic map between A and B. For a fixed element x_0 in A, let $x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$. Then the sequence $\{x_n\}$ is bounded.

Theorem 3.7. Let A and B be a nonempty subsets of a metric space (X, d) and $T: A \to B$ and $S: A \to B$ be maps. If there exists a non-decreasing function $\mu: [0,\infty) \to [0,\infty)$ and an MT-function $\varphi: [0,\infty) \to [0,1)$ such that

$$d(Tx, Sy) \leq \varphi\left(\mu(d(x,y))\right)\left[\frac{d(x,Tx)d(x,Sy) + d(y,Sy)d(y,Tx)}{d(x,Tx) + d(y,Sy)}\right] + [1-\varphi\left(\mu(d(x,y))\right)]d(A,B), \tag{5}$$

for all $x \in A$ and $y \in B$, then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = d(A,B).$$

Proof. Let $x_0 \in A$ be given. Define $x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$ for each $n \in \mathbb{N} \cup \{0\}$. Then $x_{2n} \in A$ and $x_{2n+1} \in B$ for each $n \in \mathbb{N} \cup \{0\}$. By (5), we have

$$\begin{aligned}
&d(x_1,x_2) = d(Tx_0,Sx_1)\\
&\leq \varphi\left(\mu\big(d(x_0,x_1)\big)\right)\left[\frac{d(x_0,Tx_0)d(x_0,Sx_1)+d(x_1,Sx_1)d(x_1,Tx_0)}{d(x_0,Sx_1)+d(x_1,Tx_0)}\right]\\
&+\left[1-\varphi\left(\mu\big(d(x_0,x_1)\big)\right)\right]d(A,B)\\
&\leq \varphi\left(\mu\big(d(x_0,x_1)\big)\right)\left[\frac{d(x_0,x_1)d(x_0,x_2)+d(x_1,x_2)d(x_1,x_1)}{d(x_0,x_2)+d(x_1,x_1)}\right]\\
&+\left[1-\varphi\left(\mu\big(d(x_0,x_1)\big)\right)\right]d(A,B)\\
&\leq \varphi\left(\mu\big(d(x_0,x_1)\big)\right)\left[\frac{d(x_0,x_1)d(x_0,x_2)+0}{d(x_0,x_2)}\right]\\
&+\left[1-\varphi\left(\mu\big(d(x_0,x_1)\big)\right)\right]d(A,B)\\
&\leq \varphi\left(\mu\big(d(x_0,x_1)\big)\right)d(x_0,x_1)+\left[1-\varphi\left(\mu\big(d(x_0,x_1)\big)\right)\right]d(A,B).
\end{aligned}$$

Since φ is an MT-function, then from Theorem (2.3),

$$0 \leq \sup_{n\in\mathbb{N}} \varphi\left(d(x_n,x_{n+1})\right) < 1.$$

So that we have $0 \leq \sup_{n\in\mathbb{N}} \varphi\left(\mu\big(d(x_0,x_1)\big)\right) < 1.$

Suppose $k = \sup_{n\in\mathbb{N}} \varphi\left(\mu\big(d(x_0,x_1)\big)\right) < 1.$

Therefore $0 \leq k < 1$, since $\varphi\left(\mu\big(d(x_0,x_1)\big)\right) \leq k$, we get

$$d(x_1,x_2) \leq kd(x_0,x_1)+(1-k)d(A,B). \tag{6}$$

Similarly, from inequality (5), we have

$$\begin{aligned}
&d(x_2,x_3) = d(Sx_1,Tx_2)\\
&\leq \varphi\left(\mu\big(d(x_1,x_2)\big)\right)\left[\frac{d(x_1,Sx_1)d(x_1,Tx_2)+d(x_2,Tx_2)d(x_2,Sx_1)}{d(x_1,Tx_2)+d(x_2,Sx_1)}\right]\\
&+\left[1-\varphi\left(\mu\big(d(x_1,x_2)\big)\right)\right]d(A,B)\\
&\leq \varphi\left(\mu\big(d(x_1,x_2)\big)\right)\left[\frac{d(x_1,x_2)d(x_1,x_3)+d(x_2,x_3)d(x_2,x_2)}{d(x_1,x_3)+d(x_2,x_2)}\right]\\
&+\left[1-\varphi\left(\mu\big(d(x_1,x_2)\big)\right)\right]d(A,B)\\
&\qquad\leq \varphi\left(\mu\big(d(x_1,x_2)\big)\right)\left[\frac{d(x_1,x_2)d(x_1,x_3)+0}{d(x_1,x_3)}\right]+\left[1-\varphi\left(\mu\big(d(x_1,x_2)\big)\right)\right]d(A,B)\\
&\qquad\leq \varphi\left(\mu\big(d(x_1,x_2)\big)\right)d(x_1,x_2)+\left[1-\varphi\left(\mu\big(d(x_1,x_2)\big)\right)\right]d(A,B),
\end{aligned}$$

which implies that,

$$d(x_2,x_3) \leq kd(x_1,x_2)+(1-k)d(A,B). \tag{7}$$

From (6) and (7), it follows that

$$\begin{aligned}
&d(x_2,x_3) \leq kd(x_1,x_2)+(1-k)d(A,B)\\
&\leq k[kd(x_0,x_1)+(1-k)d(A,B)]+(1-k)d(A,B)
\end{aligned}$$

$$\leq k^2 d(x_0,x_1) + (k-k^2)d(A,B) + (1-k)b(A,B)$$
$$\leq k^2 d(x_0,x_1) + (1-k^2)d(A,B).$$

Hence inductively, we have

$$d(x_n,x_{n+1}) \leq kd(x_n,x_{n-1}) + (1-k)d(A,B)$$
$$\leq k^2 d(x_{n-1},x_{n-2}) + (1-k^2)d(A,B)$$
$$\leq \ldots$$
$$\leq k^n d(x_0,x_1) + (1-k^n)d(A,B).$$

Since $0 \leq k < 1$, we obtain $\lim_{n\to\infty} k^n = 0$, that is

$$\lim_{n\to\infty} d\,(x_n,x_{n+1}) = d(A,B).$$

Corollary 3.8. Let A, B be nonempty subsets of a metric space $T: A \to B$ and $S: A \to B$. If the pair of maps T and S satisfies MT-C condition, then there exists a sequence $\{x_n\}$ in X such that
$$\lim_{n\to\infty} d\,(x_n,x_{n+1}) = d(A,B).$$

Corollary 3.9. Let A, B be nonempty subsets of a metric space. Suppose that the mapping $T: A \to B$ and $S: A \to B$ satisfy C-rational cyclic condition. For a fixed element x_0 in A, let
$x_{2n+1} = Tx_{2n}$ and $x_{2n} = Sx_{2n-1}$. Then $\mathrm{d}(x_n,x_{n+1}) \to d(A,B)$.

REFERENCES

[1] A. A. Eldred, W. A. Kirk and P. Veeramani, Proximal normal structure and relatively nonexpansive mappings, Studia Mathematica 171 (2005) 283-293.

[2] A. A. Eldered and P. Veeramani, Proximal pointwise contraction, Topology and its Aplications 156 (2009) 2942-2948.

[3] A. A. Eldred and P. Veeramani, Existence and convergence of best proximity points, J. Math. Anal. Appl. 323 (2006) 1001-1006.

[4] G.Petrushel, Cyclic representations and periodic points, Studia Uni. Babes-Bolyai Math. 50 (2005) 107-112.

[5] M.A. Al-Thafai and N. Shahzas, Convergence and existence for best proximity points, Nonlinear Analysis 70 (2009) 3665-3671.

[6] M.R. Yadav, A.K. Sharma, B.S. Thakur, Best proximity point theorems for MT-K and MT-C rational cyclic contractions in metric spaces, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering 7 (2013).

[7] S. Karpagam and S. Agrawal, Best proximity points for cyclic Meir-Keeler contraction maps, Nonlinear Analysis 74 (2011) 1040-1046.

[8] W. A. Kirk and P. S. Srinavasan and P. Veeramani, Fixed points for mapping satisfying cyclical contractive conditions, Fixed Point Theory 4 (2003) 79-89.

[9] W.S. Du, Some new results and generalizations in metric fixed point theory, Nonlinear Analysis, Theory, Methods and Applications 73 (2010) 1439-1446.

[10] W.S. Du, Nonlinear contractive conditions for coupled cone fixed point theorems, Fixed Point Theory Appl. 2010 (2010) 190606.

[11] W.S. Du, On coincidence point and fixed point theorems for nonlinear multivalued maps. Topol. Appl. 159 (2012) 49-56 .