Integration of certain Gimel-function with respect to their parameters

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ABSTRACT

The object of the present paper is to obtain some interesting results by integrating the multivariable Gimel-function with respect to its parameters. Such integrals are useful in the study of certain boundary value problems.

KEYWORDS : Multivariable Gimel-function, multiple integral contours, integration with respect to a parameter.

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1. Introduction and preliminaries.

Throughout this paper, let $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ be set of complex numbers, real numbers and positive integers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define a generalized transcendental function of several complex variables.

$$\beth(z_1, \cdots, z_r) = \beth^{0,n_2;0,n_3;\cdots;0,n_r:m^{(1)},n^{(1)};m^{(2)},n^{(2)};\cdots;m^{(r)},n^{(r)}}_{p_{i_2},q_{i_2},\tau_{i_2}:R_2;p_{i_3},q_{i_3},\tau_{i_3}:R_3;\cdots;p_{i_r},q_{i_r},\tau_{i_r}:R_r:p_{i^{(1)}},q_{i^{(1)}},\tau_{i^{(1)}};R^{(1)};\cdots;p_{i^{(r)}},q_{i^{(r)}},\tau_{i^{(r)}};R^{(r)}} \left(\begin{array}{c} z_1 \\ . \\ . \\ . \\ z_r \end{array} \right|$$

$$[(a_{2j}; \alpha^{(1)}_{2j}, \alpha^{(2)}_{2j}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha^{(1)}_{2ji_2}, \alpha^{(2)}_{2ji_2}; A_{2ji_2})]_{n_2+1,p_{i_2}}; [(a_{3j}; \alpha^{(1)}_{3j}, \alpha^{(2)}_{3j}, \alpha^{(3)}_{3j}; A_{3j})]_{1,n_3},$$
$$[\tau_{i_2}(b_{2ji_2}; \beta^{(1)}_{2ji_2}, \beta^{(2)}_{2ji_2}; B_{2ji_2})]_{1,q_{i_2}};$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha^{(1)}_{3ji_3}, \alpha^{(2)}_{3ji_3}, \alpha^{(3)}_{3ji_3}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots; [(a_{rj}; \alpha^{(1)}_{rj}, \cdots, \alpha^{(r)}_{rj}; A_{rj})_{1,n_r}],$$
$$[\tau_{i_3}(b_{3ji_3}; \beta^{(1)}_{3ji_3}, \beta^{(2)}_{3ji_3}, \beta^{(3)}_{3ji_3}; B_{3ji_3})]_{1,q_{i_3}}; \cdots; \qquad \dots$$

$$[\tau_{i_r}(a_{rji_r}; \alpha^{(1)}_{rji_r}, \cdots, \alpha^{(r)}_{rji_r}; A_{rji_r})_{n_r+1,p_r}]: [(c^{(1)}_j, \gamma^{(1)}_j; C^{(1)}_j)_{1,n^{(1)}}], [\tau_{i^{(1)}}(c^{(1)}_{ji^{(1)}}, \gamma^{(1)}_{ji^{(1)}}; C^{(1)}_{ji^{(1)}})_{n^{(1)}+1,p^{(1)}_i}]$$
$$[\tau_{i_r}(b_{rji_r}; \beta^{(1)}_{rji_r}, \cdots, \beta^{(r)}_{rji_r}; B_{rji_r})_{1,q_r}]: [(d^{(1)}_j), \delta^{(1)}_j; D^{(1)}_j)_{1,m^{(1)}}], [\tau_{i^{(1)}}(d^{(1)}_{ji^{(1)}}, \delta^{(1)}_{ji^{(1)}}; D^{(1)}_{ji^{(1)}})_{m^{(1)}+1,q^{(1)}_i}]$$

$$; \cdots; [(c^{(r)}_j, \gamma^{(r)}_j; C^{(r)}_j)_{1,n^{(r)}}], [\tau_{i^{(r)}}(c^{(r)}_{ji^{(r)}}, \gamma^{(r)}_{ji^{(r)}}; C^{(r)}_j)_{n^{(r)}+1,p^{(r)}_i}]$$
$$; \cdots; [(d^{(r)}_j, \delta^{(r)}_j; D^{(r)}_j)_{1,m^{(r)}}], [\tau_{i^{(r)}}(d^{(r)}_{ji^{(r)}}, \delta^{(r)}_{ji^{(r)}}; D^{(r)}_j)_{m^{(r)}+1,q^{(r)}_i}] \Bigg)$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \, \mathrm{d}s_1 \cdots \mathrm{d}s_r \tag{1.1}$$

with $\omega = \sqrt{-1}$

$$\psi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{n_2} \Gamma^{A_{2j}}(1 - a_{2j} + \sum_{k=1}^{2} \alpha^{(k)}_{2j} s_k)}{\sum_{i_2=1}^{R_2} [\tau_{i_2} \prod_{j=n_2+1}^{p_{i_2}} \Gamma^{A_{2ji_2}}(a_{2ji_2} - \sum_{k=1}^{2} \alpha^{(k)}_{2ji_2} s_k) \prod_{j=1}^{q_{i_2}} \Gamma^{B_{2ji_2}}(1 - b_{2ji_2} + \sum_{k=1}^{2} \beta^{(k)}_{2ji_2} s_k)]}$$

$$\frac{\prod_{j=1}^{n_3} \Gamma^{A_{3j}}(1 - a_{3j} + \sum_{k=1}^{3} \alpha_{3j}^{(k)} s_k)}{\sum_{i_3=1}^{R_3}[\tau_{i_3} \prod_{j=n_3+1}^{p_{i_3}} \Gamma^{A_{3ji_3}}(a_{3ji_3} - \sum_{k=1}^{3} \alpha_{3ji_3}^{(k)} s_k) \prod_{j=1}^{q_{i_3}} \Gamma^{B_{3ji_3}}(1 - b_{3ji3} + \sum_{k=1}^{3} \beta_{3ji3}^{(k)} s_k)]}$$

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$$\frac{\prod_{j=1}^{n_r} \Gamma^{A_{rj}}(1 - a_{rj} + \sum_{k=1}^{r} \alpha_{rj}^{(k)} s_k)}{\sum_{i_r=1}^{R_r}[\tau_{i_r} \prod_{j=n_r+1}^{p_{i_r}} \Gamma^{A_{rji_r}}(a_{rji_r} - \sum_{k=1}^{r} \alpha_{rji_r}^{(k)} s_k) \prod_{j=1}^{q_{i_r}} \Gamma^{B_{rji_r}}(1 - b_{rjir} + \sum_{k=1}^{r} \beta_{rjir}^{(k)} s_k)]} \tag{1.2}$$

and

$$\theta_k(s_k) = \frac{\prod_{j=1}^{m^{(k)}} \Gamma^{D_j^{(k)}}(d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=1}^{n^{(k)}} \Gamma^{C_j^{(k)}}(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i^{(k)}=1}^{R^{(k)}}[\tau_{i^{(k)}} \prod_{j=m^{(k)}+1}^{q_{i^{(k)}}} \Gamma^{D_{ji^{(k)}}^{(k)}}(1 - d_{ji^{(k)}}^{(k)} + \delta_{ji^{(k)}}^{(k)} s_k) \prod_{j=n^{(k)}+1}^{p_{i^{(k)}}} \Gamma^{C_{ji^{(k)}}^{(k)}}(c_{ji^{(k)}}^{(k)} - \gamma_{ji^{(k)}}^{(k)} s_k)]} \tag{1.3}$$

1) $[(c_j^{(1)}; \gamma_j^{(1)})]_{1,n_1}$ stands for $(c_1^{(1)}; \gamma_1^{(1)}), \cdots, (c_{n_1}^{(1)}; \gamma_{n_1}^{(1)})$.

2) $n_2, \cdots, n_r, m^{(1)}, n^{(1)}, \cdots, m^{(r)}, n^{(r)}, p_{i_2}, q_{i_2}, R_2, \tau_{i_2}, \cdots, p_{i_r}, q_{i_r}, R_r, \tau_{i_r}, p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}, R^{(r)} \in \mathbb{N}$ and verify :

$0 \leqslant m_2, 0 \leqslant n_2 \leqslant p_{i_2}, \cdots, 0 \leqslant m_r, 0 \leqslant n_r \leqslant p_{i_r}, 0 \leqslant m^{(1)} \leqslant q_{i^{(1)}}, \cdots, 0 \leqslant m^{(r)} \leqslant q_{i^{(r)}}$.

3) $\tau_{i_2}(i_2 = 1, \cdots, R_2) \in \mathbb{R}^+; \tau_{i_r} \in \mathbb{R}^+(i_r = 1, \cdots, R_r); \tau_{i^{(k)}} \in \mathbb{R}^+(i = 1, \cdots, R^{(k)}), (k = 1, \cdots, r)$.

4) $\gamma_j^{(k)}, C_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots, n_k); (k = 1, \cdots, r); \delta_j^{(k)}, D_j^{(k)} \in \mathbb{R}^+; (j = 1, \cdots, m_k); (k = 1, \cdots, r)$.

$\alpha_{kj}^{(l)}, A_{kj} \in \mathbb{R}^+; (j = 1, \cdots, n_k); (k = 2, \cdots, r); (l = 1, \cdots, k)$.

$\alpha_{kji_k}^{(l)}, A_{kji_k} \in \mathbb{R}^+; (j = n_k + 1, \cdots, p_{i_k}); (k = 2, \cdots, r); (l = 1, \cdots, k)$.

$\beta_{kji_k}^{(l)}, B_{kji_k} \in \mathbb{R}^+; (j = m_k + 1, \cdots, q_{i_k}); (k = 2, \cdots, r); (l = 1, \cdots, k)$.

$\delta_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r)$.

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{R}^+; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r)$.

5) $c_j^{(k)} \in \mathbb{C}; (j = 1, \cdots, n_k); (k = 1, \cdots, r); d_j^{(k)} \in \mathbb{C}; (j = 1, \cdots, m_k); (k = 1, \cdots, r)$.

$a_{kji_k} \in \mathbb{C}; (j = n_k + 1, \cdots, p_{i_k}); (k = 2, \cdots, r)$.

$b_{kji_k} \in \mathbb{C}; (j = m_k + 1, \cdots, q_{i_k}); (k = 2, \cdots, r)$.

$d_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = m^{(k)} + 1, \cdots, q_{i^{(k)}}); (k = 1, \cdots, r)$.

$\gamma_{ji^{(k)}}^{(k)} \in \mathbb{C}; (i = 1, \cdots, R^{(k)}); (j = n^{(k)} + 1, \cdots, p_{i^{(k)}}); (k = 1, \cdots, r)$.

The contour L_k is in the $s_k (k = 1, \cdots, r)$- plane and run from $\sigma - i\infty$ to $\sigma + i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}\left(1 - a_{2j} + \sum_{k=1}^{2} \alpha_{2j}^{(k)} s_k\right)(j = 1, \cdots, n_2), \Gamma^{A_{3j}}\left(1 - a_{3j} + \sum_{k=1}^{3} \alpha_{3j}^{(k)} s_k\right)$ $(j = 1, \cdots, n_3), \cdots, \Gamma^{A_{rj}}\left(1 - a_{rj} + \sum_{i=1}^{r} \alpha_{rj}^{(i)}\right)(j = 1, \cdots, n_r), \Gamma^{C_j^{(k)}}\left(1 - c_j^{(k)} + \gamma_j^{(k)} s_k\right)(j = 1, \cdots, n^{(k)})(k = 1, \cdots, r)$ to the right of the contour L_k and the poles of $\Gamma^{D_j^{(k)}}\left(d_j^{(k)} - \delta_j^{(k)} s_k\right)(j = 1, \cdots, m^{(k)})(k = 1, \cdots, r)$ lie to the left of the contour L_k. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.1) can be obtained of the corresponding conditions for multivariable H-function given by as :

$$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi \text{ where}$$

$$A_i^{(k)} = \sum_{j=1}^{m^{(k)}} D_j^{(k)} \delta_j^{(k)} + \sum_{j=1}^{n^{(k)}} C_j^{(k)} \gamma_j^{(k)} - \tau_{i^{(k)}}\left(\sum_{j=m^{(k)}+1}^{q_i^{(k)}} D_{ji^{(k)}}^{(k)} \delta_{ji^{(k)}}^{(k)} + \sum_{j=n^{(k)}+1}^{p_i^{(k)}} C_{ji^{(k)}}^{(k)} \gamma_{ji^{(k)}}^{(k)}\right) +$$

$$-\tau_{i_2}\left(\sum_{j=n_2+1}^{p_{i_2}} A_{2ji_2} \alpha_{2ji_2}^{(k)} + \sum_{j=1}^{q_{i_2}} B_{2ji_2} \beta_{2ji_2}^{(k)}\right) - \cdots - \tau_{i_r}\left(\sum_{j=n_r+1}^{p_{i_r}} A_{rji_r} \alpha_{rji_r}^{(k)} + \sum_{j=1}^{q_{i_r}} B_{rji_r} \beta_{rji_r}^{(k)}\right) \qquad (1.4)$$

Following the lines of Braaksma ([2] p. 278), we may establish the the asymptotic expansion in the following convenient form :

$$\aleph(z_1, \cdots, z_r) = 0(\ |z_1|^{\alpha_1}, \cdots, |z_r|^{\alpha_r}\ ), max(\ |z_1|, \cdots, |z_r|\ ) \to 0$$

$$\aleph(z_1, \cdots, z_r) = 0(\ |z_1|^{\beta_1}, \cdots, |z_r|^{\beta_r}\ ), min(\ |z_1|, \cdots, |z_r|\ ) \to \infty \text{ where } i = 1, \cdots, r :$$

$$\alpha_i = \min_{1 \leqslant j \leqslant m^{(i)}} Re\left[D_j^{(i)}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] \text{ and } \beta_i = \max_{1 \leqslant j \leqslant n^{(i)}} Re\left[C_j^{(i)}\left(\frac{c_j^{(i)} - 1}{\gamma_j^{(i)}}\right)\right]$$

Remark 1.
If $n_2 = \cdots = n_{r-1} = p_{i_2} = q_{i_2} = \cdots = p_{i_{r-1}} = q_{i_{r-1}} = 0$ and $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ $A_{rj} = A_{rji_r} = B_{rji_r} = 1$, then the multivariable Gimel-function reduces in the multivariable Aleph- function defined by Ayant [1].

Remark 2.
If $n_2 = \cdots = n_r = p_{i_2} = q_{i_2} = \cdots = p_{i_r} = q_{i_r} = 0$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the multivariable Gimel-function reduces in a multivariable I-function defined by Prathima et al. [4].

Remark 3.
If $A_{2j} = A_{2ji_2} = B_{2ji_2} = \cdots = A_{rj} = A_{rji_r} = B_{rji_r} = 1$ and $\tau_{i_2} = \cdots = \tau_{i_r} = \tau_{i^{(1)}} = \cdots = \tau_{i^{(r)}} = R_2 = \cdots = R_r = R^{(1)} = \cdots = R^{(r)} = 1$, then the generalized multivariable Gimel-function reduces in multivariable I-function defined by Prasad [3].

Remark 4.
If the three above conditions are satisfied at the same time, then the generalized multivariable Gimel-function reduces in the multivariable H-function defined by Srivastava and Panda [6,7].

In your investigation, we shall use the following notations.

$$\mathbb{A} = [(a_{2j}; \alpha_{2j}^{(1)}, \alpha_{2j}^{(2)}; A_{2j})]_{1,n_2}, [\tau_{i_2}(a_{2ji_2}; \alpha_{2ji_2}^{(1)}, \alpha_{2ji_2}^{(2)}; A_{2ji_2})]_{n_2+1,p_{i_2}}, [(a_{3j}; \alpha_{3j}^{(1)}, \alpha_{3j}^{(2)}, \alpha_{3j}^{(3)}; A_{3j})]_{1,n_3},$$

$$[\tau_{i_3}(a_{3ji_3}; \alpha_{3ji_3}^{(1)}, \alpha_{3ji_3}^{(2)}, \alpha_{3ji_3}^{(3)}; A_{3ji_3})]_{n_3+1,p_{i_3}}; \cdots ; [(\mathrm{a}_{(r-1)j}; \alpha_{(r-1)j}^{(1)}, \cdots, \alpha_{(r-1)j}^{(r-1)}; A_{(r-1)j})_{1,n_{r-1}}],$$

$$[\tau_{i_{r-1}}(a_{(r-1)ji_{r-1}}; \alpha_{(r-1)ji_{r-1}}^{(1)}, \cdots, \alpha_{(r-1)ji_{r-1}}^{(r-1)}; A_{(r-1)ji_{r-1}})_{n_{r-1}+1,p_{i_{r-1}}}] \tag{1.5}$$

$$\mathbf{A} = [(\mathrm{a}_{rj}; \alpha_{rj}^{(1)}, \cdots, \alpha_{rj}^{(r)}; A_{rj})_{1,n_r}], [\tau_{i_r}(a_{rji_r}; \alpha_{rji_r}^{(1)}, \cdots, \alpha_{rji_r}^{(r)}; A_{rji_r})_{\mathfrak{n}+1,p_{i_r}}] \tag{1.6}$$

$$A = [(c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,n^{(1)}}], [\tau_{i^{(1)}}(c_{ji^{(1)}}^{(1)}, \gamma_{ji^{(1)}}^{(1)}; C_{ji^{(1)}}^{(1)})_{n^{(1)}+1,p_i^{(1)}}]; \cdots ;$$

$$[(c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,n^{(r)}}], [\tau_{i^{(r)}}(c_{ji^{(r)}}^{(r)}, \gamma_{ji^{(r)}}^{(r)}; C_j^{(r)})_{n^{(r)}+1,p_i^{(r)}}] \tag{1.7}$$

$$\mathbb{B} = [\tau_{i_2}(b_{2ji_2}; \beta_{2ji_2}^{(1)}, \beta_{2ji_2}^{(2)}; B_{2ji_2})]_{1,q_{i_2}}, [\tau_{i_3}(b_{3ji_3}; \beta_{3ji_3}^{(1)}, \beta_{3ji_3}^{(2)}, \beta_{3ji_3}^{(3)}; B_{3ji_3})]_{1,q_{i_3}}; \cdots ;$$

$$[\tau_{i_{r-1}}(b_{(r-1)ji_{r-1}}; \beta_{(r-1)ji_{r-1}}^{(1)}, \cdots, \beta_{(r-1)ji_{r-1}}^{(r-1)}; B_{(r-1)ji_{r-1}})_{1,q_{i_{r-1}}}] \tag{1.8}$$

$$\mathbf{B} = [\tau_{i_r}(b_{rji_r}; \beta_{rji_r}^{(1)}, \cdots, \beta_{rji_r}^{(r)}; B_{rji_r})_{1,q_{i_r}}] \tag{1.9}$$

$$\mathrm{B} = [(\mathrm{d}_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,m^{(1)}}], [\tau_{i^{(1)}}(d_{ji^{(1)}}^{(1)}, \delta_{ji^{(1)}}^{(1)}; D_{ji^{(1)}}^{(1)})_{m^{(1)}+1,q_i^{(1)}}]; \cdots ;$$

$$[(\mathrm{d}_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,m^{(r)}}], [\tau_{i^{(r)}}(d_{ji^{(r)}}^{(r)}, \delta_{ji^{(r)}}^{(r)}; D_j^{(r)})_{m^{(r)}+1,q_i^{(r)}}] \tag{1.10}$$

$$U = 0, n_2; 0, n_3; \cdots ; 0, n_{r-1}; V = m^{(1)}, n^{(1)}; m^{(2)}, n^{(2)}; \cdots ; m^{(r)}, n^{(r)} \tag{1.11}$$

$$X = p_{i_2}, q_{i_2}, \tau_{i_2}, R_2; \cdots ; p_{i_{r-1}}, q_{i_{r-1}}, \tau_{i_{r-1}} : R_{r-1}; Y = p_{i^{(1)}}, q_{i^{(1)}}, \tau_{i^{(1)}}; R^{(1)}; \cdots ; p_{i^{(r)}}, q_{i^{(r)}}, \tau_{i^{(r)}}; R^{(r)} \tag{1.12}$$

2. Main integrals.

In this section, we evaluate three integrals with respect to their parameters involving the generalized multivariable Gimel-function.

Theorem 1.

$$\frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \Gamma(a+x)\Gamma(b-x)\Gamma(c-x)e^{\pm\omega\pi x} \daleth_{X;p_{i_r}+1,q_{i_r},\tau_{i_r}:R_r:Y}^{U;0,n_r+1:V} \left(\begin{array}{c} z_1 \\ . \\ . \\ z_r \end{array} \middle| \begin{array}{c} \mathbb{A}; \mathbf{A}: A \\ . \\ . \\ \mathbb{B}; \mathbf{B}, (\text{1-d+x};\mathrm{h}_1,\cdots,h_r;1):B \end{array} \right) \mathrm{d}x =$$

$$e^{\pm\omega\pi a}\Gamma(a+b)\Gamma(a+c) \daleth_{X;p_{i_r}+1,q_{i_r}+2,\tau_{i_r}:R_r:Y}^{U;0,n_r+1:V} \left(\begin{array}{c} z_1 \\ . \\ . \\ z_r \end{array} \middle| \begin{array}{c} \mathbb{A}; (\text{1+a+b+c-d};\mathrm{h}_1,\cdots,h_r;1), \mathbf{A}: A \\ . \\ . \\ \mathbb{B}; \mathbf{B},(\text{1+b-d};\mathrm{h}_1,\cdots,h_r;1),(1+c-d;h_1,\cdots,h_r;1):B \end{array} \right) \tag{2.1}$$

provided

$$h_i > 0 (i = 1, \cdots, r), Re(d - a - b - c) + \sum_{i=1}^{r} h_i \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[D_j^{(i)} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0.$$

$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi$ where $A_i^{(k)}$ is defined by (1.4).

Proof

To prove the theorem 1, we replace the multivariable Gimel-function by this multiple integrals contour with the help of (1.1), change the order of integrations which is justified under the conditions mentioned above. We get

$$\frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \left[\frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \frac{\Gamma(a+x)\Gamma(c-x)\Gamma(b-x)}{\Gamma(d-x+\sum_{i=1}^{r} h_i s_i - x)} e^{\pm\omega\pi x} \mathrm{d}x \right] \mathrm{d}s_1 \cdots \mathrm{d}s_r \quad (2.2)$$

Now, we evaluate the inner integral using by a integral formula ([8], p. 289) and interpreting the resulting expression with the help of (1.1), we obtain the desired result.

Theorem 2.

$$\frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \Gamma(b-x)\Gamma(a+x) e^{\pm\omega\pi x} \beth_{X;p_{i_r}+1,q_{i_r}+1,\tau_{i_r}:R_r:Y}^{U;0,n_r+1:V} \left(\begin{array}{c|c} z_1 & \mathbb{A}; \text{(1-c+x;h}_1,\cdots,h_r;1), \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B}; \mathbf{B}, \text{(1-d+x;k}_1,\cdots,k_r;1) : B \end{array} \right) \mathrm{d}x = \Gamma(a+b)$$

$$e^{\pm\omega\pi a} \beth_{X;p_{i_r}+2,q_{i_r}+2,\tau_{i_r}:R_r:Y}^{U;0,n_r+2:V} \left(\begin{array}{c|c} z_1 & \mathbb{A}; \text{(1-a-c;h}_1,\cdots,h_r;1),(1+a+b+c-d;k_1-h_1,\cdots,k_r-h_r;1), \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B}; \mathbf{B},\text{(1+b-d;k}_1,\cdots,k_r;1),(1+c-d;k_1-h_1,\cdots,k_r-h_r;1) : B \end{array} \right) \quad (2.3)$$

provided

$$h_i, k_i, k_i - h_i > 0 (i = 1, \cdots, r), Re(d - a - b - c) + \sum_{i=1}^{r} (k_i - h_i) \min_{1 \leqslant j \leqslant m^{(i)}} Re \left[D_j^{(i)} \left(\frac{d_j^{(i)}}{\delta_j^{(i)}} \right) \right] > 0.$$

$|arg(z_k)| < \frac{1}{2} A_i^{(k)} \pi$ where $A_i^{(k)}$ is defined by (1.4).

Theorem 3.

$$\frac{1}{2\pi\omega} \int_{-\omega\infty}^{\omega\infty} \frac{\Gamma(a+x)\Gamma(b-x)}{\Gamma(c-x)} e^{\pm\omega\pi x} \beth_{X;p_{i_r}+1,q_{i_r},\tau_{i_r}:R_r:Y}^{U;0,n_r+1:V} \left(\begin{array}{c|c} z_1 & \mathbb{A}; \text{(1-d+x;h}_1,\cdots,h_r;1), \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & \mathbb{B}; \mathbf{B}: B \end{array} \right) \mathrm{d}x =$$

$$\frac{\Gamma(a+b)\Gamma(c-a-b-d)}{\Gamma(c-b)} e^{\pm\omega\pi a} \beth_{X;p_{i_r}+2,q_{i_r}+1,\tau_{i_r}:R_r:Y}^{U;0,n_r+1:V} \left(\begin{array}{c|c} (\text{-z}_1)^{h_1} & \mathbb{A}; \text{(1-a-d;h}_1,\cdots,h_r;1), \mathbf{A} : A \\ \cdot & \cdot \\ \cdot & \cdot \\ (\text{-z}_r)^{h_r} & \mathbb{B}; \mathbf{B}: B \end{array} \right) \quad (2.4)$$

provided

$$h_i > 0 (i = 1, \cdots, r), Re(c - a - b - d) + \sum_{i=1}^{r} h_i \max_{1 \leqslant j \leqslant n^{(i)}} Re \left[C_j^{(i)} \left(\frac{c_j^{(i)} - 1}{\gamma_j^{(i)}} \right) \right] > 0.$$

$|arg(z_k)| < \dfrac{1}{2} A_i^{(k)} \pi$ where $A_i^{(k)}$ is defined by (1.4).

To prove the formulae (3.3) and (3.4), we use the similarly process.

4. Conclusion.

The Gimel-function of several variables presented in this paper, are quite basic in nature. Therefore , on specializing the parameters of this function, we may obtain various known and (news) integrals with respect to parameters concerning the special functions of one variable and several variables, For example, Singh et al. [5] have studied these integrals about the multivariable H-function and they have given several particular cases.

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