Heat Transfer Through Nano Fluid in a Vertical Wavy Channel with Travelling Thermal Waves

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Abstract: *The two dimensional free convection flow and heat transfer of a Cu and water nano-fluid in a vertical wavy channel is considered. Traveling thermal wave on the wavy boundaries are imposed. The channel is filled with porous medium. The set of governing equations with the above mentioned conditions leads to a system of non-linear differential equations. They are solved using RK 6th order method with the help of Mathematica 10.4 package. The flow and temperature are found with reverse behavior in either parts of the channel. The rate of heat transfer is also analyzed.*

Keywords: *Nano-fluid, Free convection, Wavy channel, Travelling Thermal Waves, Porous Medium, RK 6th order method.*

Introduction:

The heat transfer rate is more when the fluid flows on the rough surface than plain surface. The heat transfer rate is further more when flow is bounded in between two wavy walls. This complex geometry and the corresponding governing system of equations are more interesting and have lot of industrial applications like cooling of vehicles, rocket boasters, cross hatching of ablative surfaces and s film vaporization in combustion chambers etc**.**

The Channel with non –uniform wall temperatures was taken in a vertical direction with transverse magnetic field.Water with nano particles of magnegtic(Fe_3o_4) was selected as a conventional base fluid[1]. Rudraiah[2] has studied effect of magnetic field and velocity shear on the propagation of internal waves in Chiral Fluids. Mixed convective heat transfer of water/alumina nanofluid inside a vertical microchannel is investigated theoretically. A modified Buongiorno's model is employed for the nanofluid, which fully accounts for the effect of the nanoparticles migration[3].Mixed convective heat transfer of a nano fluid in a vertical channel partially filled with highly porous medium was studied by Perturbation method[4]The laminar fully developed mixed convection flow between two parallel vertical plates filled by a nano fluid is investigated by Buongiorno mathematical model.[5].The flow is generated by the periodic thermal waves persribed at the wavy walls of channel is discussed numerically and explained graphically[6].The problem of steady two-dimensional free convective flow of a walters fluid in a porous medium between a along vertical wavy wall and parallel flat wall in the presence of heat source is discussed.[8].

Mathematical Modelling:

We consider the flow of a nano fluid between the two vertical wavy walls. We choose the x – axis along the direction of flow and y – axis perpendicular to it. The two wavy walls are at $y = d + aCos\lambda x$ and $y = -d + aCos(\lambda x + \zeta)$. The upward flow is assumed due to buoyancy. The buoyancy force is due to the density variation and temperature difference along the flow. The thermal wave is imposed on the two plates. We assume that the wave length of the wavy wall is proportional to 1/λ. The flow is assumed to follow Bounessq approximation. In view of the above the governing equations in dimensional form are as follows:

Equation of Continuity,

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} = 0 \qquad (1)$$

Equation of Momentum,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}] - \frac{\mu_{nf}}{\rho_{nf}}u + g\beta_T(T - T_0)u \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{-1}{\rho_{nf}}\frac{\partial p}{\partial y}+\frac{\mu_{nf}}{\rho_{nf}}[\frac{\partial^2 v}{\partial^2 x^2}+\frac{\partial^2 v}{\partial^2 y^2}]-\frac{\mu_{nf}}{\rho_{nf}}u \quad (3)$$

Equation of Energy,

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}[\frac{\partial^2 T}{\partial^2 x^2}+\frac{\partial^2 T}{\partial^2 y^2}] \text{ Where } \alpha_{nf}=\frac{k_{nf}}{\rho_{nf}} \quad (4)$$

The boundary conditions are:

$$u=0, v=0,\ T=T_0(1+\varepsilon\cos(\lambda x+\omega t))=T_0^{'} \text{ at } y=d+a\cos\lambda x$$

$$u=0, v=0,\ T=T_1[1+\varepsilon\cos(\lambda x+\omega t)]=T_1^{'}\ at\ y=-d+a\cos(\lambda x+\zeta)$$

We introduce the following non-dimensional parameters:

$$X=\frac{x}{d},\ Y=\frac{y}{d},\ t'=\frac{t\gamma}{d^2},\ \rho=\frac{pd^2}{\rho\gamma^2},\ \theta=\frac{T-T_0^{'}}{T_1^{'}-T_0^{'}}\ U=\frac{u\,d}{y},\ V=\frac{v\,d}{y},\ \lambda'=\lambda d,\ \varepsilon=\frac{a}{d}$$

After introducing above non-dimensional parameters and the stream function the governing equations becomes,

$$\frac{\partial^3\psi}{\partial y^2\partial t}+\frac{\partial^3\psi}{\partial x^2\partial t}-\frac{\partial\psi}{\partial y}\left[\frac{\partial^3\psi}{\partial y^2\partial x}+\frac{\partial^3\psi}{\partial x^3}\right]+\frac{\partial\psi}{\partial x}\left[\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}\right]=\frac{\mu_{nf}}{\mu_f}\frac{\rho_f}{\rho_{nf}}\left[2\frac{\partial^4\psi}{\partial y^2\partial x^2}+\frac{\partial^4\psi}{\partial x^{43}}+\frac{\partial^4\psi}{\partial y^4}\right]$$

$$-Da\frac{\mu_{nf}}{\mu_f}\frac{\rho_f}{\rho_{nf}}\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right]-G\frac{\partial\theta}{\partial Y} \quad (5)$$

$$\frac{\partial T}{\partial t}-\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial X}+\frac{\partial\psi}{\partial X}\frac{\partial T}{\partial Y}=\frac{k_{nf}}{k_f}\frac{\rho_f}{\rho_{nf}}\frac{1}{\Pr}[\frac{\partial^2 T}{\partial^2 x^2}+\frac{\partial^2 T}{\partial^2 y^2}] \quad (6)$$

The non – dimensional boundary conditions are:

$$--\frac{\partial\psi}{\partial Y}=0,\frac{\partial\psi}{\partial X}=0,\theta=0.at\ \ \text{Y=1+}\varepsilon\ \text{Cos}\,\lambda^1 X$$

$$--\frac{\partial\psi}{\partial Y}=0,\frac{\partial\psi}{\partial X}=0,\theta=1.at\ \ \text{Y=--1+}\varepsilon\ \text{Cos}\,(\lambda^1 X+\xi)$$

Where, $G=\frac{d^3 g\beta_T(T_1^{'}-T_0^{'})}{\nu^2}$,.Grashoff Number,

$\Pr=\frac{\nu}{\alpha}$ Prandtl Number, Darcy number Da= $\frac{k}{d^2}$

$$(\rho\, c_p)_{nf}=(1-\phi)\,(\rho\, c_p)_f+\phi\,(\rho\, c_p)_s \quad (7)$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s \quad (8)$$

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\qquad \alpha_{nf}=\frac{k_{nf}}{(\rho\, c_p)_{nf}}$$

$$\frac{\mu_{nf}}{\mu_f}=\frac{1}{(1-\phi)^{0.25}},\qquad \frac{k_{nf}}{k_f}=\frac{k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2\ k_f)+\phi(k_f-k_s)}$$

Table-1

Physical properties	Water	Copper(cu)
$c_p(J/kg/k$	4,179	385
$\rho(kg/\mathrm{m}^3)$	997.1	8,933
K(W/mk)	0.613	400

$\beta_T X10^{-5}(1/K)$	21	1.67
μ	8.94x10^{-4}	-----

Numerical Method:

For computational purpose we approximate $f(x,y,t) = f(y)e^{i(ax+st)}$ in all the governing equations as given in Rudraiah[2]. The regular Galerkin Finite Element Method is used, the domain is divided into 10 elements. The iterative procedure has been used to attain the desired accuracy and to meet the convergence criteria.

Discussion and Results:

Figures 1- 12 exhibits the variations of u, v and θ for Pr = 0.71, λ = 0.02, λx = π/2, ωt = π/4, ς = 0 in the region $-1 \le y \le 1$. From Fig.1 the velocity u increases with increase in solid volume fraction, the reverse effect is observed for v from Fig. 5. Both figures stress the importance of solid part in the fluid. 5% or above solid volume fraction accelerates the flow. Magnitude of u increases gradually with Darcy parameter Da (Fig.2). Similarly velocity v shows gradual variations with Da from Fig. 6. Interestingly the reduction of pore diameter is inversely proportional to the velocity. The buoyancy force shows a gradual effect on u and v from Figs. 3 & 7. The buoyancy is directly proportional to the velocity. The natural convection is significant even in wavy channel. The variation of u for ε is very clear from Fig.4, the pattern for v is reversed in Fig. 8. The frequency parameter is more significant on the flow field.

The variation of absolute temperature (θ) is directly proportional to φ, Da, Gr and ϵ from Figures 9 to 12. The wavy walls of the channel reverse the effects in either parts of the channel.

Wavy channel shows its cooling trend in the left half of the channel and heating trend in the right half of the channel. All the parameters i.e. φ, Da, Gr and ϵ shows gradual variations in temperature. The buoyancy and wave frequency are directly proportional with temperature.

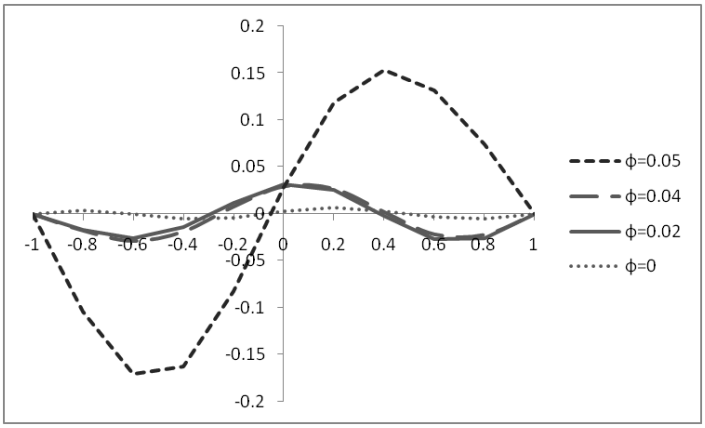

Fig.1 Variation f U with ϕ

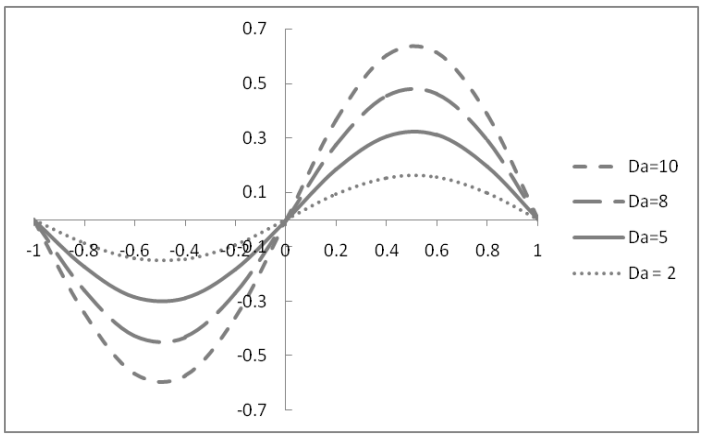

Fig.2 Variation of U with Da

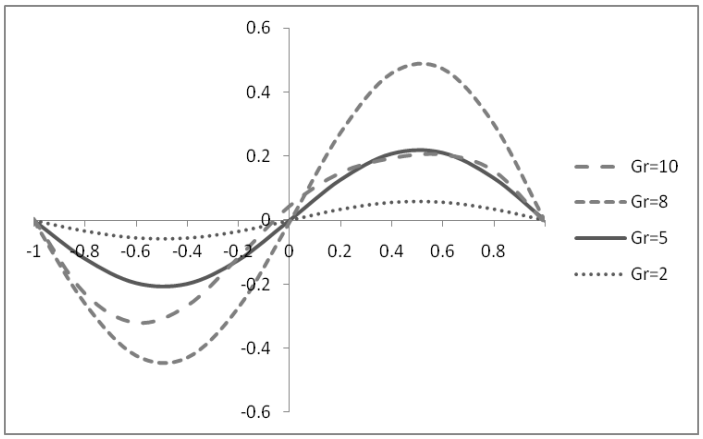

Fig.3 Variation of U with Gr

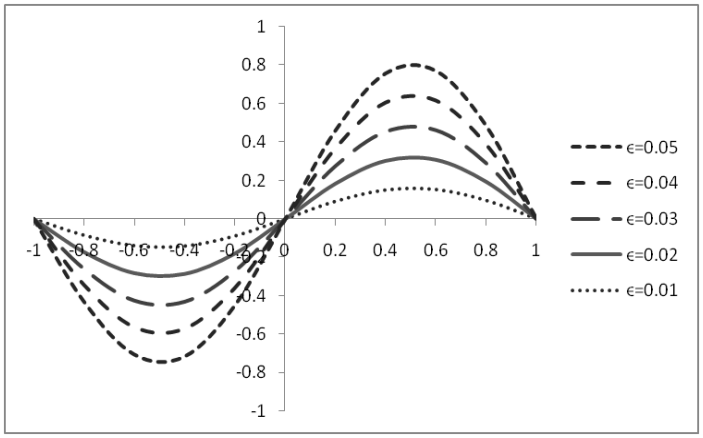

Fig.4 Variation of U with ϵ

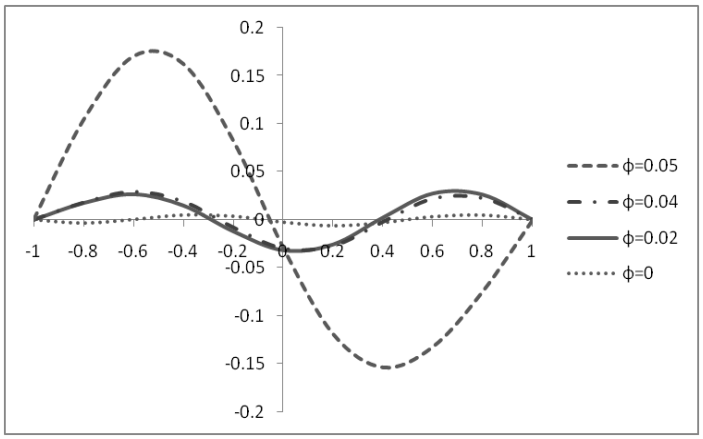

Fig.5 Variation of v with ϕ

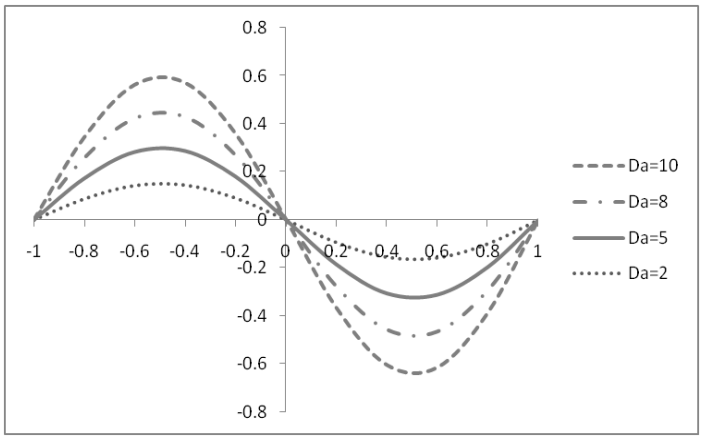

Fig.6 Variation of v with Da

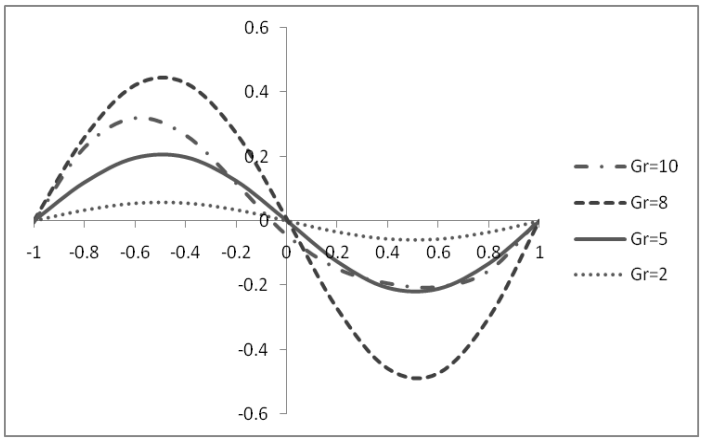

Fig.7 Variation of V with Gr

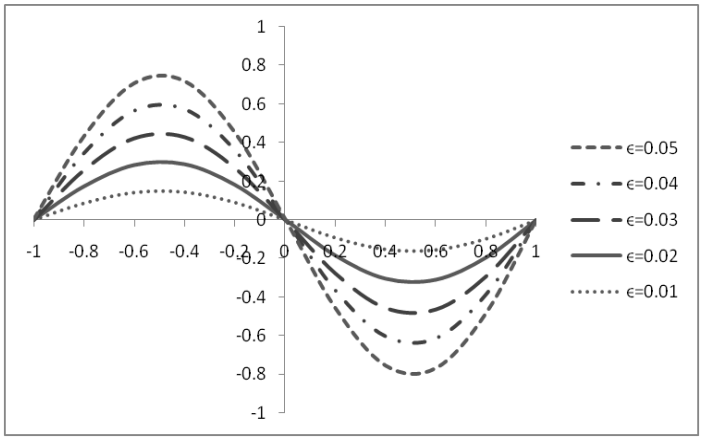

Fig.8 Variation of v with ϵ

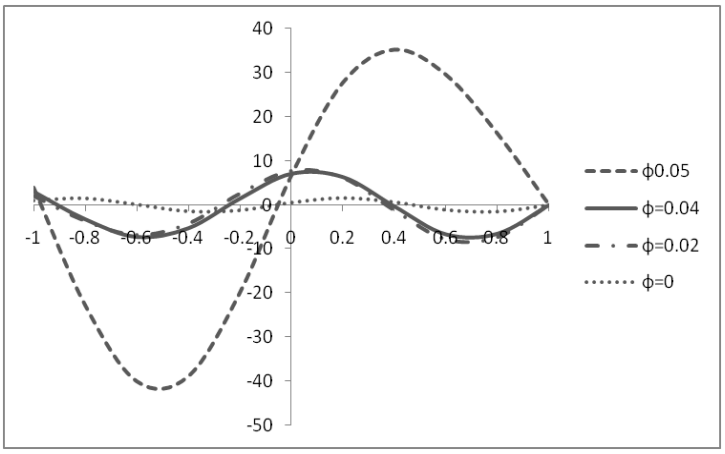

Fig.9 Variation of Θ with ϕ

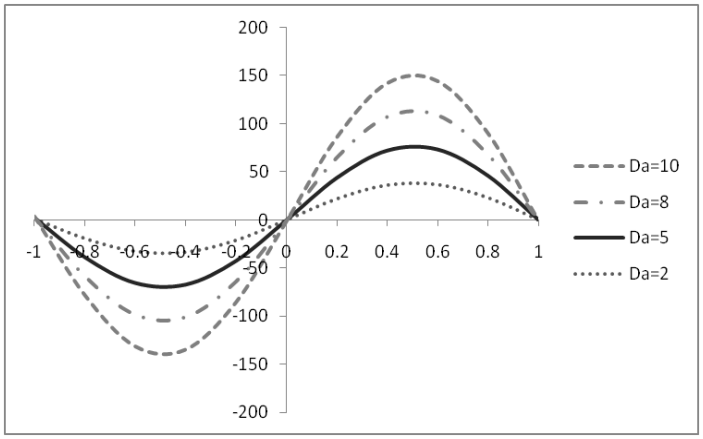

Fig.10 Variation of Θ with Da

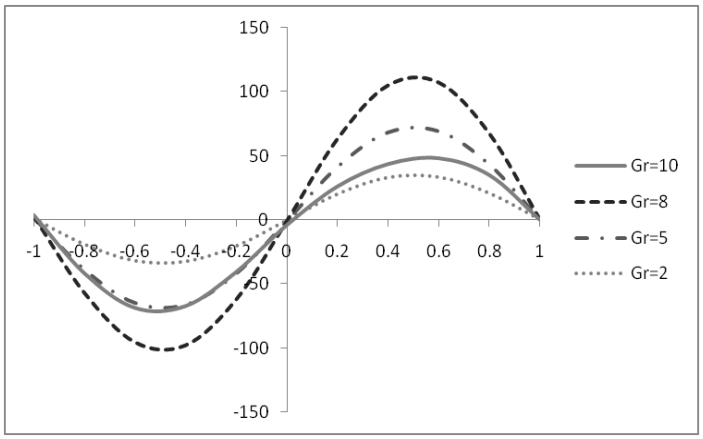

Fig.11 Variation of Θ with Gr

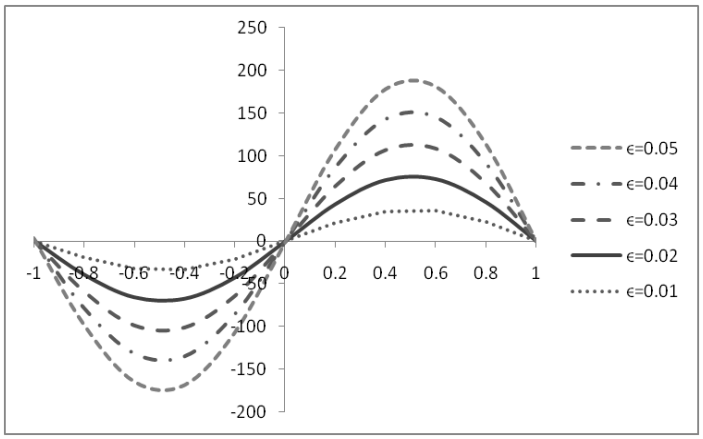

Fig. 12 Variation of Θ with ϵ

Tables for Nusselt numbers

Table-2

ϕ	0	0.02	0.04	0.05
Nu-1	1.236×10^{15}	-1.119×10^{17}	-2.1291×10^{15}	-1.272×10^{19}
Nu-2	-17.1596	-60.9359	7.3538	208.3830

Table-3

Da	2	5	8	10
Nu-1	-1.285×10^{19}	-1.272×10^{19}	-1.263×10^{19}	-1.242×10^{19}
Nu-2	210.5416	208.3830	207.0122	205.8993

Table-4

Gr	2	5	8	10
Nu-1	-6.641×10^{18}	-1.272×10^{19}	-1.270×10^{19}	-3.153×10^{18}
Nu-2	189.5614	208.3830	220.2464	−225.7471

Table-5

ϵ	0.01	0.02	0.03	0.04	0.05
Nu-1	-2.0367×10^{16}	-3.258×10^{17}	-1.649×10^{18}	-5.213×10^{18}	-1.272×10^{19}
Nu-2	−1262.215	−1033.6477	−683.0514	−252.019	208.3830

The cooling effect is found more with metal particle proportion on the left wall and heating is found more on right wall of the channel. This is biggest advantage of nano fluid. The porous medium is showing the behavior as that of solid volume fraction. The rate of heat transfer enhances with buoyancy and frequency parameter. The lesser the frequency of the wave the more the heat transfer rate.

Nomenclature:

d – Half of the distance between the wavy walls
g – Acceleration
right wall of the channel
p – Pressure
β – Molecular Diffusivity
α – Thermal Diffusivity
ω – non dimensional due to gravity
a – Constant
λ – Non dimensional wave number
ρ – Density
μ – Viscosity
u – Velocity in x - direction
v – Velocity in y - direction
T – Temperature
T_0' – Temperature on the left wall of the channel
T_1' – Temperature on the frequency parameter

Conclusions

1. The wavy walls of channel make the fluid stationery the mid region.
2. The temperature is maximum in the right half of the channel.
3. The wavy boundary reverses the flow and temperature in either part of the channel.
4. Cooling effect is observed with increase in solid volume fraction and wave frequency.

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