Unsteady Mhd Casson Fluid Flow Through Porous Medium With Heat Source/Sink And Time Dependent Suction

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Abstract

This paper presents an analytical investigation of the unsteady MHD flow of Casson fluid over a porous plate embedded in a porous medium in the presence of heat source/sink and thermal radiation. The plate is subjected to a variable suction velocity. The present problem has immediate applications in optimization of solidification processes of metals, alloys, the geothermal source investigation and nuclear fuel debris treatment. The governing equations are solved under appropriate boundary conditions using the perturbation technique. The effect of various physical parameters on the velocity and temperature profiles is discussed and presented graphically. Also the skin friction coefficient and the Nusselt number are calculated with the aid of tables.

Keywords - *Casson Fluid, MHD, Heat Source/ Sink, Porous Medium.*

1. Introduction

The flow of non- Newtonian fluid in a porous medium has received considerable attention due to its extensive applications in ceramic processing, irrigation problems, enhanced oil recovery process and filtration process. In non-Newtonian fluids, Casson fluid is a shear thinning liquid that has an infinite viscosity at zero rates of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear. Casson fluid has attracted more attention of researchers due to its enormous applications in scientific as well as in engineering areas such as in metallurgy, food processing, drilling operations and bio-engineering operations. Batra and Jena [1] studied the flow of a Casson fluid in a slightly curved tube. The secondary flow of a Casson fluid in a slightly curved tube was studied by Das and Batra [2]. Casson fluid flow in a pipe filled with a homogeneous porous medium was investigated by Dash et al. [3]. Influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction was studied by Israel-Cookey et al. [4]. The effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet was studied by Chen [5]. Hameed and Nadeem [6] given good literature on unsteady MHD flow of a non-Newtonian fluid on a porous plate. The unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate was investigated by Mustafa et al. [7]. Analytic study on non-Newtonian natural convection boundary layer flow with variable wall temperature on a horizontal plate was performed by Shahmohamadi [8]. Mukhopadhyay [9] discussed the effects of thermal radiation on Casson fluid flow and heat transfer over an unsteady stretching surface subjected to suction/blowing. Later this, Mukhopadhyay et al. [10] studied Casson fluid flow over an unsteady stretching surface. Nadeema et al. [11] studied MHD three dimensional Casson fluid flow in two lateral directions past a porous linear stretching sheet. Exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet was provided by Bhattacharya et al. [12]. Pramanik [13] studied numerically by using Runge–Kutta method, the problem of Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. Immanuel et al. [14] investigated Casson flow of MHD fluid moving steadily with constant velocity. Mahanta and Shaw [15] used Spectral Relaxation Method and analyzed three dimensional Casson fluid flows past a porous linearly stretching sheet with convective boundary condition. Some analytical solutions for flows of Casson fluid with slip boundary conditions have been presented by Ramesh and Devakar [16]. The effects of slip on free convection flow of Casson fluid over an oscillating vertical plate have been reported by Muhammad et al. [17].

The aim of the present paper is to investigate the unsteady MHD Casson fluid flow past an infinite vertical porous plate embedded in a porous medium with heat source/sink and time dependent suction. The governing equations are solved analytically using the perturbation technique. The effects of various physical parameters like Casson parameter, thermal Grashof number, permeability parameter, radiation parameter, magnetic parameter and heat source/sink parameter on fluid velocity and temperature are discussed and shown through graphs.

2. Formulation of the Problem

Consider two dimensional laminar flow of an incompressible electrically conducting Casson fluid over a semi infinite vertically porous plate embedded in porous media. Here, the $\tilde{x}-$axis is measured along the plate and $\tilde{y}-$axis normal to the plate. A constant magnetic field of strength B_0 is applied in the perpendicular direction of the plate. The magnetic field is assumed to be of small intensity so that the induced magnetic field is negligible in comparison to the applied magnetic field. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as

$$\tau_{ij} = \begin{cases} \left(\mu_B + \dfrac{\tau_y}{\sqrt{2\pi}}\right) 2e_{ij} & \pi > \pi_c \\ \left(\mu_B + \dfrac{\tau_y}{\sqrt{2\pi_c}}\right) 2e_{ij} & \pi < \pi_c \end{cases}$$

where π is the product of deformation rate with itself, namely $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j) component of the deformation rate, π_c is the critical value of π, μ_B is plastic dynamics viscosity of the Casson fluid, τ_y is the yield stress of fluid.

Under these assumptions, the governing equations for unsteady flow of the viscous incompressible fluid are written as

Equation of Continuity

$$\frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \qquad \ldots (1)$$

Equation of Motion

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \nu\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} + g\beta_T\left(\tilde{T}-\tilde{T}_\infty\right) - \frac{\sigma B_0^2 \tilde{u}}{\rho} - \frac{\nu}{\tilde{K}}\tilde{u} \qquad \ldots (2)$$

Equation of Energy

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} + \tilde{v}\frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{k}{\rho c_p}\frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} - \frac{1}{\rho c_p}\frac{\partial \tilde{q}_r}{\partial \tilde{y}} - \frac{Q_0}{\rho c_p}\left(\tilde{T}-\tilde{T}_\infty\right) \qquad \ldots (3)$$

The boundary conditions are

$$\left.\begin{array}{lll} \tilde{u} = \tilde{u}_p, & \tilde{T} = \tilde{T}_w + \varepsilon\left(\tilde{T}_w - \tilde{T}_\infty\right)e^{\tilde{n}\tilde{t}} & at\ \tilde{y}=0 \\ \tilde{u} \to 0, & \tilde{T} = \tilde{T}_\infty & as\ \tilde{y}\to\infty \end{array}\right\} \qquad \ldots (4)$$

where $\tilde{u}$ and $\tilde{v}$ are the velocity component in $\tilde{x}$ and $\tilde{y}$ direction respectively, g is the gravitation due to acceleration, $\tilde{t}$ is the dimensional time, β_T is the coefficient of thermal expansion, ν is the kinematic viscosity, $\tilde{T}$ is the temperature of the fluid, $\tilde{T}_\infty$ is the fluid temperature far away from the wall, σ is the electrical conductivity of the fluid, ρ is the fluid density, $\tilde{K}$ is the permeability of porous medium, Q_0 is the dimensional heat absorption coefficient, γ is the Casson parameter, c_p is the specific heat at constant pressure, $\tilde{q}_r$ is the radiative heat flux, $\tilde{u}_p$ is the uniform velocity of the fluid in its own plane and k is the thermal conductivity.

Using Rosseland approximation, the radiative heat flux term is given as

$$\tilde{q}_r = \frac{-4\sigma_s}{3k_e}\frac{\partial \tilde{T}^4}{\partial \tilde{y}} \qquad \dots (5)$$

where σ_s is the Stefan Boltzmann constant and k_e is the mean absorption coefficient.
We assume that the temperature difference within the fluid is sufficiently small so that $\tilde{T}^4$ can be expressed as a linear function of the temperature. This is done by expanding $\tilde{T}^4$ in a Taylor series about $\tilde{T}_\infty$ which after neglecting higher order terms take the form

$$\tilde{T}^4 \cong 4\tilde{T}_\infty^3\tilde{T} - 3\tilde{T}_\infty^4 \qquad \dots (6)$$

From the equation of continuity it is clear that the suction velocity normal to the plate is a function of the time only which is taken in the form

$$\tilde{v} = -V_0\left(1+\varepsilon A e^{\tilde{n}\tilde{t}}\right) \qquad \dots (7)$$

Introducing the following non-dimensional variables

$$u = \frac{\tilde{u}}{U_0},\ v = \frac{\tilde{v}}{V_0},\ y = \frac{V_0\tilde{y}}{\nu}, U_p = \frac{\tilde{u}_p}{U_0},\ t = \frac{\tilde{t}V_0^2}{\nu},\ n = \frac{\tilde{n}\nu}{V_0^2}, Gr = \frac{\nu g\beta_T\left(\tilde{T}_w - \tilde{T}_\infty\right)}{U_0V_0^2},\ \nu = \frac{\mu}{\rho},$$
$$\theta = \frac{\tilde{T}-\tilde{T}_\infty}{\tilde{T}_w-\tilde{T}_\infty}, \Pr = \frac{\nu\rho c_p}{k}, M = \frac{\sigma B_0^2\nu}{\rho V_0^2},\ Q = \frac{Q_0\nu}{\rho c_p V_0^2}, R = \frac{4\sigma_s\tilde{T}_\infty^3}{kk_e}, K = \frac{\tilde{K}V_0^2}{\nu^2} \qquad \dots (8)$$

Using equations (5) to (8), the equation (2) and (3) are reduced to the following non-dimensional form as follows

$$\frac{\partial u}{\partial t} - \left[1+\varepsilon A e^{nt}\right]\frac{\partial u}{\partial y} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} + Gr\theta - \left(M + \frac{1}{K}\right)u \qquad \dots (9)$$

$$\frac{\partial \theta}{\partial t} - \left[1+\varepsilon A e^{nt}\right]\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1+\frac{4R}{3}\right)\frac{\partial^2 \theta}{\partial y^2} - Q\theta \qquad \dots (10)$$

where Gr is the thermal Grashof number, M is the magnetic field parameter, $\Pr$ is the Prandtl number, Q is the heat source/sink parameter, K is the permeability parameter, R is the radiation parameter, U_p is the dimensionless velocity of the plate, t is the dimensionless time, V_0 is the scale of suction velocity at the plate, U_0 is the free stream velocity, u and θ are the dimensionless velocity and temperature respectively and n is the frequency parameter.

The corresponding boundary conditions in non-dimensional form are

$$\left.\begin{array}{lll} u = U_p, & \theta = 1+\varepsilon e^{iwt} & at\ \ y = 0 \\ u \to 0, & \theta \to 0 & as\ \ y \to \infty \end{array}\right\} \qquad \dots (11)$$

3. METHOD OF SOLUTION

Equations (9) and (10) are coupled non-linear partial differential equations whose exact solutions are not possible. However, these equations can be reduced to a set of ordinary differential equations that can be solved analytically. This can be done by representing the velocity and temperature distribution as, where $\varepsilon(<< 1)$ is a small quantity

$$u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y) + o(\varepsilon^2) \qquad \dots (12)$$
$$\theta(y,t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + o(\varepsilon^2) \qquad \dots (13)$$

Substituting equations (12) and (13) into equations (9) and (10), equating the harmonic and non-harmonic terms, and neglecting the coefficient of $o\left(\varepsilon^2\right)$ the following equations are obtained

Zeroth- order equations

$$\left(1+\frac{1}{\gamma}\right)u_0'' + u_0' - \left(M + \frac{1}{K}\right)u_0 = -Gr\theta_0 \qquad \dots (14)$$

$$(3+4R)\theta_0'' + 3\Pr\theta_0' - 3Q\Pr\theta_0 = 0 \qquad \dots (15)$$

First-order equations

$$\left(1+\frac{1}{\gamma}\right)u_1'' + u_1' - \left(M + n + \frac{1}{K}\right)u_1 = -Gr\theta_1 - Au_0' \qquad \dots (16)$$

$$(3+4R)\theta_1'' + 3\Pr\theta_1' - 3\Pr(Q+n)\theta_1 = -3A\Pr\theta_0' \qquad \dots (17)$$

The corresponding boundary conditions are

$$\left.\begin{array}{ll} u_0 = U_p,\; u_1 = 0,\; \theta_0 = 1,\; \theta_1 = 1 & at \;\; y = 0 \\ u_0 \to 0,\; u_1 \to 0,\; \theta_0 \to 0,\; \theta_1 \to 0 & as \;\; y \to \infty \end{array}\right\} \qquad \dots (18)$$

On solving equations (14) to (17) under the boundary conditions (18), we obtained the solutions $u_0, u_1, \theta_0, \theta_1$. Finally, the expressions for velocity and temperature distribution are given by

$$u(y,t) = B_1 e^{-A_1 y} + B_2 e^{-A_2 y} + \varepsilon e^{nt}\left(B_5 e^{-A_1 y} + B_6 e^{-A_2 y} + B_7 e^{-A_3 y} + B_8 e^{-A_4 y}\right) \qquad \dots (19)$$
$$\theta(y,t) = e^{-A_1 y} + \varepsilon e^{nt}\left(B_3 e^{-A_1 y} + B_4 e^{-A_3 y}\right) \qquad \dots (20)$$

The local skin friction coefficient and the Nusselt number are given by

$$C_f = -\left(1+\frac{1}{\gamma}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= \left(1+\frac{1}{\gamma}\right)\left(B_1A_1 + B_2A_2 + \varepsilon e^{nt}\left(B_5A_1 + B_6A_2 + B_7A_3 + B_8A_4\right)\right) \qquad \dots (21)$$

$$Nu = x\frac{\left(\frac{\partial \tilde{T}}{\partial \tilde{y}}\right)_{\tilde{y}=0}}{\tilde{T}_\infty - \tilde{T}_w} \Rightarrow Nu\,\mathrm{Re}_x^{-1} = -\theta'(0)$$
$$= A_1 + \varepsilon e^{iwt}\left(B_3A_3 + B_4A_1\right) \qquad \dots (22)$$

where $\mathrm{Re}_x = \dfrac{xV_o}{\nu}$

4. Results and Discussion

In this section, a parametric study is performed to illustrate the effect of various physical parameters such as thermal Grashof number, magnetic field parameter, Prandtl number, heat source/sink parameter, permeability parameter, Casson parameter and radiation parameter on fluid velocity and temperature. The obtained numerical results for fluid velocity and temperature are presented through Figs. 1 to 10.

Fig. 1 represents the variation of fluid velocity for different values of Casson parameter. It is depicted from figure that as the Casson parameter increases, the fluid velocity increases near the plate while it decreases far away from the plate. Fig. 2 and 3 illustrate the effect of varying thermal Grashof number and permeability parameter on the fluid velocity. It is observed from Fig. 2 that velocity increases with the increasing values of thermal Grashof number. This is due to enhancement of buoyancy force. From Fig. 3 it is seen that the fluid velocity increases with increasing in permeability parameter due to decrease in resistance posed by the porous matrix.

It is evident from the Fig. 4 that fluid velocity decreases with increase in magnetic parameter. Physically, this refers to the fact that the application of transverse magnetic field will result in a resistive type force known as Lorentz force that is similar to drag force which tends to resist the fluid flow and thus reducing the fluid velocity. Fig. 5 show that velocity increases with increasing in radiation parameter. Fig. 6 and 8 elucidate the effects of Prandtl number on the fluid velocity and temperature. It is observed that both velocity and temperature decreases with increase in Prandtl number. The influence of heat sink parameter on the fluid velocity and temperature is shown in Fig. 7 and 9. It is observed from the figures that velocity and temperature decreases with an increase in the heat sink parameter. Fig. 10 represents the variation of fluid temperature for different values of thermal radiation parameter. It is observed from figure that with increase in radiation parameter there is an enhancement in the fluid temperature.

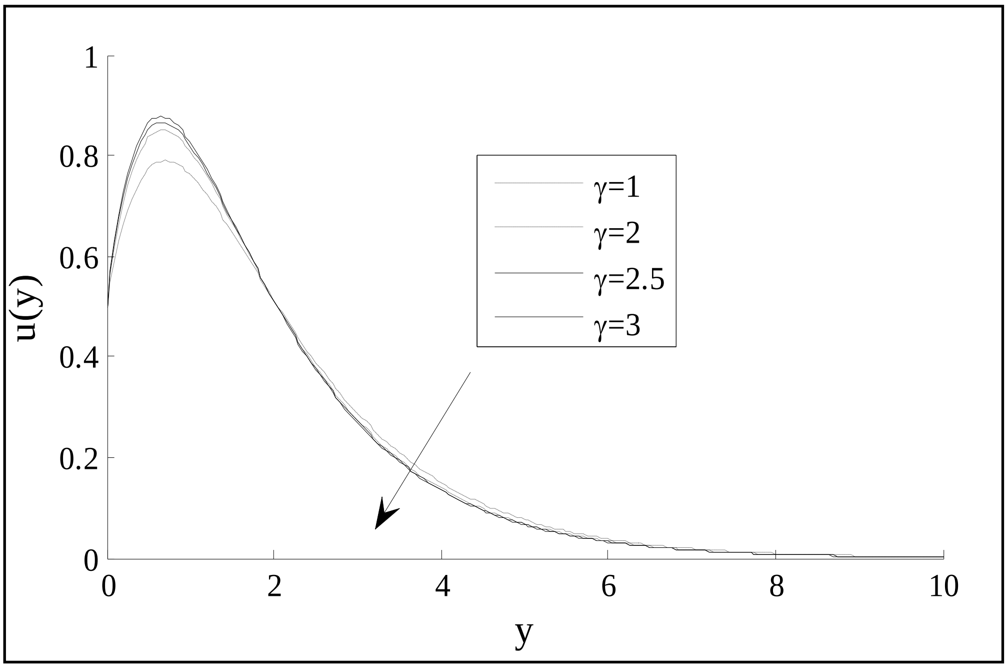

Fig. 1 Velocity distribution versus y when $Gr = 5,\ M = 1, R = 0.5, Q = 1, \Pr = 0.71, K = 1$.

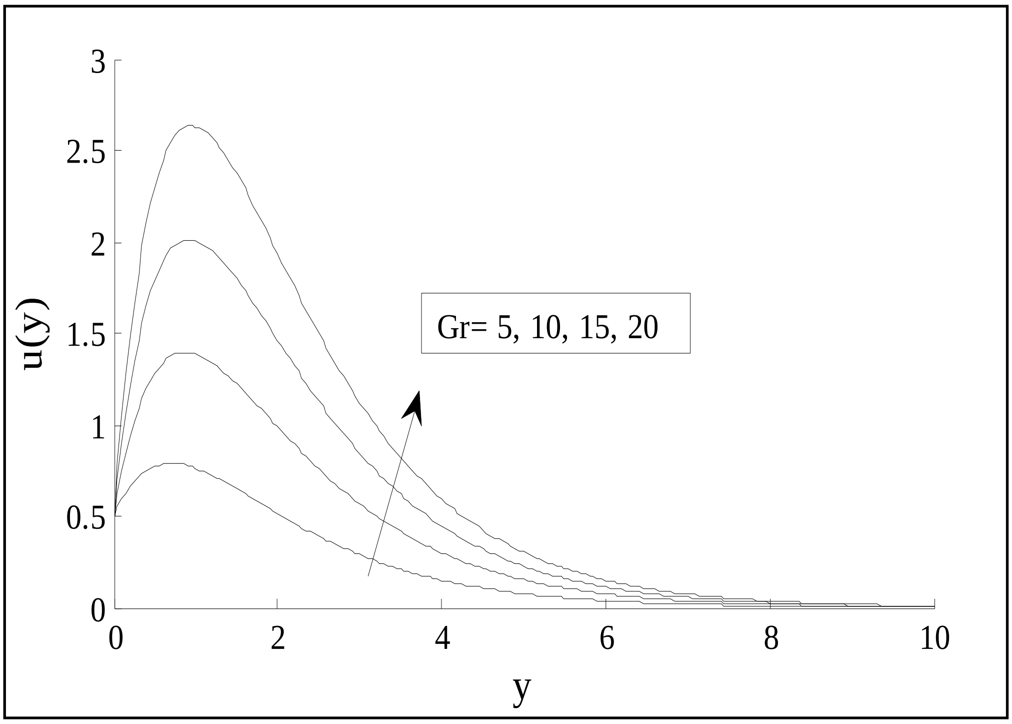

Fig. 2 Velocity distribution versus y when $\gamma = 1, M = 1, R = 0.5, Q = 1, \Pr = 0.71, K = 1$.

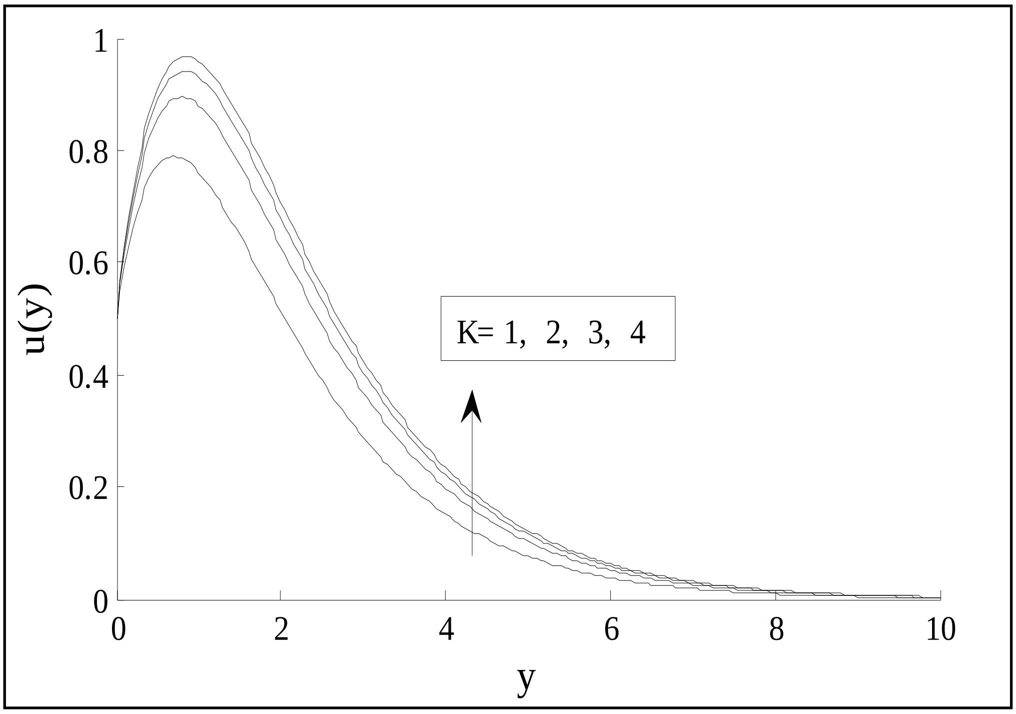

Fig. 3 Velocity distribution versus y when $\gamma = 1, M = 1, R = 0.5, Q = 1, Gr = 5, \Pr = 0.71$.

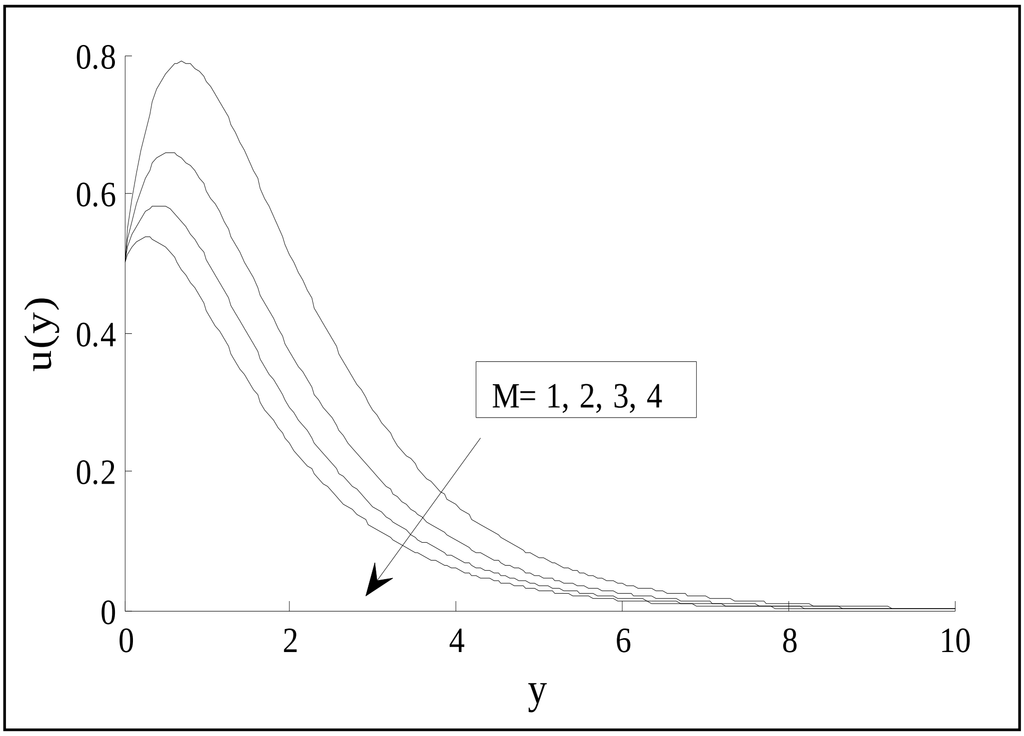

Fig. 4 Velocity distribution versus y when $\gamma = 1, Gr = 5, R = 0.5, Q = 1, \mathrm{K} = 1, \Pr = 0.71$.

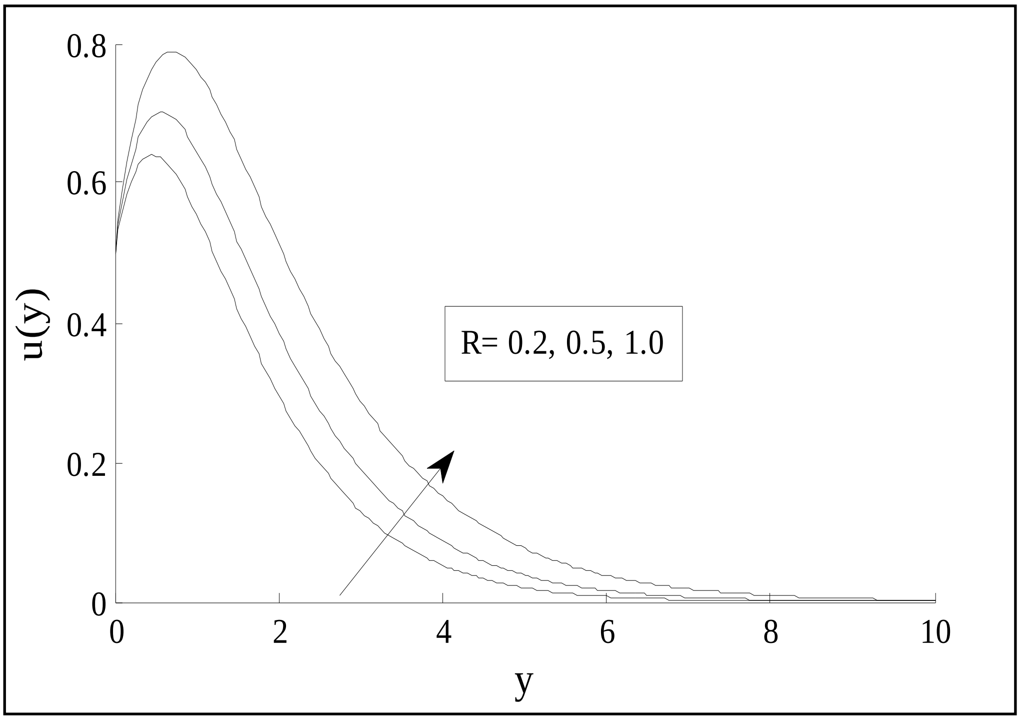

Fig. 5 Velocity distribution versus y when $\gamma = 1, Gr = 5, Q = 1, \Pr = 0.71, K = 1, M = 1$.

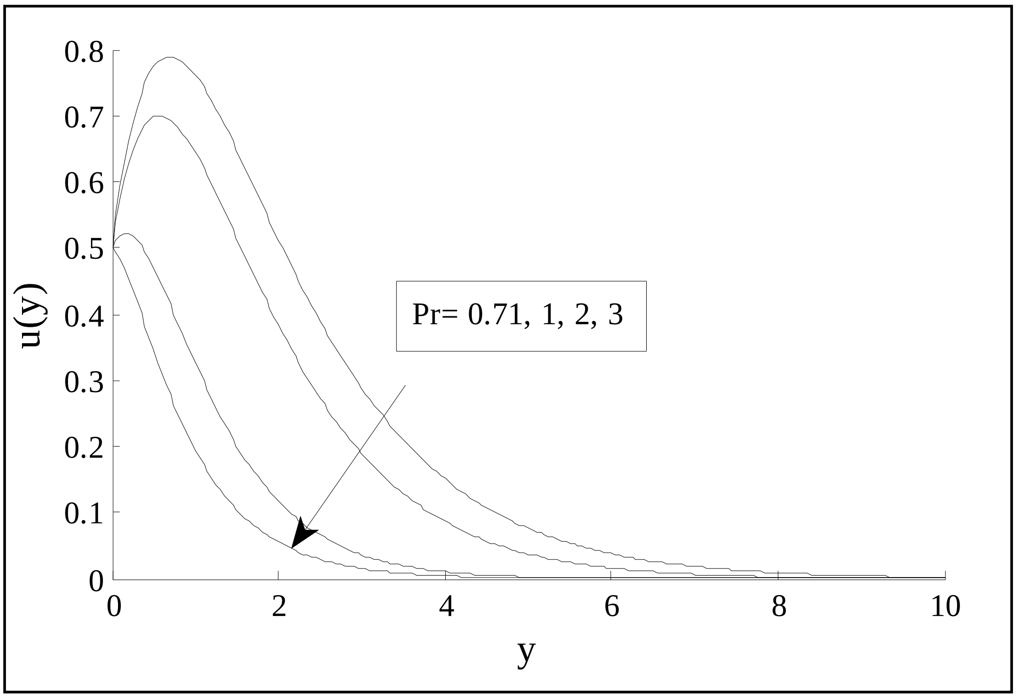

Fig. 6 Velocity distribution versus y when $\gamma = 1, Gr = 5, R = 0.5, Q = 1, K = 1, M = 1$.

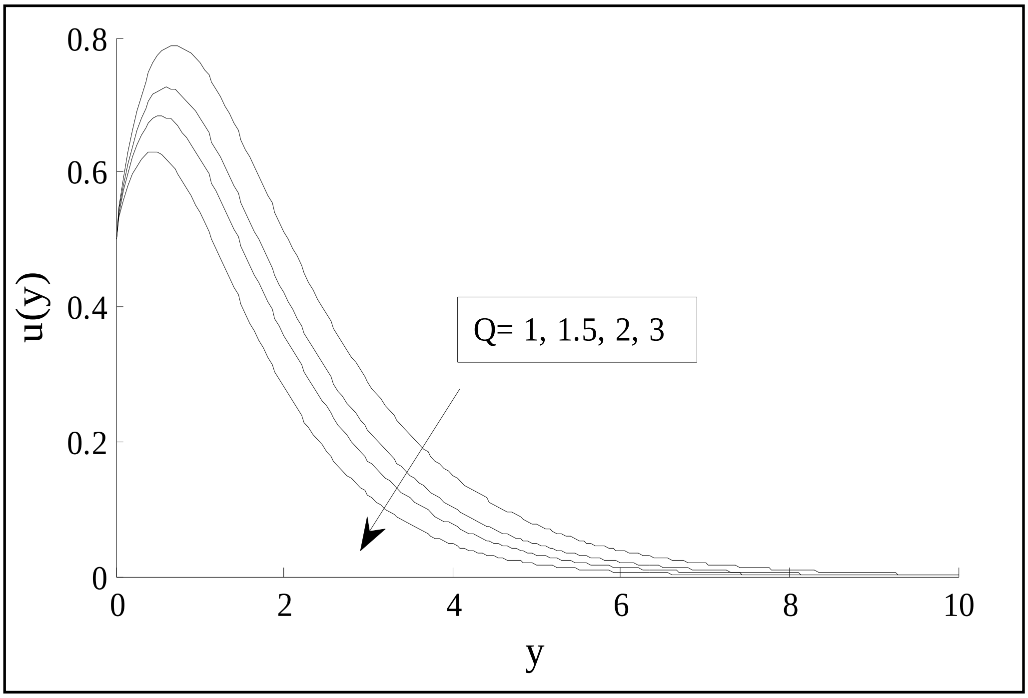

Fig. 7 Velocity distribution versus y when $\gamma = 1, Gr = 5, R = 0.5, \Pr = 0.71, M = 1, K = 1$.

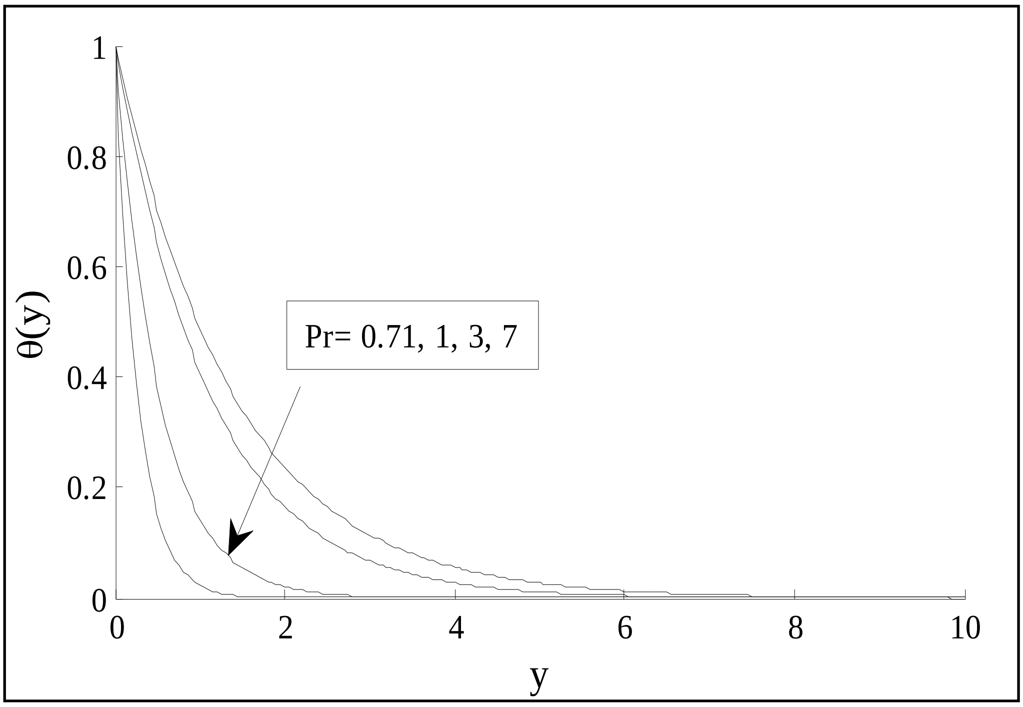

Fig. 8 Temperature distribution versus y when $\gamma = 1, Gr = 5, Q = 1, R = 0.5, M = 1, K = 1$.

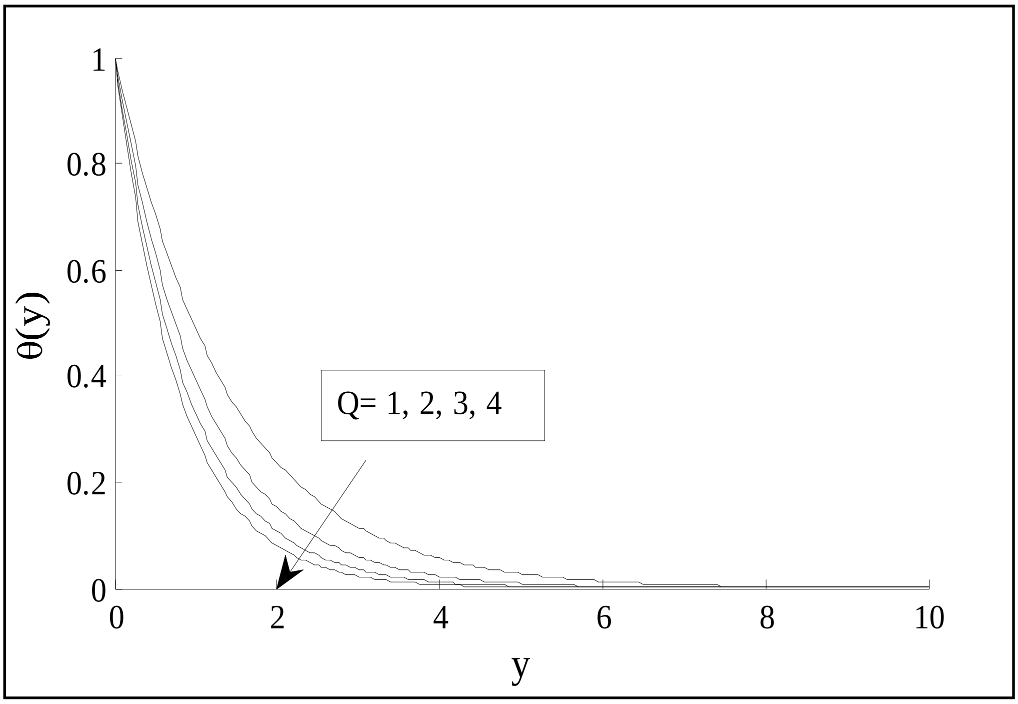

Fig. 9 Temperature distribution versus y when $\gamma = 1, Gr = 5, M = 1, R = 0.5, \Pr = 0.71, K = 1$.

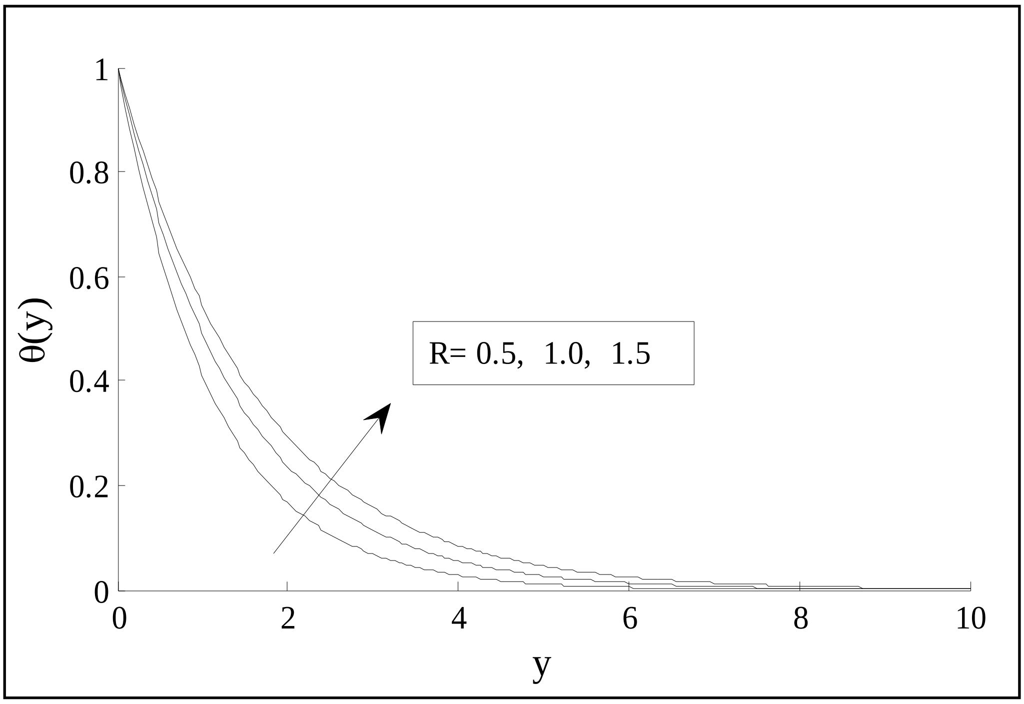

Fig. 10 Temperature distribution versus y when $\gamma=1, Gr=5, \Pr=0.71, Q=1, M=1, K=1$.

The numerical results for skin friction coefficient (C_f) and Nusselt number (Nu) for different values of physical parameters such as Casson parameter, magnetic parameter, permeability parameter, thermal Grashof number, radiation parameter, heat source/sink parameter and Prandtl number are shown in Tables 1 and 2. From Table 1 it is conclude that skin friction coefficient increases with increase in Casson parameter, magnetic parameter, Prandtl number and heat source/sink parameter whereas it show reverse effect for permeability parameter, radiation parameter and thermal Grashof number. It is noticed from Table 2 that Nusselt increases with increase in heat source parameter and Prandtl number whereas it decreases with increase in radiation parameter.

Table1: Numerical values of skin friction coefficient (C_f) for different values of physical parameters

γ	K	M	Pr	Q	R	Gr	C_f
1	1	1	0.71	1	1	5	-2.0730
1.5	1	1	0.71	1	1	5	-2.0393
2	1	1	0.71	1	1	5	-2.0181
2.5	1	1	0.71	1	1	5	-2.0036
1	2	1	0.71	1	1	5	-2.5178
1	3	1	0.71	1	1	5	-2.6964
1	4	1	0.71	1	1	5	-2.7933
1	1	2	0.71	1	1	5	--1.4274
1	1	3	0.71	1	1	5	-0.9560
1	1	4	0.71	1	1	5	-0.5816
1	1	1	1	1	1	5	-1.7161
1	1	1	3	1	1	5	-0.5670
1	1	1	7	1	1	5	0.1805
1`	1	1	0.71	2	1	5	-1.6409

1	1	1	0.71	3	1	5	-1.3753
1	1	1	0.71	4	1	5	-1.1855
1	1	1	0.71	1	2	5	-2.5212
1	1	1	0.71	1	3	5	-2.8080
1	1	1	0.71	1	4	5	-3.0133
1	1	1	0.71	1	1	7	-3.4152
1	1	1	0.71	1	1	10	-5.4284
1	1	1	0.71	1	1	15	-8.7838

Table 2: Numerical values of Nusselt number (Nu) for different values of physical parameters

Q	R	Pr	Nu
1	1	0.71	0.7327
2	1	0.71	0.9574
3	1	0.71	1.1318
4	1	0.71	1.2795
1	2	0.71	0.5535
1	3	0.71	0.4595
1	4	0.71	0.3999
1	1	1	0.9137
1	1	3	1.9709
1	1	7	3.8422

5. Conclusions

In this paper, the two dimensional flow of Casson fluid over an infinite vertical porous plate embedded in a porous medium under the influence magnetic field and heat source/sink is investigated. The numerical results for velocity and temperature distribution are obtained by using perturbation techniques. The main conclusions drawn from the above investigation are summarized as

1. The velocity of the fluid increases with increase in thermal Grashof number, permeability parameter and radiation parameter, whereas it decreases with increase in Prandtl number, magnetic parameter and heat sink parameter.
2. The velocity of the fluid shows dual behaviour for Casson parameter. As with increase in Casson parameter fluid velocity first increases and then starts decreasing.
3. The temperature of the fluid decreases with increase in Prandtl number and heat sink parameter, whereas it increases with increase in radiation parameter.
4. The skin friction coefficient increases with an increase in Casson parameter, magnetic parameter, Prandtl number and heat sink parameter, whereas it shows reverse effect for permeability parameter, radiation parameter and Grashof number.
5. The Nusselt number increases with increase in Prandtl number and heat sink parameter, whereas it decreases with increase in radiation parameter.

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