Application of Zhou's Method for Solving Oscillation and Euler's Differential Equations

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Abstract
In this paper we present zhou's method (DTM) for solving the initial value problems involving second order ordinary differential equations initial value problems involving second order ordinary differential equations we introduce the concept of DTM & applied it to obtain solution of three numerical examples for demonstration. The results are compared with exact solution & DTM method results.
There results show that the technique introduced here is accurate & easy to apply.

Keywords
Ordinary differential equitations zhou's Method (DTM), Initial value problem

I. Introduction

The purpose of this paper is to employ the DTM method on examples of ordinary differential equation of second order and compared with result obtain by exact solution by using complimentary function & particular integral. In recent years, Bizar J. used for Riccati differential equation(1), Opanuga On numerical solution of systems of ordinary differential equitations by numeriacla analytical method (2), Chen used DTM to obtain the solutions of nonlinear system of differential quotations (3), DTM was first proposed by Zhou & Proved that DTM is an iterative procedure for obtaining analytic Taylor's series solution of differential equations DTM is useful to solve ordinary diff equations. & boundary value problems (4), Ayaz F has used DTM to find the series solution of system of differential equitation(5), Duen Y use DTM for Burger's equation to obtain the series solution(6), Bert W. has applied DTM on system of linear equation and analysis of its solutions(7), Chen C.L. has applied DTM technique for steady nonlinear beat conduction problems(8), Using DTM Hassan have find out series solution and that solution compared with DTM method for linear & non linear initial value problems & proved that DTM is reliable tool to find numerical solution(9), Khaled Batiha has been used DTM to obtain the Taylor's series as solution of linear, nonlinear system of ordinary differential equations(10), Montri Thangmoon has been used to find numerical solution of ordinary differential equations(11), Edeki, A semi method for solutions of a certain class of second order ordinary differential equations

II. Basic Definitions & Properties of DTM method

v(t) can be expressed by Taylor's series, then v(t) can be represented as

$$v(t) = \sum_{k=0}^{\infty} \frac{(t-ti)^k}{k!} V(k)$$

v(t) is called inverse of V(k)

$$\therefore v(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] V(k) = D^{-1} V(k)$$

$$v(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] V(k) + R_{n+1}(t)$$

by Taylor's Series

$$V(k) = \frac{1}{k!}\left[\frac{d^k v(t)}{dt^k}\right] \quad \text{at} \quad t = t_0$$

III. Fundamental Theorems on DTM

Theorem 1 :- If $p(t) = n(t) \pm s(t)$ then
$P(k) = N(k) \pm S(k)$

Theorem 2 :- If $p(t) = \propto (t)$ then $\propto n(t)$ then
$P(k) = \propto N(k)$

Theorem 3 :- If $p(t) = \frac{dn(t)}{dt}$ then

$P(k) = (k+1)\, N(k+1)$

Theorem 4 :- If $p(t) = \frac{d^2 n(t)}{dt^2}$ then

$P(k) = (k+1)(k+2)(k+2)\, N(k+2)$

Theorem 5 :- If $p(t) = \frac{d^s n(t)}{dt^s}$ then

$P(k) = (k+1)(k+2)(k+3)\ldots(k+s)\, N(K+s)$

Theorem6 :- If $p(t) = t^s$ them

$P(K) = \sum_{l=0}^{k} S(l)\, P(k-l)$

Theorem7 :- If $p(t) = t^s$ them

$P(k) = \delta(k-s)$

$$\delta(k-s) = \begin{cases} 1 \text{ if } k = s \\ 0 \text{ if } k \neq s \end{cases}$$

Theorem 8:- If $p(t) = e^{\lambda t}$ then

$P(k) = \frac{\lambda^k}{k!}$

Theorem 9:- If $p(t) = (1+t)^s$ then

$P(k) = \frac{S(s-1)..(s-k+1)}{k!}$

Theorem 10:- if $P(t) = (1+t)^s$ then

$P(k) = \frac{w^k}{ki} \sin\left(\frac{\pi k}{2} + \propto\right)$

Theorem 11:- if $p(t) = \cos(wt + \propto)$ then

$P(k) = \frac{w^k}{ki} \cos\left(\frac{\pi k}{2} + \propto\right)$

IV. Validation of Results

Example1 : Euler's Differantial Equation:

Solve $t^2 \frac{d^2x}{d+2} - 3t \frac{dx}{dt} + 3x = 0$

With $x(1) = 0$; $x^1(1) = -2$

➔ Exact solution of

$t^2 \frac{d^2x}{d+2} - 3t \frac{dx}{dt} + 3x = 0$

With $x(1) = 0$; $x^1(1) = -2$

is $x = t - t^3$ ________ (1)

We can convert caveny's Homogeneous Qitterchial equation into linem differential equation by sub

$t = e^z$ given us

$(D^2 - 4D + 3)\ x = 0$

$$\frac{d^2x}{dz^2} - 4\frac{dx}{dz} + 3x = 0 \qquad \text{\_\_\_\_\_\_\_\_\_} \qquad (2)$$

with initial conditions $x(0) = 0$

$x^1(0) = -2$

By DTM Method

$(K + 2)(K+1)\ x(K + 2) - 4(K+1)\ x(K + 1) + 3\ x(K) = 0$

X (0) = 0

X (1) = -2

X (2) = -4

X (3) = -13/3

X (4) = -5/2

X (5) $= \frac{-107}{20}$

etc.

Solution is given by

$x(t) = x(0) + Z\,x(1) + Z^2\,x(2) + Z^3\,x(3) +$

$= 0 - 2z - 4z^2 - \frac{-13}{3}z^3 - \frac{5}{2}z^4 - \frac{107}{20}z^5 + \ldots\ldots.$

$x(t) = -2\log(t) - 4(\log(t))^2 - \frac{13}{3}(\log t)^4 - \frac{107}{20}(\log t)^5 +$

TABLE – I

t	Exact Solution	DTM Solution	Error
1	0.000000	0.000000	0.000000
1.2	-0.528000	-0.528000	0.000000
1.4	-1.344000	-1.344000	0.000000
1.6	-2.496000	-2.496000	0.000000
1.8	-4.032000	-4.032000	0.000000
2.0	-6.000000	-6.000000	0.000000

Ex 2 :- **Free Undamped Oscillations:**

An 8 *l*b weight is placed at one end of a spring suspended from the ceiling the weight is raised to 5 inches above the equilibrium position and left free. Assuming the spring constant 12 lb/ft find the equation of motion, displacement function *x*(t).

Ans. →

The equation of motion is

$\frac{w}{g}\ddot{x} = kx$ Here w = 8, g = 32, k = 12

∴ The equation is

$\frac{d^2x}{dt^2} + 48x = 0 \qquad x(0)=5/12,\ \dot{x}(0)=0$

Exact Solution is given by

$x(t) = \frac{-5}{12}\cos 4\sqrt{3}\,t$

by DTM method $= \frac{-5}{12}\left[-1 - \frac{(4\sqrt{3}t)^2}{21} + \frac{(4\sqrt{3}t)^4}{41} - \frac{(4\sqrt{3}t)^6}{61} + \cdots\ldots\right]$

$= \frac{-5}{12} + 10t^2 - 40t^4 + 64t^6 + \ldots\ldots \qquad (i)$

By DTM method

$(K+2)(K+1)\ x(K+2) = -48\ x(K)$

Put K = 0, 1, 2, 3, 4……..

$x(0) = \frac{-5}{12}$
$x(1) = 0$
$x(2) = 10$
$x(3) = 0$
$x(4) = -40$
$x(5) = 0$
$x(6) = 64$
.
.
.

By DTM solution is given by series
$x(t) = x(0) + t\,x(1) + t^2\,x(2) + t^3\,x(2) + \ldots..$
$= \frac{-5}{12} + 10t^2 - 40t^4 + 64t^6 \ldots\ldots$

TABLE II

t	Exact Solution	DTM Solution	Error
0.0	-0.41666666	-0.41666666	0.000000
0.2	-0.416544825	-0.416544825	0.000000
0.4	-0.41617937	-0.41617937	0.000000
0.6	-0.41557052	-0.41557052	0.000000
0.8	-0.41471863	-0.41471863	0.000000
1.0	-0.41362420	-0.41362420	0.000000

Ex. 3 Free, Damped Oscillation:

A 2lb weight suspended from one end of a spring stretches it to 6 Inches. A velocity of 5ft/ sec^2 upwards is imparted to the weight at its equilibrium position. Suppose a damping force βv acts on the weight Here 0< β <1 & V = x = velocity (a) Determine the position & velocity of the weight at any time (b) Discuss the care for B= 0.6 system is damped i.e. it is oscillatory.

Ans. →

By Hoke's Law
$\frac{w}{g} x + \beta x + Kx = 0$
or $\frac{2}{32} x + \beta x + 4x = 0$
or $\frac{d^2x}{d+2} + 16\beta \frac{dx}{dt} + 64x = 0$

Solving
Displacement function is
$x(t) = e^{8Bt} \left(\frac{-5}{8x}\right) \sin 8 \times t$
& Velocity is
V = x = derivative of above function
for B = 0.6 & $\propto = \sqrt{1-\beta^2} = \sqrt{1-6} = \sqrt{0.64} = \sqrt{0.8}$

$x(t) = \frac{-5}{6.5} e^{-4.8t} \sin 6.4t$ is solution

By DTM Method

$\frac{d^2x}{d+2} + 9.6 \frac{dx}{dt} + 64x = 0$ with initial condition
$x(0) = 0$

$x(0) = -5$

$(k+2)(k+1)\, x(k+2) + 9.6(k+1)\, x(k+1) + 64\, x(k) = 0$

Put K = 0, 1, 2, 3, 4, 5……

$x(0) = 0$

$x(1) = -5$

$x(2) = 24$

$x(3) = -45.6533$

$x(4) = -135.2$

$x(5) = -405.67456$

.

.

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By DTM solution is given by

$x(t) \quad = x(0) + t\, x(1) + t^2\, x(2) + t^3\, x(3) + \ldots..$

$= 0 - 5t + 24\, t^2 - -45.6533\ t^3 - -135.2\, t^4\ -405.67456t^5 + \ldots.$

TABLE III

t	Exact Solution	DTM Solution	Error
0.00000	0.000000	0.000000	0.000000
0.12265625	-0.306475106	-0.306475106	0.000000
0.245315	-0.00157724	-0.00157724	0.000000
-0.12265625	0.9949309	0.9949309	0.000000
-0.245315	2.53616124	2.53616124	0.000000

V. CONCLUSION :

In this work we applied DTM for second order ordinary differential equation, it reduces the computational difficulties of other traditional methods (Laplace Transform).

DTM is best for solving initial value problems of second order.

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