Some Allied Open Functions via Semi*Preopen and Preopen Sets in Topology

V.Ananthasuruthi[#1] S.Pasunkili Pandian[#2]
#1 M.Phil Research scholar Aditanar college of Arts and Science
, #2 Associate Professor, Department of Mathematics, Aditanar college of Arts and Science
Tiruchendur Tamil Nadu India.

Abstract:

*In this paper, we define a new classes of functions, namely p-Semi*Preopen, p*- Semi*Preopen functions and contra-p-Semi*Preopen functions using Semi*preirresolute functions, Semi*Preopen sets, and preopensets and their properties are investigated.*

Key words :

*Preopen sets, Semi*Preopen sets, Preopen functions, Semi*Preopen functions, p-Semi*Preopen, p*- Semi*Preopen functions and contra-p-Semi*Preopen functions.*

1. INTRODUCTION

In 1963, Levine [7] introduced the concept of semiopen sets in topological spaces. Andrijevic [1] introduced the class of semipreopen and semipreclosed sets in topological spaces. In 2013,S.pasunkilipandian [11] introduced new class of semi star preopen sets in topological spaces. Navalagi [8] introduced new class of functions in p-semipreopen function and contra-p-semipreopen function**.** The purpose of this paper is to study some properties of p-Semi*Preopen, p*- Semi*Preopen functions and contra-p-Semi*Preopen functions.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, γ) (or simply X, Y and Z) denote topological spaces on which no separation axioms are assumed. The closure and the interior of A $\subset$ X are denoted by *Cl*(*A*) and Int(A) respectively.

Definition 2.1[7]: A subset A of a topological space (X, τ) is called generalized closed (briefly g-closed) if C*l* (A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Definition 2.2[11]**:** Let A be a subset of (X, τ). Then generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by C*l**(A).

Definition 2.3: The subset A of a topological space (X, τ) is semiopen if A $\subseteq$ C*l* (Int(A)).

Definition 2.4: The subset A of a topological space (X, τ) is preopen if A $\subseteq$ Int (C*l*(A)). The family of all semiopen (resp.preopen,semipreopen) sets of a space X is denoted by SO(X) (resp.PO(X),SPO(X)) and that of semi closed (resp.preclosed,semipreclosed) sets of a space X is denoted by SF(X)(resp.PF(X),SPF(X)).

Definition 2.5: The preclosure of a set A of X is the intersection of all preclosed sets that contain A and A is denoted by pC*l*(A).
The union of all preopen sets which are contained in A is called the pre-interior of A is denoted by pInt(A).

Definition 2.6: The subset A of a topological space (X, τ) is called semi*preopen if A $\subseteq$ C*l** (pInt(A)).

Definition 2.7: A function $f: X \longrightarrow Y$ is called preopen, if for every open set B of X, *f*(B) is preopen in Y.

Definition 2.8: A function $f: X \longrightarrow Y$ is called semi*preopen, if the image of each open set of X, is semi*preopen in Y. It is denoted by S*PO(X).

Definition 2.9: A function $f: X \longrightarrow Y$ is called semi*preopen in the sense of Cammaroto and Noiri[CN], if the image of each semiopen set of X, is a semi*preopen in Y.

Definition 2.10: A function $f:X\longrightarrow Y$ is called contra-preopen, if for every open set B of X, f(B) is preclosed in Y.

Definition 2.11: A function $f:X\longrightarrow Y$ is said to be semi*preirresolute, if f^{-1}(V) is semi*preopen in X for every semi*preopen set V in Y.

Remark 2.12: (i). A subset A is preopen if and only if pInt(A) = A.

(ii). A subset A is preclosed if and only if pC*l* (A) = A.

Theorem 2.13: Every open set is preopen.

Theorem 2.14 : Every open set is semi*preopen.

Theorem 2.15: Every semi*preopen set is semipreopen.

3. p-SEMI*PREOPEN FUNCTIONS

In this section we define p-semi*preopen functions, p*-semi*preopen functions and contra- p-semi*preopen functions and investigate some of their properties.

Definition 3.1: A function $f:X\longrightarrow Y$ is said to be p-semi*preopen if the image of each semi*preopen set of X is an preopen set in Y.

Example 3.2 : Let X ={*a*, b, c} be a set with topology τ ={ϕ, X, {*a*}, {b}, {*a*, b}, {*a*, c} } and Y={1, 2, 3 } be a set with topology σ ={ϕ, Y, {1}, {3}, {1, 3} }.Then S*PO(X)= {ϕ, X, {*a*}, {b}, {*a*, b}, {*a*, c} } and PO(Y) = {ϕ, {1}, {3}, {1, 3}, Y }. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a function defined by $f(a) = 1 = f(c)$ and $f(b)$ =3. Then f is p-semi*preopen function.

Theorem 3.3: Every p-semi*preopen function is preopen.

Proof: Let $f:X\longrightarrow Y$ be a p-semi*preopen function. Let V be an open set in X. Since every open set is semi*preopen, V is semi*preopen in X. This implies f(V) is preopen in Y. Therefore f is preopen.

Remark 3.4: The converse of the above theorem 3.3 is not true as shown in the following example.

Example 3.5: Let X ={*a*, b, c} be a set with topology τ ={ϕ, X, {*a*}, {b}, {*a*, b} } and Y={1, 2, 3 } be a set with topology σ ={ϕ, Y, {1}, {2}, {1, 2}, {1, 3} }.Then S*PO(X)= {ϕ, X, {*a*}, {b}, {*a*, b}, {b, c}, {*a*, c} } and PO(Y)= {ϕ, {1}, {2}, {1, 2}, {1, 3}, Y }. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a function defined by $f(a)$=2 , $f(b)$=1, and $f(c)$=3. Then clearly f is preopen. but f is not p-semi*preopen function since f({*a*, c})= {2, 3} $\notin$ PO(Y).

Theorem 3.6: A function $f:X\longrightarrow Y$ is said to be p-semi*preopen if and only if for every subset A$\subset$ X, f(s*pInt (A)) $\subset$ pInt (f(A)).

Proof: Let f be a p-semi*preopen function. Then f (s*pInt (A))$\subset$ f(A) for each A$\subset$ X. Since s*pInt(A) is a semi*preopen set in X, f (s*pInt(A)) is a preopen function contained in Y. Also, pInt (f(A)) is a largest preopen set contained in f(A).That is, pInt(f(A)) $\subset$ f(A).Therefore f (s*pInt (A)) $\subset$ pInt (f(A)). On the other hand, let f (s*pInt(A)) $\subset$ pInt (f(A)) for every subset A$\subset$ X. Let G be any semi*preopen set in X. Then f(G) = f (s*pInt (G)) $\subset$ pInt (f(G)) which implies that f(G)$\subset$ pInt (f(G)).But in general, pInt (f(G))$\subset$ f(G). Therefore f(G)= pInt (f(G)), which implies that f(G) is a preopen set in Y. Hence f is a p-semi*preopen function.

Definition 3.7: A function $f:X\longrightarrow Y$ is called p*-semi*preopen functions if the image of each preopen set U of X is semi*preopen in Y.

Example 3.8: Let X={1, 2, 3} with τ ={ϕ, X, {1}, {3}, {1, 3} } and Y={*a*, b, c},with σ ={ϕ, Y, {*a*}, {b}, {*a*, b}, { *a*, c }}. Then PO(X)={ϕ, {1}, {3}, {1, 3}, X } and S*PO(Y)= {ϕ, Y, {*a*}, {b}, {*a*, b}, {*a*, c}}. Let $f:X\longrightarrow Y$ defined by $f(1) = a$, f(2)= c, f(3) = b. Then f is p*-semi*preopen.

Theorem 3.9: Every p*-semi*preopen function is semi*preopen.

Proof: Let $f:X\longrightarrow Y$ be a p*-semi*preopen function. Let U be an open set in X.Since every open set is preopen, we have U is preopen in X. This implies f(U) is semi*preopen in Y.Hence f is semi*preopen.

Remark 3.10: The converse of above theorem 3.9 is not true can be seen from the following example.

Example 3.11: Let X ={*a*, b, c} be a set with the topology τ ={ϕ, X, {*a*}} and Y={1, 2, 3}, with σ ={ϕ, Y, {1}, {2}, {1, 2}, {1, 3} }.Then PO(X) = {ϕ, X, {*a*}, {*a*, b}, {*a*, c}} and S*PO(Y)= {ϕ, Y, {1}, {2}, {1, 2}, {1, 3}}. Let f :(X, τ) ⟶(Y, σ) be a function defined by f (a) =2, f(b) =1, f(c) =3.Then f is semi*preopen. But f is not p*-semi*preopen since for the preopen set {a, c} in Y, f ({a, c})= {2, 3} $\notin$ S*PO(Y).

Remark 3.12: The concept of p*-semi*preopen function and semi*preopen (in the sense of Cammaroto and Noiri) sets are independent as seen from the following examples.

Example 3.13: Let X ={a, b, c} be a set with topology τ ={ϕ , X, {a}, {b}, {a, b}} and Y={1, 2, 3 } be a set with topology σ ={ϕ , Y, {1}, {2}, {1, 2}, {1, 3} }.Then PO(X)= {ϕ , {a}, {b}, {a, b}, X}, SO(X)= {ϕ , {a}, {b}, {a, b}, {a, c}, {b, c}, X} and S*PO(Y)= {ϕ , {1}, {2}, {1, 2}, {1, 3}, Y }. Let f :(X, τ) ⟶(Y, σ) be a function defined by f (a)=2, f(b)=1, f(c)=3. Then f is p*-semi*preopen.But f is not semi*preopen [CN] since f ({a, c})={2, 3} $\notin$ S*PO(Y).

Example 3.14: Let X ={a, b, c, d} be a set with topology τ ={ϕ , X, {a, b}, {a, b, c}} and Y = {1, 2, 3} be a set with topology σ ={ϕ , Y, {1}, {3}, {1, 3} }.Then PO(X) = {ϕ , {a}, {b}, {a, b}, {b, c}, {c, d}, {a, c}, {a, d}, {b, d}, X}, SO(X) ={ ϕ , {a, b}, {a, b, c}, {a, b, d}, X} and S*PO(Y)= {ϕ , {1}, {3}, {1, 2}, {1, 3}, {2, 3}, Y}. Let f :(X, τ) ⟶(Y, σ) be a function defined by f (a) = f(d)=1, f(b) = f(c) = 2.Then f is semi*preopen[CN]. But f is not p*-semi*preopen function since the preopen set {b} is in Y, f({b}) = {2} $\notin$ S*PO(Y).

PRE-SEMI*PREOPEN FUNCTIONS

Definition 3.15: A function $f:X\longrightarrow Y$ is called pre-semi*preopen if the image of each semi*preopen set in X is a semi*preopen set in Y.

Example 3.16: Let X = {a, b, c} be a set with topology τ ={ϕ , X, {a}, {b}, {a, b}, {a, c}} and Y={1, 2, 3} be a set with topology σ = {ϕ , Y, {1}, {3}, {1, 3} }.Then S*PO(X)= {ϕ , X, {a}, {b}, {a, b}, {a, c}} and S*PO(Y)= {ϕ , {1}, {3}, {1, 2}, {1, 3}, {2, 3}, Y}. Let f :(X, τ) ⟶(Y, σ) be a function defined by f (a)=1, f(b)=3, f(c)=2.then f is pre-semi*preopen.

Theorem 3.17: Every pre-semi*preopen function is semi*preopen.

Proof: Let $f:X\longrightarrow Y$ be a pre-semi*preopen function.let G be an open set in X.Since every open set is semi*preopen,we have G is semi*preopen in X. This implies f(G) is semi*preopen in Y.Therefore f is semi*preopen.

Remark 3.18: The converse of the above theorem 3.17 is not true as shown in the following example.

Example 3.19: Let X = {a, b, c} be a set with topology τ ={ϕ , X, {b}, {c}, {b, c}} and Y={1, 2, 3} be a set with topology σ ={ϕ , Y, {1}, {3}, {1, 3} }.Then S*PO(X)= {ϕ , X, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}} and S*PO(Y)= {ϕ , {1}, {3}, {1, 2}, {1, 3}, {2, 3}, Y}. Let f : (X, τ) ⟶ (Y, σ) be a function defined by f (a)=2, f(b) =3, f(c) =1. Then f is semi*preopen function, but f is not pre-semi*preopen function since f({a}) = {2} $\notin$ S*PO(Y).

Theorem 3.20:Let X, Y and Z be three topological spaces and let $f:X\longrightarrow Y$ and g: Y⟶Z be two functions. Then the following conditions are true.

i)if f is pre-semi*preopen function and g is p-semi*preopen function, then g$o$$f$ is p- semi* preopen function.

ii) if f is p-semi*preopen function and g is p*-semi*preopen function, then g$o$$f$ is pre-semi*preopen function.

iii) if f is semi*preopen function and g is p-semi*preopen function, then g$o$$f$ is preopen function.

iv) if f is preopen function and g is p*-semi*preopen function, then g$o$$f$ is semi*preopen function.

Proof: **(i)** Let $f:X\longrightarrow Y$ be a pre-semi*preopen function and g: Y⟶Z is p-semi*preopen function.let U be a semi*preopen subset of X. Since f is pre-semi*preopen function,we have f(U) is semi*preopen set of Y.Now, g is p-semi*preopen function, g(f(U))=g$o$$f$(U) is preopen in Z. This implies that g$o$$f$:$X\longrightarrow Z$ is p- semi* preopen function.

(ii) Let U be a semi*preopen subset of X.Since f is p-semi*preopen function, we have f(U) is preopen set of Y. Also g: Y⟶Z is p*-semi*preopen function, g(f(U))= g$o$$f$(U) is semi* preopen in Z. This implies that g$o$$f$: $X\longrightarrow Z$ is pre-semi*preopen function.

(iii) Let u be an open set in X.Since f is semi*preopen, we have f(u) is semi*preopen function in Y.Again, since g is p-semi*preopen function and f(u) is semi*preopen, g(f(u))= g$o$$f$(u) is preopen in Z.Therefore g$o$$f$ is preopen function.

(iv) Let G be an open set in X.Then f(G) is preopen in Y.[Since f is preopen function].Since g is p*-semi*preopen function and f(G) is preopen function, we have g(f(G))= g*o*f(G) is semi*preopen in Z.Therefore g*o*f is semi*preopen function.

4. **CONTRA-p-SEMI*PREOPEN FUNCTIONS**

Definition 4.1: A function $f:X\longrightarrow Y$ is said to be contra- p-semi*preopen function if the image of each semi*preopen set of X is preclosed set in Y.

Example 4.2: Let X = {a, b, c} be a set with topology τ ={ϕ , X, {a}, {a, b}} and Y={1, 2, 3} be a set with topology σ ={ϕ , Y, {1}, {3}, {1, 3} }.Then S*PO(X)= {ϕ , X, {a}, {a, b}, {a, c}} and PO(Y) = {ϕ , {1}, {3}, {1, 3}, Y} and PF(Y)= {ϕ ,{2}, {1, 2}, {2, 3}, Y}. Let f :(X,τ) ⟶(Y, σ) be a function defined by $f(a) = 2$, f(b) = 3, $f(c)$ =1. Then f is contra- p-semi*preopen function.

Theorem 4.3:Every contra- p-semi*preopen function is contra-preopen.

Proof: Let $f:X\longrightarrow Y$ be a contra- p-semi*preopen function.let U be a open set in X.Since every open set is semi*preopen, we have U is semi*preopen in X.This implies f(U) is preclosed. Therefore f is contra-preopen function.

Remark 4.4: The converse of the above theorem 3.23 is not true as shown in the following example.

Example 4.5: Let X = {a, b, c} be a set with topology τ ={ϕ , X, {c}, {a, c}} and Y={1, 2, 3} be a set with topology σ ={ϕ , Y, {2}, {2, 3}}. Then S*PO(X)= {ϕ , X, {c}, {b, c}, {a, c}} and PO(Y)= {ϕ , {2}, {1, 2}, {2, 3}, Y} and PF(Y)= {ϕ , {1, 3}, {3}, {1}, Y}. Let f :(X,τ) ⟶(Y, σ) be a function defined by $f(a)$=1= f(c), f(b) = 2. Then f is contra-preopen function. But f is not contra- p-semi*preopen function since f ({b, c})={1, 2}= f(X) $\notin$ PF(Y).

Theorem 4.6: Let X,Y be any two topological spaces.Then $f:X\longrightarrow Y$ is contra-p-semi*preopen if and only if the image of each p-semi*preclosed set of X is preopen in Y.

Proof : Let $f:X\longrightarrow Y$ be a contra- p-semi*preopen.let V be a semi*preclosed set in X. Then X\V is semi*preopen in X. By our assumption, f(X\V) is preclosed in Y. That is, Y/f(V) is preclosed in Y. This implies f(V) is preopen in Y. Conversely, assume that image of every semi*preclosed set of X is preopen in Y. Let V be a semi*preopen in X. Then X\V is semi*preclosed in X. Therefore our assumption, f(X\V) is pre-open in Y.That is, Y/ f(V) is preopen in Y. This implies f(V) is preclosed in Y. Therefore f is contra-p-semi*preopen.

Theorem 4.7: For a function $f:X\longrightarrow Y$ the following are equivalent: i) f is contra-p-semi*preopen

ii) for every subset B of Y and for every semi*preclosed subset F of X with f^{1}(B)$\subseteq$ F, then there exists a preopen subset O of Y with B$\subseteq$ O and f^{1}(O) $\subseteq$ F.

iii) for every $y\in$ Y and for every semi*preclosed subset F of X with $f^{1}(y)$ $\subseteq$ F, there exists a preopen subset O of Y with y$\in$ O and f^{1}(O) $\subseteq$ F.

Proof: (i)$\Rightarrow$ (ii): Let B be a subset of Y.let F be a semi*preclosed subset of X with f^{1}(B)$\subseteq$ F. Then F^c is semi*preopen.Since f is contra- p-semi*preopen function,we have $f(F)^c$ is preclosed in Y.This implies $[f(F^c)]^c$ is preopen in Y. Put O=$[f(F^c)]^c$. Therefore O is preopen in Y. Since f^{1}(B)$\subseteq$ F, we have $f(F^c)\subseteq B^c$and B$\subseteq$ O. Moreover $f^{1}(O) = f^{1}[f(F^c)]^c =[f^{1}[f(F^c)]^c \subseteq (F^c)^c$ = F.This implies f^{1}(O) $\subseteq$ F.

(ii) $\Rightarrow$ (iii): put B={y}.

(iii)$\Rightarrow$ (i): Let A be a semi*preopen subset of X. Then let $y \in (f(A)^c$.and let F=A^c. By (iii) there exist a preopen subset O_y of Y with $y\in O_y$ and $f^{1}(O_y) \subseteq$ F. Then we see that $y\in O_y \subseteq [f(A)]^c$. Hence $[f(A)]^c$ ={ O_y / $y\in$ $[f(A)]^c$ is preopen. Therefore f(A) is a preclosed subset of Y.Hence f is contra-p-semi*preopen function.

Theorem 4.8: Let f : (X,τ) ⟶(Y, σ) be a function. Then if f is contra-p-semi*preopen, $pCl(f(O))\subseteq f(s*pCl(O))$ for every semi*preopen subset O of X.

Proof:Let O be semi*preopen subset of X.Since f is contra-p-semi*preopen, f(O) is preclosed subset of Y. Then p$Cl(f(O))=f(O)$. Also, $(f(O)\subseteq f(s*pCl(O))$. This implies $pCl(f(O))\subseteq f(s*pCl(O))$ for every O$\in$ S*PO(X).

Theorem 4.9: Every semi*preopen function is semipreopen function.

Proof: Let f :X⟶Y be a semi*preopen function. Let V be an open set in X. Then f(V) is semi*preopen function in Y. Since every semi*preopen set is semipreopen, f(V) is semipreopen function in Y. Therefore f is semipreopen function.

Theorem 4.10: Let f :(X, τ) ⟶(Y, σ) be a function. If f is contra-p-semi*preopen,and semipreopen (ie,f(O) $\subseteq$ pInt(pCl f(O))) for every semipreopen set O of X, then f is p-semi*preopen.

Proof: Let A be a semi*preopen subset of X. Since f is semipreopen, f(A) $\subseteq$ pInt(pCl f(A)). Since f is contra-p-semi*preopen, f(A) is preclosed. This implies pInt(pCl(f(A))=pInt(f(A)) and hence f(A) $\subseteq$ pInt(f(A)). Also pInt(f(A)) $\subseteq$ f(A). That is, f(A)=pInt(f(A)). Therefore f(A) is preopen.Hence f is p-semi*preopen.

Theorem 4.11: Let f :(X, τ) ⟶(Y, σ) and g:(Y, σ)⟶(Z, γ) be two functions such that go$f$:(X, τ) ⟶(Z, γ) is contra-p-semi*preopen function. If f is a semi*preirresolute and surjective, then g is contra-p-semi*preopen function.

Proof: Suppose V is any semi*preopen set in Y. Since f is semi*preirresolute, f^{-1}(V) is semi*preopen set in X. Since go$f$ is contra-p-semi*preopen function and surjective, go$f$(f^{-1}(V))=g(V) is preclosed in Z. Therefore g is contra-p-semi*preopen function.

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