Finite source inventory system with negative customers

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Abstract

We consider a finite source queueing-inventory system with a service facility, wherein the demand of a customer is satisfied only after performing some service on the item which is assumed to be of random duration. An arriving customer turns out to be an ordinary customer with probability r and a negative customer with probability $(1-r)$, $(0 \leq r \leq 1)$. An ordinary customer, on arrival, joins the queue and the negative customer does not join the queue and takes away one waiting customer if any. The inventory is replenished according to an (s, S) policy and the replenishing times are assumed to be exponentially distributed. The server provides two types of services - one with essential service and the other with a second optional service. The service times of the 1st (essential) and 2nd (optional) services are independent and exponentially distributed. The service process is subject to interruptions, which occurs according to a Poisson process. The interrupted server is repaired at an exponential rate. The joint probability distribution of the number of customers in the system and the inventory level is obtained in the steady state case. Various system performance measures are derived and the total expected cost rate is computed under a suitable cost structure.

Keywords: (s, S) policy, Inventory with service time, Markov process, Negative Customers

1 Introduction

Research on queueing systems with inventory control has captured much attention of researchers over the last decades. Many researchers assumed that customers arrive at the service facility one by one and require service. In order to complete the customer service, an item from the inventory is needed. A served customer departs immediately from the system and the on - hand inventory decreases by one at the moment of service completion. An inventory system with service facility the reader is referred to [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

In this paper, we consider a continuous review (s, S) perishable inventory system with server interruptions and negative customers in which server provides two types of services - one with essential service and the other with a second optional service. The joint distribution of the number of customers in the system and the inventory level is obtained in the steady state case. Various system performance measures are derived and the total expected cost rate is computed under a suitable cost structure.

The rest of the paper is organized as follows. In the next section, we describe the mathematical model and the notations used in this paper are defined. Analysis of the model and the steady state solution of the model are proposed in section 3. Some key system performance measures and total expected cost rate are derived in section 4.

2 Mathematical model

Consider a service facility in which perishable items are stocked and the items are delivered to the demanding customers. The maximum capacity of the inventory is S. The demands are generated by a finite number of homogeneous sources N, $0 < N < \infty$ and the demand time points form a quasi-random distribution with parameter $\lambda(> 0)$. The probability that a customer is an ordinary is r and a negative is $(1-r)$. The removal rule adopted in this paper is RCE (Removal of a customer at the end), i.e., arrival of a negative customer removes only a customer at the end excluding the one who is receiving the service at the time of arrival of a negative customer. The arrival of a negative customer has no effect to the empty service station. The demand is for single item per customer. The items are issued to the demanding customers only after some random time due to some service on it. This type of service is referred to as first essential service (FES).

This system has a single server who provides preliminary first essential service (FES) denoted by the rate μ_1 to all arriving customers one by one according to first in first out (FIFO) discipline. As soon as the first essential service of a customer is completed, the server may provide second optional service (SOS) denoted by the rate μ_2 with probability p to only those customers who opt for it otherwise leaves the system with the complementary probability q, where p+q=1. While the server is in working state i.e., server is providing first essential service or second optional service, it may interruption at any time with interruption rate α_1 during first essential service (FES) and α_2 during second optional service (SOS). As soon as server interruption occurs, it is immediately sent for repairing where repair time denoted by η_1 for first essential service and η_2 for second optional service. After repairing, the server renders remaining service of the customers of both of the phases (FES or SOS). It is assumed that the server is under interruption, no further interruption can be fall the server. The service times of the first essential service, the second optional service and the duration of an interruption are assumed to follow an exponential distribution.

Life time of each item has negative exponential distribution with parameter $\gamma > 0$. We have assumed that an item of inventory that makes it into the service process cannot perish while in service. The reorder level for the commodity is fixed as s and an order is placed when the inventory level reaches the reorder level s. The ordering quantity for the commodity is $Q(= S - s > s + 1)$ items. The requirement $Q > s + 1$ ensures that after a replenishment the inventory level will be always above the reorder level. Otherwise it may not be possible to place reorder which leads to perpetual shortage. The lead time is assumed to be distributed as negative exponential with parameter $\beta(>0)$. The following notations are used in this paper.

$$\begin{array}{rcl}
\mathbf{e} & : & \text{a column vector of appropriate dimension containing all ones} \\
\delta_{ij} & : & \begin{cases} 1 & \text{if} \quad j=i \\ 0 & \text{otherwise} \end{cases} \\
\bar{\delta_{ij}} & : & 1-\delta_{ij} \\
H(x) & : & \text{Heaviside function} \\
k \in V_i^j & : & k=i,i+1,\dots j.
\end{array}$$

3 Analysis

Let $L(t)$, $Y(t)$ and $X(t)$ respectively, denote the inventory level, the server status and the number of customers in the waiting area at time t.

Further, server status $Y(t)$ be defined as follows:

$$Y(t) \quad : \quad \begin{cases} 0, & \text{if the server is idle at time t,} \\ 1, & \text{if the server is providing first essential service to a customer at time t,} \\ 2, & \text{if the server is providing second optional service to a customer at time t,} \\ 3, & \text{if the server is on interruption during FES at time t,} \\ 4, & \text{if the server is on interruption during SOS at time t,} \end{cases}$$

From the assumptions made on the input and output processes, it can be shown that the stochastic process $I(t)=\{(L(t),Y(t),X(t)),t\geq 0\}$ is a continuous time Markov chain with discrete state space given by $E=E_a\cup E_b\cup E_c\cup E_d$ where

$$\begin{array}{rcl}
E_a & : & \{(0,0,i_3) \mid i_3=0,1,2,\dots,N,\} \\
E_b & : & \{(i_1,0,0) \mid i_1=1,2,\dots,S,\} \\
E_c & : & \{(i_1,i_2,i_3) \mid i_1=1,2,\dots,S,\ i_2=1,3,\ i_3=1,2,\dots,N,\ \} \\
E_d & : & \{(i_1,i_2,i_3) \mid i_1=0,1,2,\dots,S,\ i_2=2,4,\ i_3=1,2,\dots,N,\ \}
\end{array}$$

the infinitesimal generator Θ can be conveniently expressed in a block partitioned matrix with entries

$$\Theta = \begin{array}{c} S \\ S-1 \\ \vdots \\ s+1 \\ s \\ s-1 \\ \vdots \\ 1 \\ 0 \end{array} \begin{pmatrix}
A_S & B_S & & & & & & & & \\
 & A_{S-1} & B_{S-1} & & & & & & & \\
 & & \cdots & & & & & & & \\
 & & & \cdots & A_{s+1} & B_{s+1} & & & & \\
C & & & & & A_s & B_s & & & \\
 & C & & & & & A_{s-1} & & & \\
 & & \cdots & & & & \cdots & & & \\
 & & C & & & & & \cdots & A_1 & B_1 \\
 & & & C_1 & & & & & & A_0
\end{pmatrix}$$

where

$$[C_1]_{i_2 j_2} = \begin{cases} C_0 & j_2 = i_2, \quad i_2 = 0, \\ C_2 & j_2 = 1, \quad i_2 = 0, \\ C_3 & j_2 = i_2, \quad i_2 = 2, 4, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_0]_{i_3 j_3} = \begin{cases} \beta & j_3 = 0, \quad i_3 = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$[C_2]_{i_3 j_3} = \begin{cases} \beta & j_3 = i_3, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[C]_{i_2 j_2} = \begin{cases} C_4 & j_2 = i_2, \quad i_2 = 0, \\ C_3 & j_2 = i_2, \quad i_2 = 1, 2, 3, 4, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[C_4]_{i_3 j_3} = \begin{cases} \beta & j_3 = 0, \quad i_3 = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$[B_1]_{i_2 j_2} = \begin{cases} E_0 & j_2 = 2, \quad i_2 = 1, \\ D_0 & j_2 = 0, \quad i_2 = 1, \\ F_1 & j_2 = 2, \quad i_2 = 2, 4, \\ B_{00}^{(1)} & j_2 = i_2, \quad i_2 = 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[B_{00}{}^{(1)}]_{i_3 j_3} = \begin{cases} \gamma & j_3 = i_3, \quad i_3 = 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[E_0]_{i_3 j_3} = \begin{cases} p\mu_1, & j_3 = i_3, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[D_0]_{i_3 j_3} = \begin{cases} q\mu_1, & j_3 = i_3 - 1, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[F_1]_{i_3 j_3} = \begin{cases} \gamma, & j_3 = i_3, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

For $i_1 = 2, 3, \ldots, S$

$$[B_{i_1}]_{i_2 j_2} = \begin{cases} E_0 & j_2 = 2, \quad i_2 = 1, \\ F_0 & j_2 = 0, \quad i_2 = 1, \\ E_{i_1} & j_2 = 0, \quad i_2 = 0 \\ D_{(i_1-1)} & j_2 = 1, \quad i_2 = 1 \\ F_{i_1} & j_2 = i_2, \quad i_2 = 2, 4 \\ G_{(i_1-1)} & j_2 = i_2, \quad i_2 = 3 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[F_0]_{i_3 j_3} = \begin{cases} q\mu_1, & j_3 = 0, \quad i_3 = 1 \\ 0, & \text{otherwise.} \end{cases}$$

$$[E_{i_1}]_{i_3j_3} = \begin{cases} i_1\gamma, & j_3 = i_3, \quad i_3 = 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$[D_{(i_1-1)}]_{i_3j_3} = \begin{cases} (i_1-1)\gamma, & j_3 = i_3, \quad i_3 \in V_1^N \\ q\mu_1, & j_3 = i_3 - 1, \quad i_3 \in V_2^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[F_{i_1}]_{i_3j_3} = \begin{cases} i_1\gamma, & j_3 = i_3, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[A_0]_{i_2j_2} = \begin{cases} V_0 & j_2 = i_2, \quad i_2 = 0, \\ V_1 & j_2 = i_2, \quad i_2 = 2, \\ V_2 & j_2 = 4, \quad i_2 = 4, \\ R & j_2 = 2, \quad i_2 = 4 \\ T_2 & j_2 = 4, \quad i_2 = 2 \\ U & j_2 = 0, \quad i_2 = 2 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[U]_{i_3j_3} = \begin{cases} \mu_2, & j_3 = i_3 - 1, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[T_2]_{i_3j_3} = \begin{cases} \alpha_2, & j_3 = i_3, \quad i_3 \in V_1^N \\ 0, & \text{otherwise.} \end{cases}$$

$$[V_0]_{i_3j_3} = \begin{cases} r(N-i_3)\lambda, & j_3 = i_3 + 1, \quad i_3 \in V_0^{N-1} \\ (1-r)i_3(N-i_3)\lambda, & j_3 = i_3 - 1, \quad i_3 \in V_1^N \\ -(r(N-i_3)\lambda\bar{\delta}_{i_3N} + (1-r)(N-i_3)\lambda i_3\bar{\delta}_{i_30} + \beta) & j_3 = i_3, \quad i_3 \in V_0^N, \\ 0, & \text{otherwise.} \end{cases}$$

$$[V_1]_{i_3j_3} = \begin{cases} r(N-i_3)\lambda, & j_3 = i_3 + 1, \quad i_3 \in V_1^{N-1} \\ (1-r)(i_3-1)(N-i_3)\lambda, & j_3 = i_3 - 1, \quad i_3 \in V_2^N \\ -(r(N-i_3)\lambda\bar{\delta}_{i_3N} + (1-r)(N-i_3)\lambda(i_3-1)\bar{\delta}_{i_30} + \beta + \mu_2 + \alpha_2 +) & j_3 = i_3, \quad i_3 \in V_1^N, \\ 0, & \text{otherwise.} \end{cases}$$

$$[V_2]_{i_3j_3} = \begin{cases} r(N-i_3)\lambda, & j_3 = i_3 + 1, \quad i_3 \in V_1^{N-1} \\ (1-r)(i_3-1)(N-i_3)\lambda, & j_3 = i_3 - 1, \quad i_3 \in V_2^N \\ -(r(N-i_3)\lambda\bar{\delta}_{i_3N} + (1-r)(N-i_3)\lambda(i_3-1)\bar{\delta}_{i_30} + \beta + \eta_2) & j_3 = i_3, \quad i_3 \in V_1^N, \\ 0, & \text{otherwise.} \end{cases}$$

For $i_1 = 1, 2, 3, \dots, S$

$$[A_{i_1}]_{i_2 j_2} = \begin{cases} H & j_2 = 1, \quad i_2 = 0, \\ H_{i_1} & j_2 = i_2, \quad i_2 = 0, \\ J & j_2 = 0, \quad i_2 = 2, \\ J_0 & j_2 = 1, \quad i_2 = 2, \\ J_{i_1} & j_2 = i_2, \quad i_2 = 1, \\ K_{i_1} & j_2 = 2, \quad i_2 = 2, \\ L_0 & j_2 = 1, \quad i_2 = 3, \\ L_{i_1} & j_2 = i_2, \quad i_2 = 3, \\ R & j_2 = 2, \quad i_2 = 4, \\ R_{i_1} & j_2 = 2, \quad i_2 = 4, \\ T_1 & j_2 = 3, \quad i_2 = 1, \\ T_2 & j_2 = 4, \quad i_2 = 2, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[H]_{i_3 j_3} = \begin{cases} r(N - i_3)\lambda & j_3 = 1, \quad i_3 = 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[H_{i_1}]_{i_3 j_3} = \begin{cases} -(r(N - i_3)\lambda + i_1\gamma + H(s - i_1)\beta) & j_3 = i_3, \quad i_3 = 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$[J]_{i_3 j_3} = \begin{cases} \mu_2 & j_3 = 0, \quad i_3 = 1, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[J_0]_{i_3 j_3} = \begin{cases} \mu_2 & j_3 = i_3 - 1, \quad i_3 \in V_2^N, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[J_{i_1}]_{i_3 j_3} = \begin{cases} r(N - i_3)\lambda & j_3 = i_3 + 1, \quad i_3 \in V_1^{N-1}, \\ (1 - r)(i_3 - 1)(N - i_3)\lambda & j_3 = i_3 - 1, \quad i_3 \in V_2^N, \\ -(r(N - i_3)\lambda\bar{\delta}_{i_3 N} + (1 - r)(i_3 - 1)(N - i_3)\lambda\bar{\delta}_{i_3 1} + \mu_1 & \\ +H(s - i_1)\beta + (i_1 - 1)\gamma + \alpha_1) & j_3 = i_3, \quad i_3 \in V_1^N, \\ 0, & \text{otherwise.} \end{cases}$$

$$[K_{i_1}]_{i_3 j_3} = \begin{cases} r(N - i_3)\lambda & j_3 = i_3 + 1, \quad i_3 \in V_1^{N-1}, \\ (1 - r)(i_3 - 1)(N - i_3)\lambda & j_3 = i_3 - 1, \quad i_3 \in V_2^N, \\ -(r(N - i_3)\lambda\bar{\delta}_{i_3 N} + (1 - r)(i_3 - 1)(N - i_3)\lambda\bar{\delta}_{i_3 1} + & \\ \mu_2 + H(s - i_1)\beta + i_1\gamma + \alpha_2) & j_3 = i_3, \quad i_3 \in V_1^N, \\ 0, & \text{otherwise.} \end{cases}$$

$$[L_0]_{i_3 j_3} = \begin{cases} \eta_1 & j_3 = i_3, \quad i_3 \in V_1^N, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[R]_{i_3 j_3} = \begin{cases} \eta_2 & j_3 = i_3, \quad i_3 \in V_1^N, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[T_1]_{i_3 j_3} = \begin{cases} \alpha_1 & j_3 = i_3, \quad i_3 \in V_1^N, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[L_{i_1}]_{i_3 j_3} = \begin{cases} r(N - i_3)\lambda & j_3 = i_3 + 1, \quad i_3 \in V_1^{N-1}, \\ (1 - r)(i_3 - 1)(N - i_3)\lambda & j_3 = i_3 - 1, \quad i_3 \in V_2^N, \\ -(r(N - i_3)\lambda\bar{\delta}_{i_3 N} + (1 - r)(i_3 - 1)(N - i_3)\lambda\bar{\delta}_{i_3 1} + & \\ \eta_1 + H(s - i_1)\beta + (i_1 - 1)\gamma) & j_3 = i_3, \quad i_3 \in V_1^N, \\ 0, & \text{otherwise.} \end{cases}$$

$$[R_{i_1}]_{i_3 j_3} = \begin{cases} r(N-i_3)\lambda & j_3 = i_3+1, \quad i_3 \in V_1^{N-1}, \\ (1-r)(i_3-1)(N-i_3)\lambda & j_3 = i_3-1, \quad i_3 \in V_2^{N}, \\ -(r(N-i_3)\lambda\bar{\delta}_{i_3 N} + (1-r)(i_3-1)(N-i_3)\lambda\bar{\delta}_{i_3 1} + & \\ \eta_2 + H(s-i_1)\beta + i_1\gamma) & j_3 = i_3, \qquad i_3 \in V_1^{N}, \\ 0, & \text{otherwise.} \end{cases}$$

It can be noted that A_{i_1}, B_{i_1}, $i_1 = 2, \ldots, S$, A_1, and C are square matrices of order $(4N+1)$. C_1 is of size $(3N+1)\times(4N+1)$, A_0 is a square matrices of order $(3N+1)$, B_1 is of size $(4N+1)\times(3N+1)$. The sub matrices C_3, E_0, F_1, T_1, T_2, R, J_0, L_0, R_1, K_1, J_1, L_1, $D_{(i_1-1)}$, F_{i_1}, $G_{(i_1-1)}$, R_{i_1}, K_{i_1}, J_{i_1}, L_{i_1}, $i_1 = 2, 3, \ldots, S$, are square matrices of order N. C_4, H_1, E_{i_1}, H_{i_1} $i_1 = 2, 3, \ldots, S$, are square matrices of order 1. C_0 and J are matrices of order $(N+1)\times 1$. D_0, C_2, $B_{00}^{(1)}$, F_0 and H are matrices of order $N \times (N+1)$, $(N+1)\times N$, $1\times(N+1)$, $N\times 1$ and $1\times N$ respectively.

3.1 Steady State Analysis

It can be seen from the structure of Θ that the homogeneous Markov process $\{(L(t), Y(t), X(t)) : t \geq 0\}$ on the finite space E is irreducible, aperiodic and persistent non-null. Hence the limiting distribution

$$\phi^{(i_1,i_2,i_3)} = \lim_{t\to\infty} Pr[L(t) = i_1, Y(t) = i_2, X(t) = i_3 | L(0), Y(0), X(0)] \text{ exists.}$$

Let $\mathbf{\Phi}$ denote the steady state probability vector of the generator Θ. Computation of the steady state probability vector $\mathbf{\Phi} = (\mathbf{\Phi^{(0)}}, \mathbf{\Phi^{(1)}}, \ldots, \mathbf{\Phi^{(S)}})$, by solving the following set of equations,

$$\begin{aligned} \mathbf{\Phi^{i_1}} B_{i_1} + \mathbf{\Phi^{i_1-1}} A_{i_1-1} &= \mathbf{0}, \qquad i_1 = 1, 2, \ldots, Q, \\ \mathbf{\Phi^{i_1}} B_{i_1} + \mathbf{\Phi^{i_1-1}} A_{i_1-1} + \mathbf{\Phi^{(i_1-1-Q)}} C_1 &= \mathbf{0}, \qquad i_1 = Q+1, \\ \mathbf{\Phi^{i_1}} B_{i_1} + \mathbf{\Phi^{i_1-1}} A_{i_1-1} + \mathbf{\Phi^{(i_1-1-Q)}} C &= \mathbf{0}, \qquad i_1 = Q+2, Q+3, \ldots, S, \\ \mathbf{\Phi^{S}} A_S + \mathbf{\Phi^{s}} C &= \mathbf{0}, \end{aligned}$$

subject to conditions $\mathbf{\Phi}\Theta = \mathbf{0}$ and $\sum\sum\sum_{(i_1,i_2,i_3)} \phi^{(i_1,i_2,i_3)} = 1.$

This is done by the following algorithm.

Step 1. Solve the following system of equations to find the value of $\mathbf{\Phi^{Q}}$

$$\mathbf{\Phi^{Q}} \left[\left\{ (-1)^Q \sum_{j=0}^{s-1} \left[\left(\mathop{\Omega}_{k=Q}^{s+1-j} B_k A_{k-1}^{-1} \right) C A_{S-j}^{-1} \left(\mathop{\Omega}_{l=S-j}^{Q+2} B_l A_{l-1}^{-1} \right) \right] \right\} B_{Q+1} + A_Q + \left\{ (-1)^Q \mathop{\Omega}_{j=Q}^{1} B_j A_{j-1}^{-1} \right\} C \right] = \mathbf{0},$$

and

$$\mathbf{\Phi^{Q}}\left[\sum_{i_1=0}^{Q-1}\left((-1)^{Q-i_1}\underset{j=Q}{\overset{i_1+1}{\Omega}}B_jA_{j-1}^{-1}\right)+I\right.$$
$$\left.+\sum_{i_1=Q+1}^{S}\left((-1)^{2Q-i_1+1}\sum_{j=0}^{S-i_1}\left[\left(\underset{k=Q}{\overset{s+1-j}{\Omega}}B_kA_{k-1}^{-1}\right)CA_{S-j}^{-1}\left(\underset{l=S-j}{\overset{i_1+1}{\Omega}}B_lA_{l-1}^{-1}\right)\right]\right)\right]\boldsymbol{\pi} \;=\; 1.$$

Step 2. Compute the values of

$$\begin{aligned}\mathbf{\Omega_{i_1}} &= (-1)^{Q-i_1}\mathbf{\Phi^{Q}}\underset{j=Q}{\overset{i_1+1}{\Omega}}B_jA_{j-1}^{-1}, && i_1=Q-1,Q-2,\ldots,0\\ &= (-1)^{2Q-i_1+1}\mathbf{\Phi^{Q}}\sum_{j=0}^{S-i_1}\left[\left(\underset{k=Q}{\overset{s+1-j}{\Omega}}B_kA_{k-1}^{-1}\right)CA_{S-j}^{-1}\left(\underset{l=S-j}{\overset{i_1+1}{\Omega}}B_lA_{l-1}^{-1}\right)\right], &&\\ & && i_1=S,S-1,\ldots,Q+1\\ &= I, && i_1=Q\end{aligned}$$

Step 3. Using, Step 1 and Step 2, calculate the value of $\mathbf{\Phi^{(i_1)}}, i_1=0,1,\ldots,S$. That is,

$$\mathbf{\Phi^{(i_1)}} \;=\; \mathbf{\Phi^{(Q)}}\Omega_{i_1}, \quad i_1=0,1,\ldots,S.$$

4 System performance measures

4.1 Expected Inventory Level

$$\eta_I=\sum_{i_1=1}^{S}i_1\Phi^{(i_1)}\mathbf{e}$$

4.2 Expected Reorder Rate

$$\begin{aligned}\eta_R \;=\; &\sum_{i_3=1}^{N}\mu_1\phi^{(s+1,1,i_3)}+(s+1)\gamma\phi^{(s+1,0,0)}\\ &+\sum_{i_3=1}^{N}(s+1)\gamma(\phi^{(s+1,2,i_3)}+\phi^{(s+1,4,i_3)})+\sum_{i_3=1}^{N}s\gamma(\phi^{(s+1,1,i_3)}+\phi^{(s+1,3,i_3)})\end{aligned}$$

4.3 Expected Perishable Rate

$$\begin{aligned}\eta_P \;=\; &\sum_{i_1=1}^{S}i_1\gamma\phi^{(s+1,0,0)}+\sum_{i_1=2}^{S}\sum_{i_3=1}^{N}(i_1-1)\gamma(\phi^{(i_1,1,i_3)}+\phi^{(i_1,3,i_3)})\\ &+\sum_{i_3=1}^{N}\sum_{i_1=1}^{S}i_1\gamma(\phi^{(i_1,2,i_3)}+\phi^{(i_1,4,i_3)})\end{aligned}$$

4.4 Expected Number of Customers in the Waiting Area

$$\begin{aligned}\Gamma_1 &= \sum_{i_3=1}^{N} i_3 \phi^{(0,0,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}) \\ &\quad + \sum_{i_3=1}^{N}\sum_{i_1=0}^{S} i_3(\phi^{(i_1,2,i_3)} + \phi^{(i_1,4,i_3)})\end{aligned}$$

4.5 Expected Waiting Time

$$\eta_W = \frac{\Gamma_1}{\Gamma_2},$$

where Γ_1 is the expected number of customers in the waiting area and the effective arrival rate of the customer (Ross [11]), Γ_2 is given by

$$\begin{aligned}\Gamma_2 &= \sum_{i_3=0}^{N-1} r(N-i_3)\lambda\phi^{(0,0,i_3)} + \sum_{i_1=1}^{S} r(N-i_3)\lambda\phi^{(i_1,0,0)} \\ &\quad + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N-1} r(N-i_3)\lambda(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}) + \sum_{i_1=0}^{S}\sum_{i_3=1}^{N-1} r(N-i_3)\lambda(\phi^{(i_1,2,i_3)} + \phi^{(i_1,4,i_3)})\end{aligned}$$

4.6 Expected Loss Rate for Customers

$$\begin{aligned}\eta_L &= (N-i_3)\lambda\phi^{(0,0,N)} + \sum_{i_1=1}^{S}(N-i_3)\lambda(\phi^{(i_1,1,N)} + \phi^{(i_1,3,N)}) \\ &\quad + \sum_{i_1=0}^{S}(N-i_3)\lambda(\phi^{(i_1,2,N)} + \phi^{(i_1,4,N)})\end{aligned}$$

4.7 Expected Arrival Rate of Negative Customers

$$\begin{aligned}\eta_N &= \sum_{i_3=0}^{N}(1-r)i_3(N-i_3)\lambda\phi^{(0,0,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}(1-r)(i_3-1)(N-i_3)\lambda(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}) \\ &\quad + \sum_{i_1=0}^{S}\sum_{i_3=1}^{N}(1-r)(i_3-1)(N-i_3)\lambda(\phi^{(i_1,2,i_3)} + \phi^{(i_1,4,i_3)})\end{aligned}$$

4.8 Effective Interruption Rate

$$\eta_{INTR} = \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\alpha_1\phi^{(i_1,1,i_3)} + \sum_{i_1=0}^{S}\sum_{i_3=1}^{N}\alpha_2\phi^{(i_1,2,i_3)}$$

4.9 Effective Repair Rate

$$\eta_{RR} = \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\eta_1\phi^{(i_1,3,i_3)} + \sum_{i_1=0}^{S}\sum_{i_3=1}^{N}\eta_2\phi^{(i_1,4,i_3)}$$

4.10 Cost Analysis

The expected total cost per unit time (expected total cost rate) in the steady state for this model is defined to be

$$TC(S,s,N) = c_h\eta_I + c_s\eta_R + c_p\eta_P + c_w\eta_W + c_l\eta_L + c_i\eta_{INTR} + c_r\eta_{RR}$$

where

c_h : The inventory carrying cost per unit item per unit time
c_s : Setup cost per order
c_p : Perishable cost per unit item per unit time
c_w : Waiting cost of a customer per unit time
c_l : Loss per unit time due to arrival of a negative customer
c_i : Cost per interruption per unit time
c_r : Cost per repair per unit time

References

[1] Berman, O. and Kim, E. (1999). *Stochastic models for inventory management at service facility,* Stochastic Models, Vol.15, No.4, 695-718.

[2] Berman, O. and Kim, E. (2004). *Dynamic inventory strategies for profit maximization in a service facility with stochastic service,* Mathematical Methods of Operations Research, Vol.60, No.3, 497-521.

[3] Berman, O. and Kim, E. (2001). *Dynamic order replenishment policy in internet-based supply chains,* Mathematical Methods of Operations Research, Vol.53, No.3, 371-390.

[4] Berman, O. and Sapna, K.P. (2000). *Inventory management at service facility for systems with arbitrarily distributed service times,* Communications in Statistics. Stochastic Models, Vol.16, No.3, 343-360.

[5] Berman, O. and Sapna, K.P. (2001). *Optimal control of service for facilities holding inventory,* Computers and Operations Research, Vol.28, No.5, 429-441.

[6] Jeganathan K. and Periyasamy, C (2014), *A perishable inventory system with repeated customers and server interruptions,* Applied Mathematics and Information Sciences Letters, Vol. 2(2), 1-11.

[7] Krishnamoorthy, A., & Anbazhagan, N (2008), *Perishable inventory system at service facility with N policy,* Stochastic Analysis and Applications, 26(1), 1-17.

[8] Krishnamoorthy, A., Nair, S.S., & Narayanan, V.C., (2010), *An inventory model with server interruptions,* Opsearch, 25(1) 1-17.

[9] Krishnamoorthy, A., Nair, S.S., & Narayanan, V.C., (2012), *An inventory model with server interruptions and retrials,* Operations Research International Journal, 12(2) 151 - 171.

[10] Schwarz, M., Sauer, C., Daduna, H., Kulik, R. and Szekli, R. (2006). *M/M/1 queueing systems with inventory,* Queueing Systems Theory and Applications, Vol.54, No.1, 55-78.

[11] Ross, S. M. *Introduction to Probability Models*, Harcourt Asia PTE Ltd, Singapore, 2000.