Identification of HPM and ADM for the (n+1)-dimensional Equal Width Wave Equation with Diffusion and Damping term

R. Asokan[1], E. Nakkeeran[2] and T. Shanmuga Priya[3]
[1,2]Department of Mathematics, Madurai Kamaraj University, Madurai Tamil Nadu, India.
[3]Department of Mathematics, Adhiyaman Arts Science College for Women, Uthangarai, Tamil Nadu, India.

Abstract:
In this paper, We present the homotopy perturbation method (HPM) and Adomian Decomposition Method (ADM) to obtain a closed form solution of the (n+1)-dimensional Equal Width wave equation with diffusion and dispersion term. These methods consider the use of the initial or boundary conditions and find the solution without any discretization, transformation or restrictive conditions and avoid the round-off errors. Few numerical examples are provided to validate the reliability and efficiency of the three methods.
Keywords: Nonlinear PDE,Adomian Decomposition Method,Differential transform method,and homotopy perturbation method.

Introduction

Many physical problems can be described by partial differential equations with variable coefficients in mathematical physics, and other areas of science and engineering. The investigation of exact or approximate solutions to these partial differential equations will help us to understand these physical phenomena better. There are some valuable efforts that focus on solving the partial differential equations arising in engineering and scientific applications [2, 6, 7, 8, 12, 13, 24, 28]. Reviewing these improvements, the Adomian decomposition method [2, 12, 24], the tanh method [6, 28], the extended tanh function method [7, 8] and other methods [13] are proposed to solve the partial differential equations. Among these solution methods, Adomian decomposition method is the most transparent method for solutions of thepartial differential equations, however, this method is involved in the calculation of complicated Adomian polynomials which narrows down its applications. To overcome this disadvantage of the Adomian decomposition method, we consider the homotopy perturbation method to solve the partial differential equations with variable coefficients. The homotopy perturbation method proposed by J.H. He [14, 15] has been the subject of extensive studies, and applied to various linear and nonlinear problems [1, 3, 4, 5, 9, 10, 11, 16, 17, 18, 19, 20, 21, 26, 27, 29]. Unlike analytical perturbation methods, the significant feature of this method which doesnt depend on a small parameter is that it provides the exact or approximate solutions to a wide range of nonlinear problems without unrealistic assumptions, linearization, discretization and the computation of the Adomian polynomials.

The well-known Korteweg and de Vries (KdV) equation, $u_t + uu_x + u_{xxx} = 0$, is a nonlinear partial differential equation (PDE) that models the time-dependent motion of shallow water waves in one space dimension. Morrison et al. [26] proposed the one-dimensional PDE,$u_t + uu_x - \mu u_{xxt} = 0$, as an equally valid and accurate model for the same wave phenomena simulated by the KdV equation. This PDE is called the equal width (EW) equation because the solutions for solitary waves with a permanent form and speed, for a given value of the parameter μ, are waves with an equal width or wavelength for all wave amplitudes.

In this work, we have employed or the identification of HPM and ADM to solve (n+1)-dimensional Equal Width wave equation with damping and damping term. Few numerical examples are carried out to validate and illustrate the above two methods.

Let us consider the (n+1)-dimensional Equal Width Wave equation with damping term as,

$$u_t = \mu_1 u_{x_1x_1t} + \mu_2 u_{x_2x_2t} + \ldots + \mu_n u_{x_nx_nt} + \nu u u_{x_1} + \delta u_{x_1x_1} + \beta u, \tag{1}$$

under the initial condition

$$u(x_1, x_2, \ldots, x_n, 0) = u_0(x_1, x_2, \ldots, x_n), \tag{2}$$

where $\mu_i, i = 1, 2, \ldots, n, \beta$ and ν are constants.

Homotopy Perturbation Method (HPM)

To describe the HPM, consider the following general nonlinear differential equation

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{3}$$

under the boundary condition

$$B(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \partial\Omega, \tag{4}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function and $\partial\Omega$ is a boundary of the domain Ω. The operator A can be divided into two parts L and N, where L is a linear operator while N is a nonlinear operator. Then Eq. (3) can be rewrittten as

$$L(u) + N(u) - f(r) = 0, \tag{5}$$

Using the homotopy technique, we construct a homotopy: V(r,p): $\Omega \times [0,1] \to R$ which satisfies H(V,p)= (1-p) [L(V)-L(u_0)] $+ p[A(V) - f(r)]$ or

$$H(V,p) = L(V) - L(u_0) + pL(u_0) + p[N(V) - f(r)], \; r \in \Omega, \quad p \in [0,1], \tag{6}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is the initial approximation of Eq. (3) which satisfies the boundary conditions. Obviously, considering Eq. (6), we will have

$$\begin{aligned} H(V,0) &= L(V) - L(u_0) = 0, \\ H(V,1) &= A(V) - f(r) = 0, \end{aligned} \tag{7}$$

changing the process of p from zero to unity is just that $V(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called the deformation also $A(V)-f(r)$ and $L(u)$ are called as homotopy. The homotopy perturbation method uses the homotopy parameter p as an expanding parameter [23-25] to obtain

$$V = v_0 + pv_1 + p^2v_2 + p^3v_3 + \ldots = \sum_{n=0}^{\infty} p^n v_n. \tag{8}$$

$p \to 1$ results the approximate solution of eq (3) as

$$u = \lim_{p\to 1} V = v_0 + v_1 + v_2 + \ldots = \sum_{n=0}^{\infty} v_n. \tag{9}$$

A comparison of like powers of p gives the solutions of various orders.

Series (9) is convergent for most of the cases. However, convergence rate depends on the nonlinear operator, $N(V)$.

He [22] suggested the following opinions:

1. The second derivative of $N(V)$ with respect to v must be small as the parameter p may be relatively large.

2. The norm of $L^{-1}\frac{\partial N}{\partial u}$ must be smaller than one so that the series converges.

Now, we implement the HPM method to Eq.(1). According to HPM, we construct the following simple homotopy

$$u_t + p[(-\mu_1 u_{x_1,x_1t} - \mu_2 u_{x_2,x_2t} - \ldots - \mu_n u_{x_n,x_nt}) - \nu u u_{x_1}] - \delta u_{x_1x_1} - \beta u = 0. \tag{10}$$

With the initial approximation $u(x_1,\ldots,x_n,0) = u_0(x_1,x_2,\ldots,x_n)$, we suppose that the solution has the following form

$$u(x_1,\ldots,x_n,t) = u_0(x_1,\ldots,x_n,t) + pu_1(x_1,\ldots,x_n,t) + p^2u_2(x_1,\ldots,x_n,t) + \ldots. \tag{11}$$

Insertion of eqn. (11) into eqn. (10) and equating the term with like powers of p, we get

$$p^0 : \frac{\partial u_0}{\partial t} = 0. \tag{12}$$

$$p^1 : \frac{\partial u_1}{\partial t} = \left(\mu_1 \frac{\partial^3 u_0}{\partial x_1^2 \partial t} + \mu_2 \frac{\partial^3 u_0}{\partial x_2^2 \partial t} + \ldots + \mu_n \frac{\partial^3 u_0}{\partial x_n^2 \partial t}\right) + \nu u_0 \frac{\partial u_0}{\partial x_1} + \beta u + \delta u_{x_1x_1}. \tag{13}$$

$$p^2 : \frac{\partial u_2}{\partial t} = \left(\mu_1 \frac{\partial^3 u_0}{\partial x_1^2 \partial t} + \mu_2 \frac{\partial^3 u_0}{\partial x_2^2 \partial t} + \ldots + \mu_n \frac{\partial^3 u_0}{\partial x_n^2 \partial t}\right) + \nu\left(u_0 \frac{\partial u_1}{\partial x_1} + u_1 \frac{\partial u_0}{\partial x_1}\right) + \beta u + \delta u_{x_1x_1}, \tag{14}$$

and so on.
We solve Eqs. (12)-(14), to get the values of u_0, u_1, u_2 etc. Thus as considering Eq. (11) and letting $p = 1$, we obtain the approximate analytic solution of Eq. (1) as

$$u(x_1,\ldots,x_n,t) = u_0(x_1,\ldots,x_n,t) + u_1(x_1,\ldots,x_n,t) + u_2(x_1,\ldots,x_n,t) + \ldots \tag{15}$$

Illustrations of HPM

In this section, we describe the above method by the following examples to validate the efficiency of the HPM.

Example : 1

Consider the (n+1)-dimensional Equal Width wave equation with damping and diffusion term and assuming the constants $\nu = \beta = \mu_i's = 1$ as,

$$u_t = u_{x_1x_1t} + u_{x_2x_2t} + \ldots + u_{x_nx_nt} + uu_{x_1} + u + u_{x_1x_1}, \tag{16}$$

under the initial condition

$$u(x_1, x_2, \ldots, x_n, 0) = u_0(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \ldots + x_n. \tag{17}$$

Applying the homotopy perturbation method to Eq. (16), we have

$$u_t + p[(-u_{x_1,x_1t} - u_{x_2,x_2t} - \ldots - u_{x_n,x_nt}) - uu_{x_1} - u - u_{x_1x_1}] = 0. \tag{18}$$

In the view of HPM, we use the homotopy parameter p to expand the solution

$$u(x_1, x_2, \ldots, x_n, t) = u_0 + pu_1 + p^2u_2 + \ldots \tag{19}$$

The approximate solution can be obtained by taking $p = 1$ in Eq. (19) as

$$u(x_1, x_2, \ldots, x_n, t) = u_0 + u_1 + u_2 + \ldots \tag{20}$$

Now substituting from Eq. (18) into Eq. (17) and equating the terms with identical powers of p, we obtain the series of linear equations, which can be easily solved. First few linear equations are given as

$$p^0 : \frac{\partial u_0}{\partial t} = 0. \tag{21}$$

$$p^1 : \frac{\partial u_1}{\partial t} = \left(\frac{\partial^3 u_0}{\partial x_1^2 \partial t} + \frac{\partial^3 u_0}{\partial x_2^2 \partial t} + \ldots + \frac{\partial^3 u_0}{\partial x_n^2 \partial t}\right) + u_0\frac{\partial u_0}{\partial x_1} + u_0 + u_{x_0x_0}. \tag{22}$$

$$p^2 : \frac{\partial u_2}{\partial t} = \left(\frac{\partial^3 u_0}{\partial x_1^2 \partial t} + \frac{\partial^3 u_0}{\partial x_2^2 \partial t} + \ldots + \frac{\partial^3 u_0}{\partial x_n^2 \partial t}\right) + \left(u_0\frac{\partial u_1}{\partial x_1} + u_1\frac{\partial u_0}{\partial x_1}\right) + u_1 + u_{x_1x_1}. \tag{23}$$

Using the initial condition (17), the solution of Eq. (21) is given by

$$u(x_1, x_2, \ldots, x_n, 0) = u_0(x_1, x_2, \ldots, x_n) = (x_1 + x_2 + \ldots + x_n). \tag{24}$$

Then the solution of Eq. (22) will be

$$u_1(x_1, \ldots, x_n, t) = \int_0^t \left(\left(\frac{\partial^3 u_0}{\partial x_1^2 \partial t} + \frac{\partial^3 u_0}{\partial x_2^2 \partial t} + \ldots + \frac{\partial^3 u_0}{\partial x_n^2 \partial t}\right) + u_0\frac{\partial u_0}{\partial x_1}\right) dt. \tag{25}$$

$$u_1(x_1, x_2, \ldots, x_n, t) = 2(x_1 + x_2 + \ldots + x_n)t. \tag{26}$$

Also, we can find the solution of Eq. (23) by using the following formula

$$u_2(x_1, \ldots, x_n, t) = \int_0^t \left(\frac{\partial^3 u_0}{\partial x_1^2 \partial t} + \frac{\partial^3 u_0}{\partial x_2^2 \partial t} + \ldots + \frac{\partial^3 u_0}{\partial x_n^2 \partial t} \right) + \left(u_0 \frac{\partial u_1}{\partial x_1} + u_1 \frac{\partial u_0}{\partial x_1} \right) + u_1 + u_{x_1 x_1} dt. \tag{27}$$

$$u_2(x_1, x_2, \ldots, x_n, t) = 3(x_1 + x_2 + \ldots + x_n)t^2. \tag{28}$$

etc. Therefore, from Eq. (24), the approximate solution of Eq.(16) is given as

$$u(x_1, \ldots, x_n, t) = (x_1 + \ldots + x_n) + 2(x_1 + \ldots + x_n)t + 3(x_1 + \ldots + x_n)t^2 + \ldots \tag{29}$$

Hence the exact solution can be expressed as

$$u(x_1, x_2, \ldots, x_n, t) = \frac{(x_1 + x_2 + \ldots + x_n)}{(1-t)^2}, \tag{30}$$

provided that $0 \leq t < 1$.

Illustration of HPM for (3+1)-dimensional Equal Width wave equation with damping and diffusion term

Consider the (1+3)-dimensional Equal Width wave equation with damping and diffusion term as,

$$u_t = u_{xxt} + u_{yyt} + u_{zzt} + uu_x + u + u_{xx}, \tag{31}$$

under the initial condition

$$u(x, y, z, 0) = u_0(x, y, z) = x + y + z. \tag{32}$$

Applying the homotopy perturbation method to Eq. (31), we have

$$u_t + p[(-u_{xxt} - u_{yyt} - u_{zzt}) - uu_x - u - u_{xx}] = 0. \tag{33}$$

In the view of HPM, we use the homotopy parameter p to expand the solution

$$u(x, y, z, t) = u_0 + pu_1 + p^2 u_2 + \ldots \tag{34}$$

The approximate solution can be obtained by taking $p = 1$ in Eq. (34) as

$$u(x, y, z, t) = u_0 + u_1 + u_2 + \ldots \tag{35}$$

Now, substituting Eq.(34) into Eq.(33) and equating the terms with identical powers of p, we obtain the series of linear equations. First few linear equations are given as

$$p^0 : \frac{\partial u_0}{\partial t} = 0. \tag{36}$$

$$p^1 : \frac{\partial u_1}{\partial t} = \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t} + \frac{\partial^3 u_0}{\partial z^2 \partial t} \right) + u_0 \frac{\partial u_0}{\partial x} + u_0 + u_{x_0,x_0}. \tag{37}$$

$$p^2 : \frac{\partial u_2}{\partial t} = \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t} + \frac{\partial^3 u_0}{\partial z^2 \partial t} \right) + \left(u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x} \right) + u_1 + u_{x_1,x_1}. \tag{38}$$

Then the solution of Eq. (36) using the initial condition (32) is given by

$$u(x, y, z, 0) = u_0(x, y, z) = (x + y + z). \tag{39}$$

We derive the solution of Eq. (37) in the following form

$$u_1(x, y, z, t) = \int_0^t \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t} + \frac{\partial^3 u_0}{\partial z^2 \partial t} \right) + u_0 \frac{\partial u_0}{\partial x} + u_0 + u_{x_0,x_0} dt. \tag{40}$$

$$u_1(x, y, z, t) = 2(x + y + z)t. \tag{41}$$

Also, we can find the solution of Eq. (38) by using the following formula

$$u_2(x, y, z, t) = \int_0^t \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t} + \frac{\partial^3 u_0}{\partial z^2 \partial t} \right) + \left(u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x} \right) + u_1 + u_{x_1,x_1} dt. \tag{42}$$

$$u_2(x, y, z, t) = 3(x + y + z)t^2. \tag{43}$$

etc. Therefore, from Eq. (35), the approximate solution of Eq.(31) is given as

$$u(x, y, z, t) = (x + y + z) + 2(x + y + z)t + 3(x + y + z)t^2 + \dots \tag{44}$$

The exact solution is

$$u(x, y, z, t) = \frac{(x + y + z)}{(1 - t)^2}, \tag{45}$$

provided that $0 \leq t < 1$.

HPM for (2+1)-dimensional Equal Width wave equation with damping and diffusion term

Consider the (2+1)-dimensional Equal Width wave equation as,

$$u_t = u_{xxt} + u_{yyt} + uu_x + u + u_{xx}, \tag{46}$$

under the initial condition

$$u(x, y, 0) = u_0(x, y) = x + y. \tag{47}$$

Applying the homotopy perturbation method to Eq. (46), we have

$$u_t + p[(-u_{xxt} - u_{yyt}) - uu_x - u - u_{xx}] = 0. \tag{48}$$

In the view of HPM, we use the homotopy parameter p to expand the solution

$$u(x,y,t) = u_0 + pu_1 + p^2u_2 + \dots \tag{49}$$

The approximate solution can be obtained by taking $p = 1$ in Eq. (49) as

$$u(x,y,t) = u_0 + u_1 + u_2 + \dots \tag{50}$$

Now, substituting Eq.(49) into Eq.(48) and equating the terms with identical powers of p, we obtain the series of linear equations. First few linear equations are given as

$$p^0 : \frac{\partial u_0}{\partial t} = 0. \tag{51}$$

$$p^1 : \frac{\partial u_1}{\partial t} = \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t}\right) + u_0\frac{\partial u_0}{\partial x} + u_0 + u_{x_0,x_0}. \tag{52}$$

$$p^2 : \frac{\partial u_2}{\partial t} = \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t}\right) + \left(u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x}\right) + u_1 + u_{x_1,x_1}. \tag{53}$$

Then the solution of Eq. (51) using the initial condition (47) is given by

$$u(x,y,0) = u_0(x,y) = (x+y). \tag{54}$$

We derive the solution of Eq. (52) in the following form

$$u_1(x,y,t) = \int_0^t \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t}\right) + u_0\frac{\partial u_0}{\partial x} + u_0 + u_{x_0,x_0} dt. \tag{55}$$

$$u_1(x,y,t) = 2(x+y)t. \tag{56}$$

Also, we can find the solution of Eq. (53) by using the following formula

$$u_2(x,y,t) = \int_0^t \left(\frac{\partial^3 u_0}{\partial x^2 \partial t} + \frac{\partial^3 u_0}{\partial y^2 \partial t}\right) + \left(u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x}\right) + u_1 + u_{x_1,x_1}.dt. \tag{57}$$

$$u_2(x,y,t) = 3(x+y)t^2. \tag{58}$$

etc. Therefore, from Eq. (50), the approximate solution of Eq.(46) is given as

$$u(x,y,t) = (x+y) + 2(x+y)t + 3(x+y)t^2 + \dots. \tag{59}$$

The exact solution can be expressed as

$$u(x,y,t) = \frac{(x+y)}{(1-t)^2}, \tag{60}$$

provided that $0 \leq t < 1$.

Adomian Decomposition Method (ADM)

Consider the following linear operator and their inverse operators: $\mathrm{L}_t = \frac{\partial}{\partial t}$; $L_{x_i,x_i,t} = \frac{\partial^3}{\partial x_i^2 \partial t}$, $i = 1, 2, \ldots, n$. $\mathrm{L}_t^{-1} = \int_0^t (.)dt$, $L_{x_i,x_i,t} = \int_0^{x_i}\int_0^t (.)d\tau d\tau d\gamma$, $i = 1, 2, \ldots, n$. Using the above notations, Eq.() becomes

$$L_t(u) = \left(\sum_{i=1}^{n} \alpha_i L_{x_i,x_i,t}(u)\right) + \nu u \frac{\partial u}{\partial x_i} + \beta L(u),, \tag{61}$$

operating the inverse operators L_t^{-1} to eqn. (61) and using the initial condition gives

$$u(x_1, \ldots, x_n, t) = u_0(x_1, \ldots, x_n, t) + L_t^{-1}\left(\sum_{i=1}^{n} \alpha_i L_{x_i,x_i,t}(u)\right) + \nu L_t^{-1} u \frac{\partial u}{\partial x_1} + \beta L^{-1}(u) + \delta L^{-1} u(x_1, x_1). \tag{62}$$

The decomposition method consists of representing the solution $u(x_1, x_2, \ldots, x_n, t)$ by the decomposition series

$$u(x_1, x_2, \ldots, x_n, t) = \sum_{q=0}^{\infty} u_q(x_1, x_2, \ldots, x_n, t). \tag{63}$$

The nonlinear term $u\frac{\partial u}{\partial x_1}$ is represented by a series of the so called Adomian polynomials, given by

$$u\frac{\partial u}{\partial x_i} = \sum_{q=0}^{\infty} A_q(x_1, x_2, \ldots, x_n, t). \tag{64}$$

The component $u_q(x_1, x_2, \ldots, x_n, t)$ of the solution $u(x_1, x_2, \ldots, x_n, t)$ is determined in a recursive manner. Replacing the decomposition series (63) and (64) for u into eqn. (62) gives

$$\begin{aligned} \sum_{q=0}^{\infty} u_q(x_1, x_2, \ldots, x_n, t) &= u_0(x_1, \ldots, x_n, t) + L_t^{-1}\left(\sum_{i=1}^{n} \alpha_i L_{x_i,x_i,t}(u)\right) \\ &+ \nu L_t^{-1} \sum_{q=0}^{\infty} A_q(x_1, x_2, \ldots, x_n, t) + \beta L^{-1}(u) + +\delta_i L_(t)^{-1} x_i, x_i(u) \end{aligned} \tag{65}$$

According to ADM the zero-th component $u_0(x_1, \ldots, x_n, t)$ is identified from the initial or boundary conditions and from the source terms. The remaining components of $u(x_1, \ldots, x_n, t)$ are determined in a recursion manner as follows

$$u_0(x_1, \ldots, x_n, t) = u_0(x_1, \ldots, x_n), \tag{66}$$

$$u_k(x_1, \ldots, x_n, t) = L_t^{-1}\left(\sum_{i=1}^{n} \alpha_i L_{x_i,x_i,t}(u)\right) + \nu L_t^{-1}(A_k) + \gamma L^{-1}(u_k) + \beta L^{-1}(u_k) + \delta L^{-1}(u_{x_i,x_i}),\ k \geq 0, \tag{67}$$

where the Adomian polynomials for the nonlinear term $u\frac{\partial u}{\partial x_1}$ are derived from the following recursive formulation,

$$A_k = \frac{1}{k!}\frac{d^k}{d\lambda^k}\left(\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right)_{\lambda=0}, \; k = 0, 1, 2, \dots. \tag{68}$$

First few Adomian polynomials are given as

$$A_0 = u_0\frac{\partial u_0}{\partial x_1}, \; A_1 = u_0\frac{\partial u_1}{\partial x_1} + u_1\frac{\partial u_0}{\partial x_1} + \beta L^{-1}(u_0) + \delta L^{-1}(u_{x_0,x_0}),$$
$$A_2 = u_2\frac{\partial u_0}{\partial x_1} + u_1\frac{\partial u_1}{\partial x_1} + u_0\frac{\partial u_2}{\partial x_1} + \beta L^{-1}(u_1) + \delta L^{-1}(u_{x_1,x_1}), \tag{69}$$

using eq.(67) for the adomian polynomials A_k, we get

$$u_0(x_1, \dots, x_n, t) = u_0(x_1, \dots, x_n), \tag{70}$$

$$u_1(x_1, \dots, x_n, t) = L_t^{-1}\left(\sum_{i=1}^{n}\alpha_i L_{x_i,x_i,t}(u_0)\right) + \nu L_t^{-1}(A_0) + \gamma L^{-1}(u_0) + \delta L^{-1}(u_{x_0,x_0}), \tag{71}$$

$$u_2(x_1, \dots, x_n, t) = L_t^{-1}\left(\sum_{i=1}^{n}\alpha_i L_{x_i,x_i,t}(u_1)\right) + \nu L_t^{-1}(A_1) + \gamma L^{-1}(u_1)\delta L^{-1}(u_{x_1,x_1})+, \tag{72}$$

and so on. Then the q-th term, u_q can be determined from

$$u_q = \sum_{0}^{q-1} u_k(x_1, \dots, x_n, t). \tag{73}$$

Knowing the components of u, the analytical solution follows immediately.

Computations of ADM for (n+1)-dimensional Equal Width Wave equaion with damping and diffusion term

Using Eqns.(68) and (69), first few components of the decomposition series are given by

$$u_0(x_1, \dots, x_n, t) = (x_1 + \dots + x_n), \tag{74}$$

$$u_1(x_1, \dots, x_n, t) = 2(x_1 + \dots + x_n)t, \tag{75}$$
$$u_2(x_1, \dots, x_n, t) = 3(x_1 + \dots + x_n)t^2, \tag{76}$$
$$u_3(x_1, \dots, x_n, t) = 4(x_1 + \dots + x_n)t^3, \tag{77}$$

Then by the decomposition series, we get the solution

$$\begin{aligned} u(x_1, \dots, x_n, t) &= \sum_{k=0}^{\infty} u_k(x_1, \dots, x_n, t), \\ &= u_0(x_1, \dots, x_n, t) + u_1(x_1, \dots, x_n, t) + u_2(x_1, \dots, x_n, t) + \dots, \\ &= (x_1 + \dots + x_n)(1 + 2t + 3t^2 + \dots) \end{aligned} \tag{78}$$

Hence, the exact solution is

$$u(x_1, \ldots, x_n, t) = \frac{(x_1 + \ldots + x_n)}{(1-t)^2}. \tag{79}$$

Adomian Decomposition Method for (3+1)-dimensional Equal Width wave equation with damping and diffusion term

Using Eqns.(68) and (69), first few components of the decomposition series are given by

$$u_0(x,y,z,t) = (x+y+z), \tag{80}$$

$$u_1(x,y,z,t) = 2(x+y+z)t, \tag{81}$$

$$u_2(x,y,z,t) = 3(x+y+z)t^2, \tag{82}$$

$$u_3(x,y,z,t) = 4(x+y+z)t^3, \tag{83}$$

and so on. By the decomposition series, we obtain the solution

$$\begin{aligned} u(x,y,z,t) &= \sum_{k=0}^{\infty} u_k(x,y,z,t), \\ &= u_0(x,y,z,t) + u_1(x,y,z,t) + u_2(x,y,z,t) + \ldots, \\ &= (x+y+z)(1+2t+3t^2+4t^3+\ldots) \end{aligned} \tag{84}$$

Therefore the exact solution is

$$u(x,y,z,t) = \frac{(x+y+z)}{(1-t)^2}, \tag{85}$$

provided that $0 \leq t < 1$.

Adomian Decomposition Method for (2+1)-dimensional Equal Width wave equation

Using Eqns.(68) and (69), first few components of the decomposition series are given by

$$u_0(x,y,t) = (x+y), \tag{86}$$

$$u_1(x,y,t) = 2(x+y)t, \tag{87}$$

$$u_2(x,y,t) = 3(x+y)t^2, \tag{88}$$

$$u_3(x,y,t) = 4(x+y)t^3, \tag{89}$$

and so on. By the decomposition series, we get the solution

$$\begin{aligned} u(x,y,t) &= \sum_{k=0}^{\infty} u_k(x,y,t), \\ &= u_0(x,y,t) + u_1(x,y,t) + u_2(x,y,t) + \ldots, \\ &= (x+y)(1+2t+3t^2+4t^3+\ldots) \end{aligned} \tag{90}$$

Therefore the exact solution can be expressed as

$$u(x,y,t) = \frac{(x+y)}{(1-t)^2}, \tag{91}$$

provided that $0 \leq t < 1$.

Conclusion

In this chapter, homotopy perturbation method and adomian decomposition method have been successfully applied for solving (n+1)-dimensional Equal Width wave equation with damping and diffusion term. The solutions obtained by these methods are an infinite power series for an appropriate initial condition, which can, in turn, be expressed in a closed form, the exact solution. The results reveal that the methods are very effective, convenient and quite accurate mathematical tools for solving (n+1)-dimensional Equal Width wave equation with damping and diffusion. These methods, which can be used without any need to complex computations except simple and elementary operations, are also promising techniques for solving other nonlinear problems.

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