R Index of Some Bridge Graphs

Dr.A.Sumathi
Assistant Professor, Department of Mathematics
Seethalakshmi Ramaswami College,Tiruchirappalli- 620002, India

Abstract: *The degree of a vertex of a molecular graph is the number of first neighbors of the vertex. Sum degree and Multiplication degree of the vertex of a molecular graph is the sum of the degree of the vertices of the neighborhood vertices of the vertex and product of the degree of the vertices of the neighborhood vertices of a vertex respectively. The R degree of the vertex of a molecular graph is the sum of the sum degree of the vertex and the Multiplication degree of the vertex. The concept of degree in graph theory is closely interconnected to the concept of valence in chemistry. In this paper, some formulas are obtained for calculating the vertex based topological R index of the some Bridge graphs.*

Keywords: *Degree of vertex, Neighbourhood, Topological indices*

I. INTRODUCTION

A topological representation of a molecule is called molecular graph. A molecular graph is a collection of points representing the atoms and set of lines representing the covalent bondsin the molecule.The first degree based topological index was introduced by Randic in 1975. His index was defined as $R(G) = \sum_{uv} \frac{1}{\sqrt{d_u(G)d_v(G)}}$ [15].First and Second Zagreb indices, $M_1(G) = \sum_{v} d_v(G)^2$, $M_2(G) = \sum_{uv} d_v(G)d_v(G)$ [1,11]. Narumi- Katayama index, $NK(G) = \prod_{v} d_v(G)$ [9]. Multiplicative versions of the Zagreb indices $\prod_1(G) = \prod_{v} d_v(G)^2$, $\prod_2(G) = \prod_{uv} d_u(G)d_v(G)$, $\prod_1^*(G) = \prod_{uv} [d_u(G) + d_v(G)]$ [10,5,14]. Ernesto Estrada conceived the Atom-BondConnectivityindex, $ABC(G) = \sum_{uv} \sqrt{\frac{d_u(G) + d_v(G) - 2}{d_u(G)d_v(G)}}$ [7,8,12] .Augmented Zagreb index is the Modified version of ABC index, it was introduced by Furtula*et al.*[3]. It is defined as , $AZI(G) = \sum_{uv} \left(\frac{d_u(G)d_v(G)}{d_u(G) + d_v(G) - 2} \right)^3$ .Geometric-arithmetic indexwas invented by Vukicevicand Furtula[6]. It is defined as $GA(G) = \sum_{uv} \frac{\sqrt{d_u(G)d_v(G)}}{\frac{1}{2}[d_u(G) + d_v(G)]}$ .Zhang re-introduced the Harmonic index in [17], $H(G) = \sum_{uv} \frac{2}{d_u(G) + d_v(G)}$.Sum-connectivity indexwas introduced by Bo Zhou andNenadTrinajstić [4]. It was defined as, $SCI(G) = \sum_{uv} \frac{1}{\sqrt{d_u(G) + d_v(G)}}$ .The concept of R degree of a vertex and R index of a graph were introduced by the Author SileymanEdiz[18].The R degree of a vertex and R index of some well-known graphs given in[2].

II. DEFINITIONS

Throughout this paper, we consider only simple connected graphs, i.e. connected graphs without self-loops and parallel edges. For a graph G, V(G) and E(G) denote the set of all vertices and edges respectively. The degree of the vertex v is defined as the number of edges incident with v and denoted by d(v). The set of all vertices which are adjacent to vis called the neighborhood of vand denoted by N(v). For a vertex v, the sum degree of v is defined as $S_v = \sum_{u \in N(v)} \deg(u)$ and for a vertex v, the multiplication degree of v is defined as $M_v = \prod_{u \in N(v)} \deg(u)$ [18].Bridge is an edge of a graph whose deletion increases its number of connected components. Bridge graph (Tree) is a graph whose every edge is a Bridge. The subdivided star($K_{1,n}$:n) is a graph obtained as one point union of n paths of path of length two.[13].The $P_{n,p,k}$tree is a graph obtained from $P_n$by adding p neighbors to each of its nonterminal vertices and k neighbors to each of its terminal vertices[13].The Bistar$B_{n,n}$ is a graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge[13].The Thorn rod $P_{p,t}$is a graph, which includes a linear chain of pvertices and degree-*t* terminal vertices at each of the two rod ends[13].The Corona $G_1 \Theta G_2$ of two graphs G_1 and $G_2$is defined as the graph G obtained by taking one copy of G_1 (which has nvertices) and n copies of G_2 and then joining the i^{th}vertex of G_1 to every vertex in the i^{th} copy of G_2. The graph $P_n \Theta K_1$ is called a comb. It is denoted as P_n^+ [13].The R degree of a vertex v of a simple connected graph Gis defined as $r(v) = S_v + M_v$. The first R index of a simple connected graph Gdefined as $R^1(G) = \sum_{v \in G} (r(v))^2$. The Second R index of a simple connected graph Gdefined as $R^2(G) = \sum_{uv \in E} [r(u)\, r(v)]$.The Third R index of a simple connected graph Gdefined as $R^3(G) = \sum_{uv \in E} [r(u) + r(v)]$.Our notation is standard and mainly taken from standard books of graph theory [16].

III. R INDEX OF SOME BRIDGE GRAPHS

Theorem:3.1

$$R^1(K_{1,n}:n) = (2n + 2^n)^2 + n(16 + (2n+1)^2)$$

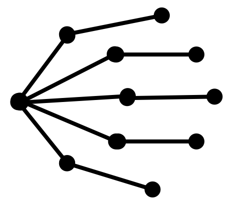
$$R^2(K_{1,n}:n) = 2n(2n+1)(n + 2 + 2^{n-1})$$

$$R^3(K_{1,n}:n) = n(6(n+1) + 2^n)$$

Proof: Let $|V(K_{1,n}:n)| = 2n+1$ and $|E(K_{1,n}:n)| = 2n$

For the vertex v_1 , $S_{v_1} = 2n$, $M_{v_1} = 2^n$, For the internal vertices,

$S_{v_2} = \dots = S_{v_{n+1}} = n+1$, $M_{v_2} = \dots = M_{v_{n+1}} = n$



$(K_{1,5} : 5)$

Fig 3.1

For the rest of the pendent vertices $S_{v_{n+2}} = \dots = S_{v_{2n+1}} = M_{v_{n+2}} = \dots = M_{v_{2n+1}} = 2$.

Hence $r(v_1) = 2n + 2^n$, $r(v_2) = \dots = r(v_{n+1}) = 2n+1$, $r(v_{n+2}) = \dots = r(v_{2n+1}) = 4$

After Simplification,

$$R^1(K_{1,n}:n) = (2n + 2^n)^2 + n(16 + (2n+1)^2)$$

$$R^2(K_{1,n}:n) = 2n(2n+1)(n + 2 + 2^{n-1})$$

$R^3(K_{1,n}:n) = n(6(n+1)+2^n)$

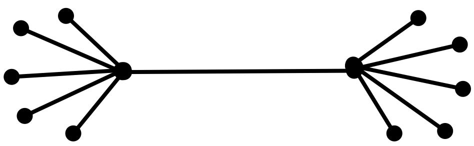
Theorem:3.2

$R^1(B_{n,n}) = 8n(n+1)^2 + 2(3n+2)^2$

$R^2(B_{n,n}) = (3n+2)(4n^2+7n+2)$

$R^3(B_{n,n}) = (5n+2)(2n+2)$

Proof: Let $|V(B_{n,n})| = 2n+2$ and $|E(B_{n,n})| = 2n+1$



$B_{5,5}$

Fig.3.2

For the pendent vertices, $S_{v_1} = \ldots = S_{v_{2n}} = n+1,\ M_{v_1} = \ldots = M_{v_{2n}} = n+1$.

For the internal vertices v_{2n+1}, v_{2n+2}, $S_{v_{2n+1}} = S_{v_{2n+2}} = 2n+1,\ M_{v_{2n+1}} = M_{v_{2n+2}} = n+1$

Hence, $r(v_1) = \ldots = r(v_{2n}) = 2n+2, r(v_{2n+1}) = .r(v_{2n+2}) = 3n+2$

After Simplification,

$R^1(B_{n,n}) = 8n(n+1)^2 + 2(3n+2)^2$

$R^2(B_{n,n}) = (3n+2)(4n^2+7n+2)$

$R^3(B_{n,n}) = (5n+2)(2n+2)$

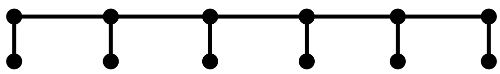
Theorem:3.3

$R^1(P_n^+) = 418 + 256(n-4) + 36(n-2)$

$R^2(P_n^+) = 81(n-5) + 54(n-2) + 132$

$R^3(P_n^+) = 18(n-5) + 15(n-2) + 56$

Proof: Let $|V(P_n^+)| = 2n$ and $|E(P_n^+)| = 2n-1$



P_6^+

Fig. 3.3

For the vertices $v_1, \ldots, v_n$ on the path P_n, $S_{v_1} = S_{v_n} = 4, S_{v_2} = S_{v_{n-1}} = 6, S_{v_3} = \ldots = S_{v_{n-2}} = 7,$

$M_{v_1} = M_{v_n} = 3, M_{v_2} = M_{v_{n-1}} = 6, M_{v_3} = \ldots = M_{v_{n-2}} = 9,$

For the pendent vertices $v_{n+1}, \ldots, v_{2n}$,

$S_{v_{n+1}} = S_{v_{2n}} = 2, S_{v_{n+2}} = \ldots = S_{v_{2n-1}} = 3,\ M_{v_{n+1}} = M_{v_{2n}} = 2, M_{v_{n+2}} = \ldots = M_{v_{2n-1}} = 3$

Hence, $r(v_1) = r(v_n) = 7, r(v_2) = r(v_{n-1}) = 12, r(v_3) = r(v_{n-2}) = 16$,

After Simplification,

$R^1(P_n^+) = 418 + 256(n-4) + 36(n-2)$

$R^2(P_n^+) = 81(n-5) + 54(n-2) + 132$

$R^3(P_n^+) = 18(n-5) + 15(n-2) + 56$

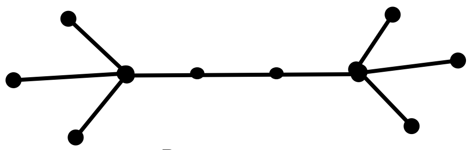
Theorem:3.4

$R^1(P_{p,t}) = 2(t+3)^2 + 2(3t+2)^2 + 64(p-4) + 8t^2(t-1)$

$R^2(P_{p,t}) = 2(3t+2)(t+11) + 4t(t-1)(t+3)$

$R^3(P_{p,t}) = 2(7t+5) + 16(p-5) + 6(t^2-1)$

Proof: Let $|V(P_{p,t})| = p + 2t - 2$ and $|E(P_{p,t})| = p + 2t - 3$



$P_{4,4}$

Fig.3.4

For the vertices $v_1,\dots, v_p$ on the path P_p, $S_{v_1} = S_{v_p} = t+1, S_{v_2} = S_{v_{p-1}} = t+2, S_{v_3} = \dots = S_{v_{p-2}} = 4,$

$M_{v_1} = M_{v_p} = 2, M_{v_2} = M_{v_{p-1}} = 2t, M_{v_3} = \dots = M_{v_{p-2}} = 4,$

For the pendent vertices $v_{p+1},\dots, v_{p+2t-3}$, $S_{v_{p+1}} = \dots = S_{v_{p+2t-3}} = t, M_{v_{p+1}} = \dots = M_{v_{p+2t-3}} = t$

Hence, $r(v_1) = r(v_p) = t+3, r(v_2) = r(v_{p-1}) = 3t+2, r(v_3) = r(v_{p-2}) = 8$

$r(v_{p+1}) = \dots = r(v_{p+2t-3}) = 2t,$

After Simplification,

$R^1(P_{p,t}) = 2(t+3)^2 + 2(3t+2)^2 + 64(p-4) + 8t^2(t-1)$

$R^2(P_{p,t}) = 2(3t+2)(t+11) + 4t(t-1)(t+3)$

$R^3(P_{p,t}) = 2(7t+5) + 16(p-5) + 6(t^2-1)$

IV. CONCLUSION

In this paper, some formulas are obtained for calculating the new vertex based topological R index of the certain Bridge graphs.

REFERENCES

[1] A.T. Balaban, I. Motoc, D. Bonchev, and O. Mekenyan, Topics Curr. Chem. **114** (1983) 21.
[2] A.Sumathi, R index of some graphs, Annals of Pure and Applied Mathematics, 2279-0888 (O) Vol. 16, No. 1, pp 63-67, Jan 2018.
[3] Furtula, A. Graovac, D. Vukičević, J. Math. Chem. **48** (2010)370.
[4] B. Zhou and N. Trinajstić, J. Math. Chem. **46** (2009) 1252.
[5] D. J. Klein and V. R. Rosenfeld, MATCH Commun. Math. Comput.Chem. **64** (2010) 607
[6] D. Vukičević and B. Furtula, J. Math. Chem. **46** (2009) 1369.
[7] E. Estrada, Chem. Phys. Lett. **463** (2008) 422.
[8] E. Estrada, L. Torres, L. Rodríguez, and I. Gutman, Indian J.Chem. **37A** (1998) 849.
[9] H. Narumi and M. Katayama, Mem. Fac. Engin. Hokkaido Univ.**16** (1984) 209.
[10] I. Gutman and M. Ghorbani, Appl. Math. Lett. **25** (2012) 1435.
[11] I. Gutman and N. Trinajstić, Chem. Phys. Lett. **17** (1972) 535.
[12] I. Gutman, J. Tošović, S. Radenković, and S. Marković, Indian J.Chem. **51A** (2012) 690.
[13] J.A. Gallian., A dynamic survey of graph labeling, The Electronics Journal of combinations 17 (2010)
[14] M. Eliasi, A. Iranmanesh, and I. Gutman,MATCHCommun.Math. Comput. Chem. **68** (2012)217.
[15] M. Randić, J. Am. Chem. Soc. **97** (1975) 6609.
[16] R. Balakrishnan , K.Renganathan, A Text book of graph theory, springer – verlag New York, (2000)
[17] S. Fajtlowicz, Congr. Numer. **60** (1987) 187.
[18] SiileymanEdiz ,On R degrees of vertices and R indices of graphs, International Journal of Advanced chemistry,5(2),70-72,2017