On solving a Fuzzy Optimal Subdivision Problem using Fuzzy Dynamic Programming

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ABSTRACT-*The mathematical technique of optimizing a sequence of interrelated decisions over a period of time is called Dynamic Programming. In many real life situations the decision making process consists of selecting a combination of plans from a large number of alternative combinations in fuzzy environments. In this paper, a new fuzzy approach is proposed to solve a fuzzy optimal subdivision problem where the positive quantity which is to be divided is taken as trapezoidal fuzzy number. The solution is obtained by the method of Mathematical Induction. The specific feature of this proposed approach is that the vagueness and imprecision in the optimal subdivision models is easily eliminated by fuzzy dynamic programming.*

Key words - *Fuzzy dynamic Programming, Trapezoidal Fuzzy numbers, Mathematical Induction*

I.INTRODUCTION

Many real life situations that arises in this world are dealt by the dynamic programming technique by dividing the given problem in to sub problems or stages. Only one stage is considered at a time and the various infeasible combinations are eliminated with the objective of reducing the volume of computations. The solution is obtained by moving from one stage to the next and is completed when the final stage is reached. It has been applied in many fields such as production, scheduling, inventory, salesman allocation, advertising media, equipment replacement, Markovian decision models, probabilistic decision problems etc.,

In the current scenario, multi stage decision making persist in all our day to day activities. DP is used to deal those problems. But many factors such as vagueness and uncertainty prevail in the field of MCDM problems. Zadeh[10] introduced the concept of fuzzy theory in the year 1960. Later on, it has wide applications in all the fields. In 1970, Bellman and Zadeh[3] applied the concept of the fuzzy set theory in decision making process. The process of applying fuzzy set theory in the field of Dynamic Programming is called Fuzzy Dynamic Programing. Many researchers contributed their work to the field of FDP.

Baldwin and Pilsworth[2] applied the concepts of DP for fuzzy systems with fuzzy environment. Chang[4] also made use of FDP in the decision making process. Kacprzyk and Esogbue[7] discussed the main developments and applications of FDP in many fields. Later on, Li and Lai[8] shared the FDP approach to hybrid multi objective multistage decision making problems. Abo-Sinna[1] contributed some applications of multiple objective fuzzy dynamic programming problems. Yuan and Wu[9] created an algorithm of FDP in AGV scheduling. Many such researchers contributed their enormous work to the literature[5,6] of FDP.

This paper is proposed to find an optimal solution to a fuzzy optimal subdivision problem with uncertain and vague parameters. More specifically, the positive quantity $\tilde{b}$ which is to be factorized into 'n' factors such that their sum is minimum is taken as Trapezoidal Fuzzy Number. It is solved by the method of Mathematical Induction.

II. DEFINITION AND PRELIMINARIES

A.*Fuzzy set*

Let X denotes a universal set. Then, the membership function μ_A by which a fuzzy set A is usually defined has the form $\mu_A: X \rightarrow [0,1]$, where [0,1] denotes the interval of real numbers from 0 to 1, both inclusive.

B.*Trapezoidal Fuzzy Number:*

A Trapezoidal fuzzy number (TrFN) denoted by $\tilde{A}$ is defined as (p_1, q_1, r_1, s_1) where the membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le p_1 \\ \frac{x-p_1}{q_1-p_1}, & p_1 \le x \le q_1 \\ 1, & q_1 \le x \le r_1 \\ \frac{s_1-x}{s_1-r_1}, & r_1 \le x \le s_1 \\ 0, & x \ge s_1 \end{cases}$$

C.*Arithmetic Operations of Trapezoidal Fuzzy Numbers:*

Let $\tilde{A} = (p_1, q_1, r_1, s_1)$ and $\tilde{B} = (p_2, q_2, r_2, s_2)$ be two trapezoidal fuzzy numbers. Then,

(i) $\tilde{A} + \tilde{B} = (p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2)$

(ii) $\tilde{A} - \tilde{B} = (p_1 - s_2, q_1 - r_2, r_1 - q_2, s_1 - p_2)$

(iii) $k\tilde{A} = (kp_1, kq_1, kr_1, ks_1)\ for\ k \ge 0$

(iv) $k\tilde{A} = (ks_1, kr_1, kq_1, kp_1)\ for\ k < 0$

(v) $\tilde{A} * \tilde{B} = (t_1, t_2, t_3, t_4)$

where$t_1 = minimum\{p_1p_2, p_1s_2, s_1p_2, s_1s_2\}$

$$t_2 = minimum\{q_1q_2, q_1r_2, r_1q_2, r_1r_2\}$$

$$t_3 = maximum\{q_1q_2, q_1r_2, r_1q_2, r_1r_2\}$$

$$t_4 = maximum\{p_1p_2, p_1s_2, s_1p_2, s_1s_2\}$$

III. DYNAMIC PROGRAMMING

A.*Optimal Subdivision Problem*

The Dynamic Programming technique is used to solve optimal subdivision problems. For example, a positive quantity can be divided into some sub-quantities whose sum of cubes or squared sums or product may be maximum or minimum. In the same way, a known product can be split into a certain number of factors where the squared sum or the sum of factors may be minimum or maximum. Hence a wide range of generalized problems can be solved and made as templates to enable the researchers to apply them to specific problems.

B.*Fuzzy Optimal Subdivision Problem*

Consider the following optimal subdivision problem. The objective of this paper is to factorize a positive quantity $\tilde{b}$ which is taken as Trapezoidal Fuzzy Number into 'n' factors such that their sum is minimum. Its Linear Programming Problem model is given by

Min$\tilde{z} = \tilde{y}_1 + \tilde{y}_2 + \tilde{y}_3 + \cdots + \tilde{y}_n$

such that $\tilde{y}_1\tilde{y}_2\tilde{y}_3 \ldots \tilde{y}_n = \tilde{b}$

and$\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \dots \tilde{y}_n \geq 0$

First we shall develop a recursive equation connecting the optimal decision function for the n-stage problem with the optimal decision function for the (n-1) stage sub-problem. This problem comprises of factorizing a positive quantity $\tilde{b}$ into 'n' factors such that the sum of the factors is minimum. So, it can be treated as an n-stage fuzzy dynamic programming problem. Let $\tilde{y}_i$ be the i^{th} part of $\tilde{b}$ and each i may be regarded as a stage. Since $\tilde{y}_i$ assume any non-negative value which satisfies$\tilde{y}_1\tilde{y}_2\tilde{y}_3 \dots \tilde{y}_n = \tilde{b}$, the alternative at each stage are infinite. This means $\tilde{y}_i$ is continuous. Therefore, the optimal decisions at each stage are obtained by using usual classical method of differentiation.

Let $\tilde{f}_n(\tilde{b})$ be the minimum attainable sum $\tilde{y}_1 + \tilde{y}_2 + \tilde{y}_3 + \cdots + \tilde{y}_n$when the positive quantity '$\tilde{b}$' is factorized into 'n' factors$\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \dots \tilde{y}_n$. Let $\tilde{b} = (b_1, b_2, b_3, b_4)$ where $b_1 < b_2 < b_3 < b_4$ and $\tilde{x} = (x, x, x, x)$

Stage 1:

For $n = 1$

$\tilde{y}_1 = \tilde{b}$ i.e., $\tilde{y}_1 = (b_1, b_2, b_3, b_4)$ is the only factor. Then,

$\tilde{f}_1(\tilde{b}) = \min_{\tilde{y}_1 = \tilde{b}} (b_1, b_2, b_3, b_4)$

$\Rightarrow \tilde{f}_1(\tilde{b}) = (b_1, b_2, b_3, b_4)$

$\Rightarrow \tilde{f}_1(\tilde{b}) = \tilde{b}$---[3.2.1]

This is a trivial case.

Stage 2:

For $n = 2$

We take $\tilde{y}_1 = (x, x, x, x)$ and $\tilde{y}_2 = \left(\frac{b_1}{x}, \frac{b_2}{x}, \frac{b_3}{x}, \frac{b_4}{x}\right)$

Here, $\tilde{b}$is factorised into two factors $\tilde{y}_1$and $\tilde{y}_2$ such that $\tilde{y}_1\tilde{y}_2 = \tilde{b}$

The above statement is proved in the following manner.

$\tilde{y}_1\tilde{y}_2 = (x, x, x, x)\left(\frac{b_1}{x}, \frac{b_2}{x}, \frac{b_3}{x}, \frac{b_4}{x}\right)$

$\Rightarrow \tilde{y}_1\tilde{y}_2 = (t_1, t_2, t_3, t_4)$

By the Property of Trapezoidal Fuzzy Numbers, we have

Consider t_1 =min $\left\{x\frac{b_1}{x}, x\frac{b_4}{x}, x\frac{b_1}{x}, x\frac{b_4}{x}\right\}$ = min(b_1, b_4, b_1, b_4) = b_1

Similarly,

t_2 =min$\left\{x\frac{b_2}{x}, x\frac{b_3}{x}, x\frac{b_2}{x}, x\frac{b_3}{x}\right\}$ = min(b_2, b_3, b_2, b_3) = b_2

t_3 =max$\left\{x\frac{b_2}{x}, x\frac{b_3}{x}, x\frac{b_2}{x}, x\frac{b_3}{x}\right\}$ = max(b_2, b_3, b_2, b_3) = b_3

t_4 =max$\left\{x\frac{b_1}{x}, x\frac{b_4}{x}, x\frac{b_1}{x}, x\frac{b_4}{x}\right\}$ = max(b_1, b_4, b_1, b_4) = b_4

Therefore , we have

$\tilde{y}_1\tilde{y}_2 = (t_1, t_2, t_3, t_4) = (b_1, b_2, b_3, b_4) \Rightarrow \tilde{y}_1\tilde{y}_2 = \tilde{b}$

$\tilde{f}_2(\tilde{b}) = \min\limits_{0\le\tilde{x}\le\tilde{b}}\{\tilde{y}_1 + \tilde{y}_2\}$

$\Rightarrow \tilde{f}_2(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{(x,x,x,x) + \left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)\right\}$ [From 3.2.1]

$\Rightarrow \tilde{f}_2(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{(x,x,x,x) + \tilde{f}_1\left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)\right\}$--[3.2.2]

Stage 3:

For $n = 3$

We take $\tilde{y}_1 = (x,x,x,x)$ and $\tilde{y}_2\tilde{y}_3 = \left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)$

Now, $\tilde{b}$must be factorized into three factors $\tilde{y}_1, \tilde{y}_2$and $\tilde{y}_3$ such that $\tilde{y}_1\tilde{y}_2\tilde{y}_3 = \tilde{b}$

i.e., $\left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)$ is further factorized into two factors whose minimum attainable sum is $\tilde{f}_2\left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)$.

Then $\tilde{f}_3(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\{\tilde{y}_1 + \tilde{y}_2 + \tilde{y}_3\}$

$\Rightarrow \tilde{f}_3(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{(x,x,x,x) + \tilde{f}_2\left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)\right\}$ [From 3.2.2]

Stage n:

In general, the recursive equation for the n-stage problem is

$\tilde{f}_n(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{(x,x,x,x) + \tilde{f}_{n-1}\left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)\right\}$---[3.2.3]

The above recursive equation is solved by the method of Mathematical Induction.

For $n = 2$, Equation [3.2.3] becomes

$$\tilde{f}_2(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{(x,x,x,x) + \tilde{f}_1\left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)\right\}$$

$\Rightarrow \tilde{f}_2(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{(x,x,x,x) + \left(\frac{b_1}{x},\frac{b_2}{x},\frac{b_3}{x},\frac{b_4}{x}\right)\right\}$ [From 3.2.1]

$$\Rightarrow \tilde{f}_2(\tilde{b}) = \min_{0\le\tilde{x}\le\tilde{b}}\left\{x + \frac{b_1}{x}, x + \frac{b_2}{x}, x + \frac{b_3}{x}, x + \frac{b_4}{x}\right\}$$

Let $h_1 = x + \frac{b_1}{x}$. To find maximum or minimum value, we take $\frac{dh_1}{dx} = 0$ to get the solution as $x = \sqrt{b_1}$

Similarly if we take h_2, h_3, h_4 as $x+\frac{b_2}{x}$, $x+\frac{b_3}{x}$, $x+\frac{b_4}{x}$ respectively, then we have the solutions as $x=\sqrt{b_2}$, $x=\sqrt{b_3}, x=\sqrt{b_4}$

This function $\left\{x+\frac{b_1}{x}, x+\frac{b_2}{x}, x+\frac{b_3}{x}, x+\frac{b_4}{x}\right\}$ will attain its minimum when $\tilde{x}=\left(\sqrt{b_1}, \sqrt{b_2}, \sqrt{b_3}, \sqrt{b_4}\right)$

Therefore, $\tilde{f}_2(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x)+\left(\frac{b_1}{x}, \frac{b_2}{x}, \frac{b_3}{x}, \frac{b_4}{x}\right)\right\}$

$$\Rightarrow \tilde{f}_2(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{\left(\sqrt{b_1}, \sqrt{b_2}, \sqrt{b_3}, \sqrt{b_4}\right)+\left(\frac{b_1}{\sqrt{b_1}}, \frac{b_2}{\sqrt{b_2}}, \frac{b_3}{\sqrt{b_3}}, \frac{b_4}{\sqrt{b_4}}\right)\right\}$$

$$\Rightarrow \tilde{f}_2(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{\sqrt{b_1}+\frac{b_1}{\sqrt{b_1}}, \sqrt{b_2}+\frac{b_2}{\sqrt{b_2}}, \sqrt{b_3}+\frac{b_3}{\sqrt{b_3}}, \sqrt{b_4}+\frac{b_4}{\sqrt{b_4}}\right\}$$

$$\Rightarrow \tilde{f}_2(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{2\sqrt{b_1}, 2\sqrt{b_2}, 2\sqrt{b_3}, 2\sqrt{b_4}\right\}$$

$$\Rightarrow \tilde{f}_2(\tilde{b})=2\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{\sqrt{b_1}, \sqrt{b_2}, \sqrt{b_3}, \sqrt{b_4}\right\}$$

$\Rightarrow \tilde{f}_2(\tilde{b})=2\left(b_1^{\frac{1}{2}}, b_2^{\frac{1}{2}}, b_3^{\frac{1}{2}}, b_4^{\frac{1}{2}}\right)$

The optimal policy is given by

$\left[\left(b_1^{\frac{1}{2}}, b_2^{\frac{1}{2}}, b_3^{\frac{1}{2}}, b_4^{\frac{1}{2}}\right), \left(b_1^{\frac{1}{2}}, b_2^{\frac{1}{2}}, b_3^{\frac{1}{2}}, b_4^{\frac{1}{2}}\right)\right]$ and $\tilde{f}_2(\tilde{b})=2\left(b_1^{\frac{1}{2}}, b_2^{\frac{1}{2}}, b_3^{\frac{1}{2}}, b_4^{\frac{1}{2}}\right)$

For $n=3$, Equation [3.2.3] becomes

$$\tilde{f}_3(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x)+\tilde{f}_2\left(\frac{b_1}{x}, \frac{b_2}{x}, \frac{b_3}{x}, \frac{b_4}{x}\right)\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x)+2\left(\left(\frac{b_1}{x}\right)^{\frac{1}{2}}, \left(\frac{b_2}{x}\right)^{\frac{1}{2}}, \left(\frac{b_3}{x}\right)^{\frac{1}{2}}, \left(\frac{b_4}{x}\right)^{\frac{1}{2}}\right)\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x)+\left(2\left(\frac{b_1}{x}\right)^{\frac{1}{2}}, 2\left(\frac{b_2}{x}\right)^{\frac{1}{2}}, 2\left(\frac{b_3}{x}\right)^{\frac{1}{2}}, 2\left(\frac{b_4}{x}\right)^{\frac{1}{2}}\right)\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b})=\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{x+2\left(\frac{b_1}{x}\right)^{\frac{1}{2}}, x+2\left(\frac{b_2}{x}\right)^{\frac{1}{2}}, x+2\left(\frac{b_3}{x}\right)^{\frac{1}{2}}, x+2\left(\frac{b_4}{x}\right)^{\frac{1}{2}}\right\}$$

Let $h_1 = x+2\left(\frac{b_1}{x}\right)^{\frac{1}{2}}$ To find maximum or minimum value, we take $\frac{dh_1}{dx}=0$ to get the solution as $x=\sqrt[3]{b_1}$

Similarly if we take h_2, h_3, h_4 as $x+2\left(\frac{b_2}{x}\right)^{\frac{1}{2}}$, $x+2\left(\frac{b_3}{x}\right)^{\frac{1}{2}}$, $x+2\left(\frac{b_4}{x}\right)^{\frac{1}{2}}$ respectively, then we have the solutions as $x=\sqrt[3]{b_2}, x=\sqrt[3]{b_3}, x=\sqrt[3]{b_4}$

The function will attain its minimum when $\tilde{x} = \left(b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right)$

Therefore, $\tilde{f}_3(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x) + 2\left(\left(\frac{b_1}{x}\right)^{\frac{1}{2}}, \left(\frac{b_2}{x}\right)^{\frac{1}{2}}, \left(\frac{b_3}{x}\right)^{\frac{1}{2}}, \left(\frac{b_4}{x}\right)^{\frac{1}{2}}\right)\right\}$

$$\Rightarrow \tilde{f}_3(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{b_1^{\frac{1}{3}} + 2\left(\frac{b_1}{b_1^{\frac{1}{3}}}\right)^{\frac{1}{2}}, b_2^{\frac{1}{3}} + 2\left(\frac{b_2}{b_2^{\frac{1}{3}}}\right)^{\frac{1}{2}}, b_3^{\frac{1}{3}} + 2\left(\frac{b_3}{b_3^{\frac{1}{3}}}\right)^{\frac{1}{2}}, b_4^{\frac{1}{3}} + 2\left(\frac{b_4}{b_4^{\frac{1}{3}}}\right)^{\frac{1}{2}}\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{b_1^{\frac{1}{3}} + 2\left(b_1^{\frac{2}{3}}\right)^{\frac{1}{2}}, b_2^{\frac{1}{3}} + 2\left(b_2^{\frac{2}{3}}\right)^{\frac{1}{2}}, b_3^{\frac{1}{3}} + 2\left(b_3^{\frac{2}{3}}\right)^{\frac{1}{2}}, b_4^{\frac{1}{3}} + 2\left(b_4^{\frac{2}{3}}\right)^{\frac{1}{2}}\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{3b_1^{\frac{1}{3}}, 3b_2^{\frac{1}{3}}, 3b_3^{\frac{1}{3}}, 3b_4^{\frac{1}{3}}\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b}) = 3\min_{0\leq\tilde{x}\leq\tilde{b}}\left\{b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right\}$$

$$\Rightarrow \tilde{f}_3(\tilde{b}) = 3\left\{b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right\}$$

Therefore, the optimal policy is given by

$\left[\left(b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right), \left(b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right), \left(b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right)\right]$ and $\tilde{f}_3(\tilde{b}) = 3\left(b_1^{\frac{1}{3}}, b_2^{\frac{1}{3}}, b_3^{\frac{1}{3}}, b_4^{\frac{1}{3}}\right)$

Let us assume that the optimal policy for $n = m$ is

$\left[\left(b_1^{\frac{1}{m}}, b_2^{\frac{1}{m}}, b_3^{\frac{1}{m}}, b_4^{\frac{1}{m}}\right), \left(b_1^{\frac{1}{m}}, b_2^{\frac{1}{m}}, b_3^{\frac{1}{m}}, b_4^{\frac{1}{m}}\right), \ldots \left(b_1^{\frac{1}{m}}, b_2^{\frac{1}{m}}, b_3^{\frac{1}{m}}, b_4^{\frac{1}{m}}\right)\right]$ and

$$\tilde{f}_m(\tilde{b}) = m\left(b_1^{\frac{1}{m}}, b_2^{\frac{1}{m}}, b_3^{\frac{1}{m}}, b_4^{\frac{1}{m}}\right)$$

For $n = m + 1$, Equation [3.2.3] becomes

$$\tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x) + \tilde{f}_m\left(\frac{b_1}{x}, \frac{b_2}{x}, \frac{b_3}{x}, \frac{b_4}{x}\right)\right\}$$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x) + m\left(\left(\frac{b_1}{x}\right)^{\frac{1}{m}}, \left(\frac{b_2}{x}\right)^{\frac{1}{m}}, \left(\frac{b_3}{x}\right)^{\frac{1}{m}}, \left(\frac{b_4}{x}\right)^{\frac{1}{m}}\right)\right\}$$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(x,x,x,x) + \left(m\left(\frac{b_1}{x}\right)^{\frac{1}{m}}, m\left(\frac{b_2}{x}\right)^{\frac{1}{m}}, m\left(\frac{b_3}{x}\right)^{\frac{1}{m}}, m\left(\frac{b_4}{x}\right)^{\frac{1}{m}}\right)\right\}$$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{x + m\left(\frac{b_1}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_2}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_3}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_4}{x}\right)^{\frac{1}{m}}\right\}$$

Let $h_1 = x + m\left(\frac{b_1}{x}\right)^{\frac{1}{m}}$ To find maximum or minimum value, we take $\frac{dh_1}{dx} = 0$ to get the solution as $x = \sqrt[m+1]{b_1}$

Similarly if we take h_2, h_3, h_4 as $x + m\left(\frac{b_2}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_3}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_4}{x}\right)^{\frac{1}{m}}$ respectively, then we have the solutions as $x = \sqrt[m+1]{b_2}, x = \sqrt[m+1]{b_3}, x = \sqrt[m+1]{b_4}$

The function will attain its minimum when $\tilde{x} = \left(b_1^{\frac{1}{m+1}}, b_2^{\frac{1}{m+1}}, b_3^{\frac{1}{m+1}}, b_4^{\frac{1}{m+1}}\right)$

Therefore, $\tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{x + m\left(\frac{b_1}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_2}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_3}{x}\right)^{\frac{1}{m}}, x + m\left(\frac{b_4}{x}\right)^{\frac{1}{m}}\right\}$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{b_1^{\frac{1}{m+1}} + m\left(\frac{b_1}{b_1^{\frac{1}{m+1}}}\right)^{\frac{1}{m}}, b_2^{\frac{1}{m+1}} + m\left(\frac{b_2}{b_2^{\frac{1}{m+1}}}\right)^{\frac{1}{m}}, b_3^{\frac{1}{m+1}} + m\left(\frac{b_3}{b_3^{\frac{1}{m+1}}}\right)^{\frac{1}{m}}, b_4^{\frac{1}{m+1}} + m\left(\frac{b_4}{b_4^{\frac{1}{m+1}}}\right)^{\frac{1}{m}}\right\}$$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{b_1^{\frac{1}{m+1}} + m\left(b_1^{\frac{m}{m+1}}\right)^{\frac{1}{m}}, b_2^{\frac{1}{m+1}} + m\left(b_2^{\frac{m}{m+1}}\right)^{\frac{1}{m}}, b_3^{\frac{1}{m+1}} + m\left(b_3^{\frac{m}{m+1}}\right)^{\frac{1}{m}}, b_4^{\frac{1}{m+1}} + m\left(b_4^{\frac{m}{m+1}}\right)^{\frac{1}{m}}\right\}$$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = \min_{0\leq\tilde{x}\leq\tilde{b}}\left\{(m+1)b_1^{\frac{1}{m+1}}, (m+1)b_2^{\frac{1}{m+1}}, (m+1)b_3^{\frac{1}{m+1}}, (m+1)b_4^{\frac{1}{m+1}}\right\} \quad \text{Since } m+1 > 0$$

$$\Rightarrow \tilde{f}_{m+1}(\tilde{b}) = (m+1)\left\{b_1^{\frac{1}{m+1}}, b_2^{\frac{1}{m+1}}, b_3^{\frac{1}{m+1}}, b_4^{\frac{1}{m+1}}\right\}$$

i.e., The result is true for $n = m + 1$. Hence by mathematical induction, the optimal policy is

$$\left[\left(b_1^{\frac{1}{n}}, b_2^{\frac{1}{n}}, b_3^{\frac{1}{n}}, b_4^{\frac{1}{n}}\right), \left(b_1^{\frac{1}{n}}, b_2^{\frac{1}{n}}, b_3^{\frac{1}{n}}, b_4^{\frac{1}{n}}\right), \ldots \left(b_1^{\frac{1}{n}}, b_2^{\frac{1}{n}}, b_3^{\frac{1}{n}}, b_4^{\frac{1}{n}}\right)\right] \text{and} \tilde{f}_n(\tilde{b}) = n\left(b_1^{\frac{1}{n}}, b_2^{\frac{1}{n}}, b_3^{\frac{1}{n}}, b_4^{\frac{1}{n}}\right)$$

IV. CONCLUSION

A new approach has been proposed in this paper for solving fuzzy optimal subdivision problem in which the positive quantity which is to be divided into ‘n’ factors is taken as trapezoidal fuzzy number. The solution is obtained by the method of Mathematical Induction.The optimal solution is obtained by using the fuzzy recursive equations. Many real-world problems involve sequential or multistage decision making. Sometimes, the parameters may not be known precisely due to some uncontrollable factors. If the obtained results are crisp values then it might lose some helpful information. Fuzzy Dynamic Programming is a powerful optimization procedure that is particularly applicable to many complex problems requiring a sequence of interrelated decisions in a fuzzy environment and hence it has wide range of applications in the future.

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