E-Cordiality of Tail C_3Related One Point Union Graphs

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1. Abstract:

In this paper we obtain e-cordial labeling of one point union of k copies of tail graph namely $G^{(k)}$ where G = tail $C_3(P_t)$. We have taken different values of t as 2,3,4. If we change point of union on G then different structures of $G^{(k)}$ are obtained. Of these we have taken pairwise non isomorphic structures of $G^{(k)}$ and have proved that they all are e-cordial under certain conditions. We have also considered the case that more than one tails are attached to G such that sum of edges is t-1 for given t and the family is e-cordial.

Key words: *graph, E-cordial, tail graph, C_3, labeling.*

Subject Classification: 05C78

2. Introduction:

In 1997 Yilmaz and Cahit introduced weaker version of edge graceful labeling E-cordial labeling [4]. Let G be a (p,q) graph. f:E→{0,1}be a function. Define f on V by f(v) $=\sum\{f(vu)(vu)\in E(G)\}(mod\ 2)$.The function f is called as E-cordial labeling if $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$ where $v_f(i)$ is the number of vertices labeled with i =0,1. And $e_f(i)$ is the number of edges labeled with i= 0,1,We follow the convention that $v_f(0,1)$ =(a, b) for $v_f(0)$=a and $v_f(1)$=b further $e_f(0,1)$=(x,y)for $e_f(0)$=x and $e_f(1)$=y. A graph that admits E-cordial labeling is called as E-cordial graph.Yilmaz and Cahit prove that trees $T_n$are E-cordial iff for n not congruent to 2(mod 4), K_n are E-cordial iff n not congruent to 2(mod 4), Fans F_n are E-cordial iff for n not congruent to 1(mod 4).Yilmaz and Cahit observe that A graph on n vertices cannot be E-cordial if n is congruent to 2 (mod 4).One should refer Dynamic survey of graph labeling by Joe Gallian [2] for more results on E-cordial graphs.

The graphs we consider are finite, undirected, simple and connected. For terminology and definitions we refer Harary[3] and Dynamic survey of graph labeling by Joe Gillian [2]. The families we discuss are obtained by taking acopies of graph G and fuse it with one end of path P_t.Further take one point union on different points to get $G^{(k)}$. If there are more pathsthat all are attached at the same fixed point on G we represent these families by G(tPn).We take t = 1 and G = C_3, n=2,3,4.

3. Preliminaries:

3.1 Fusion of vertex. Let G be a (p,q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has at least p-1vertices and q-1 edges.[5]

3.2 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies.If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

3.3 A tail graph (also called as antenna graph) is obtained by fusing a path p_k to some vertex of G. This is denoted by tail(G, P_k). If there are t number of tails of equal length say (k-1) then it is denoted by tail(G, tp_k).If there are two or more tails attached at same vertex of G we denote it by tailG(P_t,P_k..)If G is a (p,q) graph and a tail P_k is attached to it then tail(G, P_k) has p+k-1 vertices and q+k-1 edges.

4. Main Results:

Theorem 4.1 All structures of one point union of k copies ofG = tail (C_3,P_2)i.e. $G^{(K)}$ are e-cordial provided k is not congruent to 3(mod 4).

Proof: The different structures are due to the common vertex is changed.

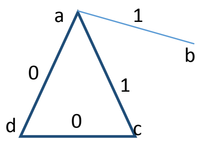

Fig.4.1 common vertex can be 'a', 'b', and 'c' or 'd' . $v_f(0,1)= (2,2)$, $e_f(0,1)= (2,2)$

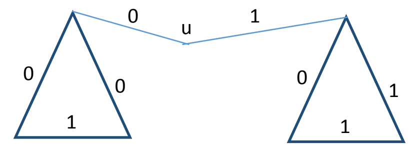

Figure 4.2 : edge labels are shown.$v_f(0,1)= (3,4)$, $e_f(0,1)= (4,4)$

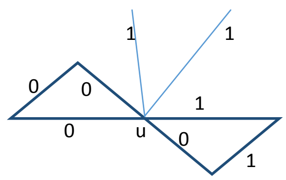

Figure 4.3 : edge labels are shown.$v_f(0,1)= (3,4)$, $e_f(0,1)= (4,4)$

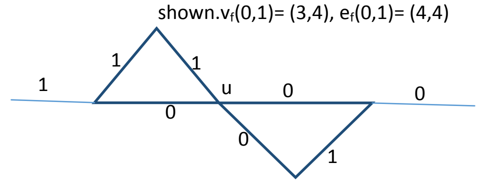

Figure 4.4 : edge labels are shown.$v_f(0,1)= (3,4)$, $e_f(0,1)= (4,4)$

It follows from figure 4.1 that there are only three non-isomorphic structures possible and are at verices 'a', 'b', and 'c' or'd'. In $G^{(K)}$ when k =1 the copy in figure 4.1 will work.

Define a functionf:$E((G)^{(k)})\rightarrow\{0,1\}$. This gives us four types of labeling as shown in figure 4.1, fig.4.2, fig.4.3 and fig 4.4.

For all other K>1 we first obtain $G^{(K)}$ for k = 2x, x=1,2,..,. This is done by fusing x times at vertex 'u' the block in figure 4.2 or figure 4.3 or figure 4.4 depending on if one point union is to be taken at 'b', at 'a',or at 'c' respectively.$G^{(K)}$ for k = 2x + 1 , x=1,2,..,.is obtained by fusing a copy in figure 4.1 on respective vertex at vertex u.

The label number distribution for all structures is as follows:

For k = 2x, x=1,3,5,..we have $v_f(0,1)= (3+6x,4+6x)$, $e_f(0,1)= (4x,4x)$ and label of common vertex '1'.
For k = 2x, x=2, 4, 6,..we have $v_f(0,1)= (7+6x, 6+6x)$, $e_f(0,1)= (4x,4x)$ and label of common vertex '0'.When the common vertex is 'b' or 'c' for k = 2x+1, for x = 2,4,6,,..we have label distribution as $v_f(0,1)= (8+6x,8+6x)$, $e_f(0,1)= (2+4x, 2+4x)$ and *label of common vertex is '1'*.and when common vertex is 'a' for k = 2x+1, for x = 2,4,6,,..we have label distribution as $v_f(0,1)= (8+6x,8+6x)$, $e_f(0,1)= (2+4x, 2+4x)$. The only difference is at *label of common vertex is '0'*.
Thus the graph is e-cordial. #

Theorem 4.2 All structures of one point union of k copies of G = tail (C_3, $2P_2$)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..

Proof: There are four possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'a', 'b' or 'c'. This is clear from fig 4.5.

The function f:$E(G^{(K)})\rightarrow\{0,1\}$ gives following three types of labels. We combine it to obtain a labeled copy of $G^{(K)}$. The numbers in diagrams are edge labels.

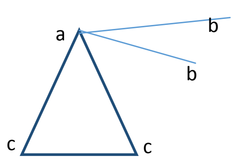

Fig.4.5 common vertex can be 'a', 'b', and 'c' or 'd' . $v_f(0,1)= (2,2)$, $e_f(0,1)= (2,2)$

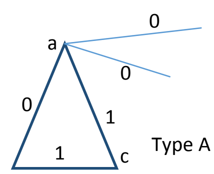

Fig.4.6 $v_f(0,1)= (3,2)$, $e_f(0,1)= (3,2)$

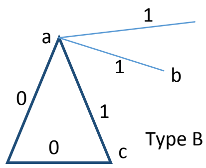

Fig.4.7 $v_f(0,1)= (1,4)$, $e_f(0,1)= (2,3)$

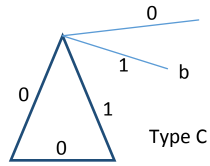

Fig.4.8 $v_f(0,1)= (3,2)$, $e_f(0,1)= (3,2)$

In structure 1 we fuse Type A and Type B label at vertex 'a' whose label is '1'. For k≡1 (mod2) we use Type A label and Type B label for k ≡0 (mod2).
In structure 2 we fuse Type A and Type B label at vertex 'c'whose label is '1'. For k≡1 (mod2) we use Type A label and Type B label for k ≡0 (mod2).
In structure 3 we fuse Type C and Type B label at vertex 'b'whose label is '1'. For k≡1 (mod2) we use Type C label and Type B label for k ≡0 (mod2).

The resultant label numbers are $v_f(0,1)= (3+4x,2+4x)$, $e_f(0,1)= (3+5x,2+5x))$ for k =2x + 1 such that x=0, 1, 2 …and $v_f(0,1)= (5+4x,4+4x)$, $e_f(0,1)= (5x,5x)$ for k =2x such that x= 1, 2 …Thus for all values of k we have the graph is e-cordial. #

Theorem 4.3 All structures of one point union of k copies of G = tail (C_3, P_3)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..

Proof: There are four possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'a', 'b','c'or 'd'. This is clear from fig 4.5.The function $f:E(G^{(K)})\rightarrow\{0,1\}$ gives following two types of labels. We combine it to obtain a labeled copy of $G^{(K)}$.

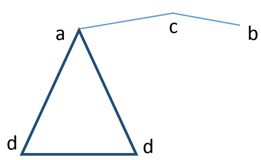

Fig.4.9 common vertex can be 'a', 'b', and 'c' or 'd' .

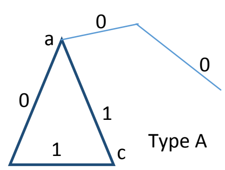

Fig.4.10 $v_f(0,1)= (3,2)$, $e_f(0,1)= (3,2)$

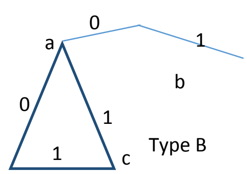

Fig.4.11 $v_f(0,1)= (1,4)$, $e_f(0,1)= (2,3)$

In structure 1 we fuse Type A and Type B label at vertex 'b'. For k≡1 (mod2) we use Type A label and Type B label for k ≡0 (mod2).
In structure 2 we fuse Type A and Type B label at vertex 'c'. For k≡1 (mod2) we use Type A label and Type B label for k ≡0 (mod2).
In structure 3 we fuse Type A and Type B label at vertex 'a'. For k≡1 (mod2) we use Type A label and Type B label for k ≡0 (mod2).
In structure 4 we fuse Type A and Type B label at vertex 'd'. For k≡1 (mod2) we use Type A label and Type B label for k ≡0 (mod2).
The resultant label numbers are $v_f(0,1)= (3+4x,2+4x)$, $e_f(0,1)= (3+5x,2+5x)$ for k =2x + 1 such that x=0, 1, 2 …and $v_f(0,1)= (5+4x,4+4x)$, $e_f(0,1)= (5x,5x)$ for k =2x such that x= 1, 2 …Thus for all values of k we have the graph is e-cordial. #

Theorem 4.4 All structures of one point union of k copies of G = tail (C_3,3P_2)i.e. $G^{(K)}$ are e-cordial for all knot congruent to 1(mod 4).

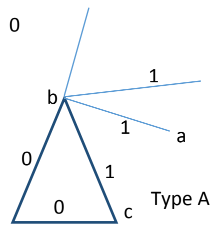

Fig.4.12 $v_f(0,1)$= (2,4), $e_f(0,1)$= (3,3).This is **not** e-cordial.

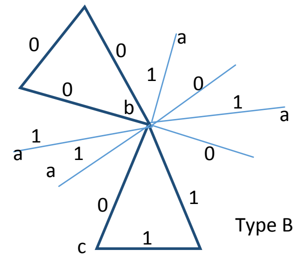

Fig.4.13 $v_f(0,1)$= (5,6), $e_f(0,1)$= (6,6).label of vertex 'b'= 1

Proof: There are 3 possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'a', 'b',' or 'c' on G. This is clear from fig 4.12.When k ≡1 (mod 4), the structures are not e-cordial.The function f:E($G^{(K)}$)→{0,1} gives types B label as above shown in fig 4.13. We combine it to obtain a labeled copy of $G^{(K)}$.To obtain any structure we first use Type B label repeatedly for k times where k is even number given by k = 2x; x=1, 2, …

To obtain labeled copy of $G^{(K)}$ for k = 2x+1 and k not congruent to 1(mod 4) ,we first obtain the structure for k = 2x and at the common point of it fuse Type A label with vertex on it , same as the common vertex.

For structure 1 the fusion point on G is vertex 'a'.

For structure 2 the fusion point on G is vertex 'b'.

For structure 3 the fusion point on G is vertex 'c' .

In all structures we get label number distribution as follows: $v_f(0,1)$= (5x+1,5x), $e_f(0,1)$= (6x,6x) when k = 2x, x = 2,4,6..and fusion vertex label as '0'.When k = 2x; x= 1, 3, 5 …we have $v_f(0,1)$= (5x,5x+1), $e_f(0,1)$= (6x,6x) and the vertex label of common vertex is '1'.When k is of type K = 3+4x, x= 0,1,2,3..$v_f(0,1)$= (10x+8, 10x+8), $e_f(0,1)$= (6x,6x).And fusion vertex label as '0'

Thus the graph is e-cordial. #

Theorem 4.5 All structures of one point union of k copies of G = tail (C_3,P_3,P_2)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod 4).

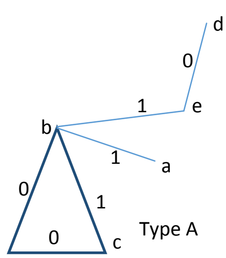

Fig.4.14 $v_f(0,1)$= (2,4), $e_f(0,1)$= (3,3).This is **not** e-

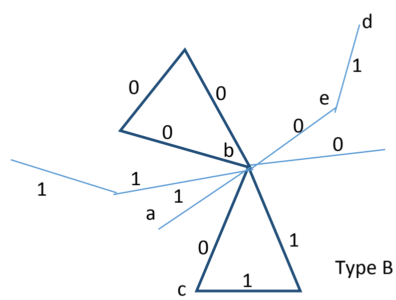

Fig.4.15 $v_f(0,1)$= (5,6), $e_f(0,1)$= (6,6)

Proof: There are 5 possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'a', 'b',' d' , 'e' or 'c' on G. This is clear from fig 4.14.When k ≡1 (mod 4), the structures are not e-cordial.

The function f:E($G^{(K)}$)→{0,1} gives type B label as shown in fig. 4.15. We combine it to obtain a labeled copy of $G^{(K)}$.To obtain any structure we first use Type B label repeatedly for k times where k is even number given by k = 2x; x=1, 2, …

To obtain labeled copy of $G^{(K)}$ for k = 2x+1 and k not congruent to 1(mod 4) ,we first obtain the structure for k = 2x and at the common point of it fuse Type A label with vertex on it , same as the common vertex.

For structure 1 the fusion point on G is vertex 'a'.

For structure 2 the fusion point on G is vertex 'b'.

For structure 3 the fusion point on G is vertex 'd'.
For structure 1 the fusion point on G is vertex 'e'.
For structure 2 the fusion point on G is vertex 'c'.

In all structures we get label number distribution as follows: $v_f(0,1)$= (5x+1,5x), $e_f(0,1)$= (6x,6x) when k = 2x, x = 2,4,6..and fusion vertex label as '0'.When k = 2x; x= 1, 3, 5 …we have $v_f(0,1)$= (5x,5x+1), $e_f(0,1)$= (6x,6x) and the vertex label of common vertex is '1'.

When k is of type K = 3+4x, x = 0,1,2,3..$v_f(0,1)$= (10x+8, 10x+8), $e_f(0,1)$= (6x,6x).With fusion vertex label as '0'
Thus the graph is e-cordial. #

Theorem 4.6 All structures of one point union of k copies of G = tail (C_3, P_4)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod 4).

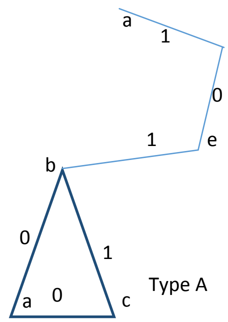

Fig.4.16$v_f(0,1)$= (2,4), $e_f(0,1)$= (3,3).This copy is **not** e-cordial.

Proof: There are 5 possible structures on $G^{(k)}$ which are pairwise non-isomorphic depending on common point 'a', 'b',' d' , 'e' or 'c' on G. This is clear from fig 4.16.When k ≡1 (mod 4), the structures are not e-cordial. The function f:E($G^{(K)}$)→{0,1} gives type B label as shown in fig. 4.17. We combine it to obtain a labeled copy of $G^{(K)}$.To obtain any structure we first use Type B label repeatedly for k times where k is even number given by k = 2x; x=1, 2, …

To obtain labeled copy of $G^{(K)}$ for k = 2x+1 and k not congruent to 1(mod 4) ,we first obtain the structure for k = 2x and at the common point of it fuse Type A label with vertex on it , same as the common vertex.

For structure 1 the fusion point on G is vertex 'a'.
For structure 2 the fusion point on G is vertex 'b'.
For structure 3 the fusion point on G is vertex 'd'.
For structure 1 the fusion point on G is vertex 'e'.
For structure 2 the fusion point on G is vertex 'c'.

In all structures we get label number distribution as follows: $v_f(0,1)$= (5x+1,5x), $e_f(0,1)$= (6x,6x) when k = 2x, x = 2,4,6..and fusion vertex label as '0'.When k = 2x; x= 1, 3, 5 …we have $v_f(0,1)$= (5x,5x+1), $e_f(0,1)$= (6x,6x) and the vertex label of common vertex is '1'.

When k is of type K = 3+4x, x = 0,1,2,3..$v_f(0,1)$= (10x+8, 10x+8), $e_f(0,1)$= (6x,6x).
Thus the graph is e-cordial. #

Conclusions: In this paper we have obtained one point union of k copies of tail graph tail(G,P_t) .We have taken G = C_3 and t = 2, 3, 4. Doing so we also consider tail $P_t$attached to G is splited in a different tails of smaller length such that sum of all edges will be t-1.In obtaining $G^{(k)}$ we get non isomorphic structures .these all are e-cordial depending on main result that number of vertices must not be congruent to 2 (mod 4).

The results we have obtained are : that All structures of one point union of k copies of

1) G = tail (C_3, P_2)i.e. $G^{(K)}$ are e-cordial provided k is not congruent to 3(mod 4).
2)G = tail (C_3, $2P_2$)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..
3) G = tail (C_3, P_3)i.e. $G^{(K)}$ are e-cordial for all k=1, 2, ..
4) G= tail (C_3, $3P_2$)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod
5)G = tail (C_3, P_3, P_2)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod 4).
6)G= tail (C_3, P_4)i.e. $G^{(K)}$ are e-cordial for all k not congruent to 1(mod 4).

We obtain different structures on same graph and show that they all are e-cordial.This property og G is called as invariance under e-cordiality.

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