On Square Difference Labelled Graphs

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Abstract

Let $G(V,E)$ be a graph with p vertices and q edges. A (p,q) graph $G(V,E)$ is said to be a square difference graph if there exists a bijection $f:V(G)\rightarrow\{0,1,2,...,p-1\}$ such that the induced function $f^*:E(G)\rightarrow N$, N is a natural number, given by $f^*(uv)=|[f(u)]^2-[f(v)]^2|$ for every edges uv in G and are all distinct and the function f is called a Square difference labeling of the graph G. In this paper, we prove $S_m\cup S_n$, $P_m\cup L_n$, $C_m\cup C_n$, $L_m\cup S_n$, $C_m\cup S_n$ are the square difference graphs.

Key Words: *Square difference labeling, Square difference graph*.

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1. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected, simple graph. The vertex set and the edge set of a graph G are denoted by V (G) and E(G) respectively. Let G(p,q) be a graph with p =|V(G |) vertices and q =| E(G |) edges. Graph labeling ,where the vertices and edges are assigned real values or subsets of a set are subject to certain conditions. A detailed survey of graph labeling can be found in [2]. Terms not defined here are used in the sense of Harary in [3].There are different kinds of labelings in the graph labeling such as Graceful, Harmonious, Cordial, Fibonacci, Square sum, etc. The concept of square difference labeling was first introduced in [1] and some results on square difference labeling of graphs are discussed in [1,4,6]. In this paper we investigate some more graphs for square difference labeling. We use the following definitions in the subsequent sections.

Definition 1.1[1]: A graph $G(p,q)$ is said to be a square difference graph if there exists a bijection $f:V(G)\rightarrow\{0,1,2,...,p-1\}$ such that the induced function $f^*:E(G)\rightarrow N$, N is a natural number given by $f^*(uv)=|[f(u)]^2-[f(v)]^2|$ for every edges uv in G and are all distinct and the function f is called a square difference labeling of the graph G.

Definition 1.2[2]: A complete bipartite graph $K_{1,n}$ is called a star and it has $n+1$ vertices and n edges and also it is denoted as S_n.

2. Main Results

Theorem 2.1: All the graph $C_m\cup C_n$ is a square difference graph.

Proof: Let C_m be the cycle graph with m vertices and m edges. Let C_n be the cycle graph

with n vertices and n edges. Let $V(C_m)=\{u_i\ :\ 1\le i\le m\}$. Let $V(C_n)=\{v_i\ :\ 1\le i\le n\}$.

Therefore $V(C_m \cup C_n) = \{u_i : 1 \le i \le m \; ; \; v_i : 1 \le i \le n\}$.

Let $E(C_m) = \{u_i u_{i+1} : 1 \le i \le m-1 ; u_1 u_m\}$. Let $E(C_n) = \{v_i v_{i+1} : 1 \le i \le n-1 ; \; v_1 v_n\}$.

Therefore $E(C_m \cup C_n) = \begin{cases} u_i u_{i+1} \; : \; 1 \le i \le m-1 \; , \; u_1 u_m \\ v_i v_{i+1} \; : \; 1 \le i \le n-1 \; , \; v_1 v_n \end{cases}$.

Then we have $|V(C_m \cup C_n)| = m+n$ and $|E(C_m \cup C_n)| = m+n$.

Case (i) $m < n$

Subcase (i) (Both m and n are odd)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., \; m+n-1\}$

as follows: $f(u_i) = i-1 \;\; if \;\; 1 \le i \le m$; $f(v_i) = m-1+i \;\; if \;\; 1 \le i \le n$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{i+1}) = 2i-1 \;\; if \; 1 \le i \le m-1$; $f^*(u_1 u_m) = (m-1)^2$;

$f^*(v_i v_{i+1}) = 2m+2i-1 \;\; if \;\; 1 \le i \le n-1$; $f^*(v_1 v_n) = (2m+n-1)(n-1)$.

Subcase (ii) (m is odd and n is even)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., \; m+n-1\}$

as follows: $f(u_i) = i-1 \;\; if \;\; 1 \le i \le m$; $f(v_i) = m-2+2i \;\; if \;\; 1 \le i \le \frac{n}{2}$

$f(v_{\frac{n+2i}{2}}) = m-1+2i \;\; if \;\; 1 \le i \le \frac{n}{2}$. Let f^* be the induced edge labeling of f.

The induced edge labels by f^* as follows: $f^*(u_i u_{i+1}) = 2i-1 \;\; if \; 1 \le i \le m-1$;
$f^*(u_1 u_m) = (m-1)^2$; $f^*(v_i v_{i+1}) = 4(m+2i-1) \;\; if \;\; 1 \le i \le \left(\frac{n-2}{2}\right)$;
$f^*(v_{\frac{n}{2}} v_{\frac{n+2}{2}}) = (2m+n-1)(n-3)$; $f^*(v_{\frac{n+2i}{2}} v_{\frac{n+2+2i}{2}}) = 4(m+2i) \;\; if \;\; 1 \le i \le \left(\frac{n-2}{2}\right)$;
$f^*(v_1 v_n) = (2m+n-1)(n-1)$.

Subcase (iii) (m is even and n is odd)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., \; m+n-1\}$

as follows: $f(u_i)=i-1 \quad if \quad 1\le i\le m$; $f(v_i)=m-2+2i \quad if \quad 1\le i\le \left(\frac{n+1}{2}\right)$

$f(v_{\frac{n+1+2i}{2}})=m-1+2i \quad if \quad 1\le i\le \left(\frac{n-1}{2}\right)$. Let f^* be the induced edge labeling of f.

The induced edge labels by f^* as follows: $f^*(u_iu_{i+1})=2i-1 \quad if \; 1\le i\le m-1$;

$f^*(u_1u_m)=(m-1)^2$; $f^*(v_iv_{i+1})=4(m+2i-1) \quad if \quad 1\le i\le \left(\frac{n-1}{2}\right)$;

$f^*(v_{\frac{n+1}{2}}v_{\frac{n+3}{2}})=(2m+n)(n-2)$; $f^*(v_{\frac{n+1+2i}{2}}v_{\frac{n+3+2i}{2}})=4(m+2i) \quad if \quad 1\le i\le \left(\frac{n-3}{2}\right)$;

$f^*(v_1v_n)=(2m+n-2)(n-2)$.

Subcase (iv) (Both m and n are even)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,....,\; m+n-1\}$

as follows: $f(u_i)=i-1 \quad if \quad 1\le i\le m$; $f(v_i)=m-2+2i \quad if \quad 1\le i\le \frac{n}{2}$

$f(v_{\frac{n+2i}{2}})=m-1+2i \quad if \quad 1\le i\le \frac{n}{2}$. Let f^* be the induced edge labeling of f.

The induced edge labels by f^* as follows: $f^*(u_iu_{i+1})=2i-1 \quad if \; 1\le i\le m-1$;

$f^*(u_1u_m)=(m-1)^2$; $f^*(v_iv_{i+1})=4(m+2i-1) \quad if \quad 1\le i\le \left(\frac{n-2}{2}\right)$;

$f^*(v_{\frac{n}{2}}v_{\frac{n+2}{2}})=(2m+n-1)(n-3)$; $f^*(v_{\frac{n+2i}{2}}v_{\frac{n+2+2i}{2}})=4(m+2i) \quad if \quad 1\le i\le \left(\frac{n-2}{2}\right)$;

$f^*(v_1v_n)=(2m+n-1)(n-1)$.

Case (ii) $(m=n)$

Subcase (i) $(m=3 \quad and \quad n=3)$

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,....,\; m+n-1\}$

as follows: $f(u_i)=i-1 \quad if \quad 1\le i\le 3$; $f(v_i)=2+i \quad if \quad 1\le i\le 3$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_iu_{i+1})=2i-1 \quad if \; 1\le i\le 2$; $f^*(u_1u_3)=4$;

$f^*(v_i v_{i+1}) = 2i+5 \quad if \quad 1 \le i \le 2$; $f^*(v_1 v_3) = 16$.

Subcase (ii) $(m > 3 \quad and \quad n > 3)$

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., \ m+n-1\}$

as follows: $f(u_i) = 2i-2 \quad if \quad 1 \le i \le m$; $f(v_i) = 2i-1 \quad if \quad 1 \le i \le n$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{i+1}) = 8i-4 \quad if \quad 1 \le i \le (m-1)$; $f^*(u_1 u_m) = (m+n-2)^2$;

$f^*(v_i v_{i+1}) = 8i \quad if \quad 1 \le i \le (n-1)$; $f^*(v_1 v_n) = (m+n)(m+n-2)$.

Case (iii) $(m > n)$

Subcase (i) (Both m and n are odd)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., \ m+n-1\}$

as follows: $f(u_i) = n+i-1 \quad if \quad 1 \le i \le m$; $f(v_i) = i-1 \quad if \quad 1 \le i \le n$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{i+1}) = 2n+2i-1 \quad if \quad 1 \le i \le m-1$; $f^*(u_1 u_m) = (m-1)(2n+m-1)$;

$f^*(v_i v_{i+1}) = 2i-1 \quad if \quad 1 \le i \le n-1$; $f^*(v_1 v_n) = (n-1)^2$.

Subcase (ii) (m is odd and n is even)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., \ m+n-1\}$

as follows: $f(u_i) = n+2i-2 \quad if \quad 1 \le i \le \frac{m}{2}$; $f(v_{\frac{m+2i}{2}}) = n-1+2i \quad if \quad 1 \le i \le \frac{m}{2}$

$f(v_i) = i-1 \quad if \quad 1 \le i \le n$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{i+1}) = 4(n+2i-1) \quad if \quad 1 \le i \le \left(\frac{m-2}{2}\right)$; $f^*(u_{\frac{m}{2}} u_{\frac{m+2}{2}}) = (2n+m-1)(m-3)$

$f^*(v_{\frac{m+2i}{2}} v_{\frac{m+2+2i}{2}}) = 4(n+2i) \quad if \quad 1 \le i \le \left(\frac{m-2}{2}\right)$; $f^*(u_1 u_m) = (m-1)(2n+m-1)$;

$f^*(v_i v_{i+1}) = 2i - 1 \quad if \quad 1 \le i \le n-1; \quad f^*(v_1 v_n) = (n-1)^2$.

Subcase (iii) (m is even and n is odd)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., m+n-1\}$

as follows: ; $f(u_i) = n - 2 + 2i \quad if \quad 1 \le i \le \left(\frac{m+1}{2}\right)$; $f(u_{\frac{m+1+2i}{2}}) = n - 1 + 2i \quad if \quad 1 \le i \le \left(\frac{m-1}{2}\right)$.

$f(v_i) = i - 1 \quad if \quad 1 \le i \le n$. Let f^* be the induced edge labeling of f .

The induced edge labels by f^* as follows: $f^*(u_i u_{i+1}) = 4(n + 2i - 1) \quad if \quad 1 \le i \le \left(\frac{m-1}{2}\right)$;

$f^*(u_{\frac{m+1}{2}} v_{\frac{m+3}{2}}) = (m+2n)(m-2)$; $f^*(v_{\frac{m+1+2i}{2}} v_{\frac{m+3+2i}{2}}) = 4(n+2i) \quad if \quad 1 \le i \le \left(\frac{m-3}{2}\right)$;

$f^*(u_1 u_m) = (m-2)(2n+m-2)$; $f^*(v_i v_{i+1}) = 2i - 1 \quad if \; 1 \le i \le n-1$;

$f^*(v_1 v_n) = (n-1)^2$.

Subcase (iv) (Both m and n are even)

Define a bijection f from the vertices of $C_m \cup C_n$ to $\{0,1,2,...., m+n-1\}$

as follows: $f(u_i) = n - 2 + 2i \quad if \quad 1 \le i \le \frac{m}{2}$; $f(u_{\frac{m+2i}{2}}) = n - 1 + 2i \quad if \quad 1 \le i \le \frac{m}{2}$;

$f(v_i) = i - 1 \quad if \quad 1 \le i \le n$ .. Let f^* be the induced edge labeling of f .

The induced edge labels by f^* as follows: $f^*(u_i u_{i+1}) = 4(n + 2i - 1) \quad if \quad 1 \le i \le \left(\frac{m-2}{2}\right)$

$f^*(v_{\frac{m}{2}} v_{\frac{m+2}{2}}) = (m+2n-1)(m-3)$; $f^*(v_{\frac{m+2i}{2}} v_{\frac{m+2+2i}{2}}) = 4(n+2i) \quad if \quad 1 \le i \le \left(\frac{m-2}{2}\right)$

$f^*(u_1 u_m) = (m-1)(2n+m-1)$; $f^*(v_i v_{i+1}) = 2i - 1 \quad if \; 1 \le i \le n-1; f^*(v_1 v_n) = (n-1)^2$.

Example.2.2: A square difference labeling of $C_6 \cup C_{15}$ is shown in the Fig 2.1.

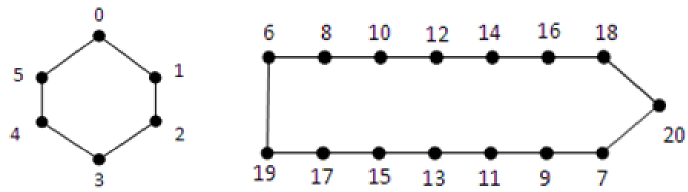

Fig 2.1

Theorem 2.3: The graph $S_m \cup S_n$ is a square difference graph.

Proof: Let S_m be the star graph with $m+1$ vertices and m edges. Let S_n be the star graph with $n+1$ vertices and n edges. Let $V(S_m)=\{u_i : 1 \le i \le m+1\}$. Let $V(S_n)=\{v_i : 1 \le i \le n+1\}$.

Therefore $V(S_m \cup S_n)=\{u_i : 1 \le i \le m+1 \ ; \ v_i : 1 \le i \le n+1\}$.

Let $E(S_m)=\{u_i u_{m+1} : 1 \le i \le m\}$. Let $E(S_n)=\{v_i v_{n+1} : 1 \le i \le n\}$.

Therefore $E(S_m \cup S_n)=\{u_i u_{m+1} : 1 \le i \le m \ ; \ v_i v_{n+1} : 1 \le i \le n\}$.

Then we have $|V(S_m \cup S_n)|=m+n+2$ and $|E(S_m \cup S_n)|=m+n$.

Define a bijection f from the vertices of $S_m \cup S_n$ to $\{0,1,2,....,m+n+1\}$ as follows:

$f(u_i)=i+1 \quad if \quad 1 \le i \le m \ ; \ f(u_{m+1})=0 \ ; \ f(v_i)=m+i+1 \quad if \quad 1 \le i \le n \ ; \ f(v_{n+1})=1$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{m+1})=(i+1)^2 \quad if \quad 1 \le i \le m; \ f^*(v_i v_{n+1})=(m+i)(m+2+i) \quad if \quad 1 \le i \le n$.

***Theorem 2.4*:** Any graph $P_m \cup L_n$ admits a square difference labeling.

Proof: Let P_m be the path graph with m vertices and $m-1$ edges.

Let L_n be the Ladder graph with $2n$ vertices and $3n-2$ edges.

Let $V(P_m)=\{u_i : 1 \le i \le m\}$. Let $V(L_n)=\{v_i , w_i : 1 \le i \le n\}$.

Therefore $V(P_m \cup L_n)=\{u_i : 1 \le i \le m \ ; \ v_i , w_i : 1 \le i \le n\}$.

Let $E(P_m)=\{u_iu_{i+1}\ :\ 1\le i\le m-1\}$. Let $E(L_n)=\begin{cases} v_iv_{i+1} & :1\le i\le n-1 \\ w_iw_{i+1} & :1\le i\le n-1 \\ v_iw_i & :1\le i\le n \end{cases}$

Therefore $E(P_m\cup L_n)=\begin{cases} u_iu_{i+1} & :1\le i\le m-1 \\ v_iv_{i+1} & :1\le i\le n-1 \\ w_iw_{i+1} & :1\le i\le n-1 \\ v_iw_i & :1\le i\le n \end{cases}$

Then we have $|V(P_m\cup L_n)|=m+2n$ and $|E(P_m\cup L_n)|=m+3n-3$.

Define a bijection f from the vertices of $P_m\cup L_n$ to $\{0,1,2,....,m+2n\}$

as follows: $f(u_i)=i-1\quad if\quad 1\le i\le m$; $f(v_i)=m-1+i\quad if\quad 1\le i\le n$;
$f(w_i)=m+n-1+i\quad if\quad 1\le i\le n$. Let f^* be the induced edge labeling of f.

The induced edge labels by f^* as follows: $f^*(u_iu_{i+1})=2i-1\quad if\quad 1\le i\le m-1$
$f^*(v_iv_{i+1})=2m-1+2i\quad if\quad 1\le i\le n-1$; $f^*(w_iw_{i+1})=2m+2n-1+2i\quad if\quad 1\le i\le n-1$;
$f^*(v_iw_i)=n(2m+n-2+2i)\quad if\quad 1\le i\le n$.

***Theorem 2.5*:** Every graph $L_m\cup S_n$ is a square difference graph.

Proof: Let L_m be the Ladder graph with $2m$ vertices and $3m-2$ edges.

Let S_n be the star graph with $n+1$ vertices and n edges. Let $V(L_m)=\{u_i\ ,\ v_i:\ 1\le i\le m\}$.

Let $V(S_n)=\{w_i\ :\ 1\le i\le n+1\}$. Let $V(L_m\cup S_n)=\begin{cases} u_i,v_i & :\ 1\le i\le m \\ w_i & :\ 1\le i\le n+1 \end{cases}$

. Let $E(L_m)=\begin{cases} u_iu_{i+1} & :\ 1\le i\le m-1 \\ v_iv_{i+1} & :\ 1\le i\le m-1 \\ u_iv_i & :\ 1\le i\le m \end{cases}$

Let $E(S_n)=\{w_i\ w_{n+1}:\ 1\le i\le n\}$

Therefore $E(L_m)=\begin{cases} u_iu_{i+1} & :\ 1\le i\le m-1 \\ v_iv_{i+1} & :\ 1\le i\le m-1 \\ u_iv_i & :\ 1\le i\le m \\ w_iw_{n+1} & :\ 1\le i\le n \end{cases}$

Then we have $|V(L_m\cup S_n)|=2m+n+1$ and $|E(P_m\cup L_n)|=3m+n-2$.

Define a bijection f from the vertices of $L_m \cup S_n$ to $\{0,1,2,...., 2m+n+1\}$

as follows: $f(u_i)=i$ $\quad if \quad 1 \le i \le m$; $f(v_i)=m+i$ $\quad if \quad 1 \le i \le m$;
$f(w_i)=2m+i$ $\quad if \quad 1 \le i \le n$; $f(w_{n+1})=0$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{i+1})=2i+1$ $\quad if \quad 1 \le i \le m-1$; $f^*(v_i v_{i+1})=2m+1+2i$ $\quad if \quad 1 \le i \le m-1$;

. $f^*(u_i v_i)=m(m+2i)$ $\quad if \quad 1 \le i \le m$; $f^*(w_i w_{n+1})=(2m+i)^2$ $\quad if \quad 1 \le i \le n$.

Example 2.6: A square difference labeling of $L_7 \cup C_{16}$ is shown in the Fig 2.2.

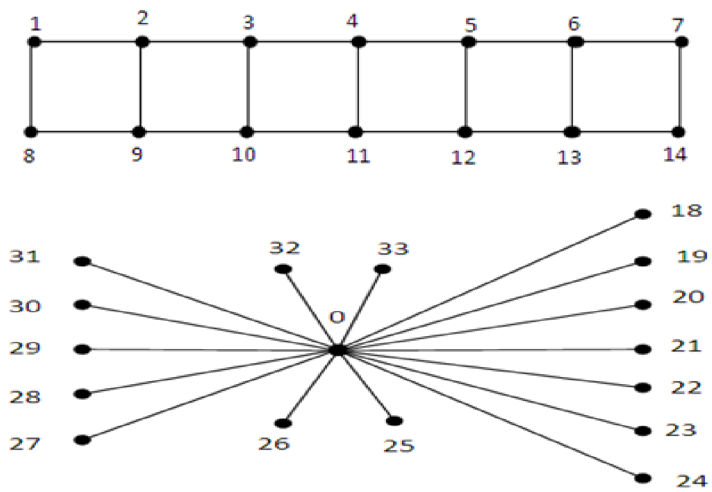

Fig 2.1

***Theorem 2.7*:** The graph $C_m \cup S_n$ is a square difference graph.

Proof: Let C_m be the cycle graph with m vertices and m edges. Let S_n be the star graph with $n+1$ vertices and n edges. Let $V(C_m)=\{ u_i : 1 \le i \le m\}$. Let $V(S_n)=\{v_i : 1 \le i \le n+1\}$.

Therefore $V(C_m \cup S_n)=\{ u_i : 1 \le i \le m ; v_i : 1 \le i \le n+1\}$. Let
$E(C_m)=\{ u_i u_{i+1} : 1 \le i \le m-1 , u_1 u_m \}$. Let $E(S_n)=\{v_i v_{n+1} : 1 \le i \le n \}$.

Therefore $E(C_m \cup S_n)=\begin{cases} u_i u_{i+1} & : 1 \le i \le m-1, \ u_1 u_m \\ v_i v_{n+1} & : 1 \le i \le n \end{cases}$

Then $|V(C_m \cup S_n)| = m+n+1$ and $|E(C_m \cup S_n)| = m+n$.

Define a bijection f from the vertices of $C_m \cup S_n$ to $\{0,1,2,...., m+n\}$

as follows: $f(u_i) = i$ *if* $1 \le i \le m$; $f(v_i) = m+i$ *if* $1 \le i \le n$; $f(v_{n+1}) = 0$.

Let f^* be the induced edge labeling of f. The induced edge labels by f^* as follows:

$f^*(u_i u_{i+1}) = 2i+1$ *if* $1 \le i \le m-1$; $f^*(u_1 u_m) = m^2 - 1$; $f^*(v_i v_{n+1}) = (m+i)^2$ *if* $1 \le i \le n$.

3.Conclusion

In this paper, we investigated the square difference labeling behavior of some union related graphs. We have planned to investigate the square difference labeling of some more special graphs in the next paper.

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