# On Neutrosophic Soft Field 

Tuhin Bera ${ }^{1}$ and Nirmal Kumar Mahapatra ${ }^{2}$<br>${ }^{1}$ Department of Mathematics, Boror S. S. High School, Bagnan, Howrah-711312,WB, India, $2^{2}$ Department of Mathematics, Panskura Banamali College, Panskura RS-721152,WB, India,


#### Abstract

The concept of 'Field' plays an important role in defining the algebraic structure 'Linear Space'. Following the definition of neutrosophic soft field and it's element namely neutrosophic soft scalar introduced in [29], their structural characteristics have been investigated in the present paper. Then the neutrosophic soft function over a crisp field is defined and illustrated by suitable examples. In continuation, the nature of neutrosophic soft homomorphic image and pre-image of a neutrosophic soft field are studied here. Finally, the concept of neutrosophic soft algebra over a neutrosophic soft field has been proposed along with the establishment of some related theorems.


Keywords Neutrosophic soft field, Cartesian product of neutrosophic soft fields, Neutrosophic soft subfield, Neutrosophic soft homomorphism, Neutrosophic soft algebra.

## 1 Introduction

Once upon a time, the uncertainties appearing in several real world problem like in law, medicine, engineering, management, industrial, IT sector etc were handled by practice of probability theory, theory of fuzzy set, intuitionistic fuzzy set theory, theory of interval mathematics, rough set theory etc. Because of the insufficiency in the available information situation, evaluation of membership values and nonmembership values estimated in intuitionistic fuzzy set theory are not always possible. So there exists an indeterministic part upon which hesitation survives. The Neutrosophic set (NS) theory by Smarandache [16], [17] meets that fact which is a generalisation of classical set, fuzzy set, intuitionistic fuzzy set. The neutrosophic logic includes the information about the percentage of truth, indeterminacy and falsity grade which are not available in intuitionistic fuzzy set theory.

Because of the inadequacy of parametrization tools, each of these theories suffers from inherent difficulties. Molodtsov [1] introduced the concept of soft set theory which is free from the parametrization inadequacy syndrome of different theories dealing with uncertainty present in most of our real life situation. The parametrization tool of soft set theory makes it very convenient and easy to apply in practice. The classical algebraic structures were extended over fuzzy set, intuitionistic fuzzy set and soft set by many authors for instance Rosenfeld [2], Mukherjee and Bhattacharya [3], Sharma [4], Aktas and Cagman [5], Maji et al. [6]-[9], Augunoglu and Aygun [10], Yaqoob et al. [11], Varol et al. [12], Zhang [13], Nanda [14], Wenxiang and Tu [15] and others.

Maji [18] has brought a combined concept Neutrosophic soft set (NSS) theory. Upon this concept Broumi et al. [19], Cetkin et al. [20], [21], Deli and Broumi [22], [23], Bera and Mahapatra [24]-[29] and others have designed their research works on some fundamental algebraic structures. Deli and Broumi [22] also modified the operations related to indeterminacy membership function as given by Maji [18].

This paper investigates the characteristics of neutrosophic soft field and develops some of it's related properties and theorems. The organisation of the paper is as follows. Section 2 gives some preliminary useful definitions related to it. In Section 3, the structural characteristics of neutrosophic soft field have been investigated. Section 4 and Section 5 deal with the Cartesian product of neutrosophic soft fields and the concept of neutrosophic soft subfield, respectively. The nature of neutrosophic soft homomorphic image and pre-image of neutrosophic soft fields are studied in Section 6. In Section 7, the concept of neutrosophic soft algebra has been introduced along with the development of some related theorems. Finally, the conclusion has been drawn in Section 8 .

## 2 Preliminaries

We recall some basic definitions related to fuzzy set, soft set, neutrosophic soft set for the sake of completeness.

### 2.1 Definition [28]

1. A binary operation $*:[0,1] \times[0,1] \rightarrow[0,1]$ is said to be continuous $t$ - norm if $*$ satisfies the following conditions :
(i) $*$ is commutative and associative.
(ii) $*$ is continuous.
(iii) $a * 1=1 * a=a, \forall a \in[0,1]$.
(iv) $a * b \leq c * d$ if $a \leq c, b \leq d$ with $a, b, c, d \in[0,1]$.

A few examples of continuous $t$-norm are $a * b=a b, a * b=\min \{a, b\}, a * b=\max \{a+b-1,0\}$.
2. A binary operation $\diamond:[0,1] \times[0,1] \rightarrow[0,1]$ is said to be continuous $t$-conorm ( $s-$ norm ) if $\circ$ satisfies the following conditions :
(i) $\circ$ is commutative and associative.
(ii) $\circ$ is continuous.
(iii) $a \diamond 0=0 \diamond a=a, \forall a \in[0,1]$.
(iv) $a \diamond b \leq c \circ d$ if $a \leq c, b \leq d$ with $a, b, c, d \in[0,1]$.

A few examples of continuous $s$-norm are $a \diamond b=a+b-a b, a \diamond b=\max \{a, b\}, a \diamond b=\min \{a+b, 1\}$.

### 2.2 Definition [16]

Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A neutrosophic set $A$ in $X$ is characterized by a truth-membership function $T_{A}$, an indeterminacy-membership function $I_{A}$ and a falsity-membership function $F_{A} \cdot T_{A}(x), I_{A}(x)$ and $F_{A}(x)$ are real standard or non-standard subsets of $]^{-} 0,1^{+}\left[\text {. That is } T_{A}, I_{A}, F_{A}: X \rightarrow\right]^{-} 0,1^{+}[$. There is no restriction on the sum of $T_{A}(x), I_{A}(x), F_{A}(x)$ and so, $-0 \leq \sup T_{A}(x)+\sup I_{A}(x)+\sup F_{A}(x) \leq 3^{+}$.

### 2.3 Definition [1]

Let $U$ be an initial universe set and $E$ be a set of parameters. Let $P(U)$ denote the power set of $U$. Then for $A \subseteq E$, a pair $(F, A)$ is called a soft set over $U$, where $F: A \rightarrow P(U)$ is a mapping.

### 2.4 Definition [18]

Let $U$ be an initial universe set and $E$ be a set of parameters. Let $N S(U)$ denote the set of all NSs of $U$. Then for $A \subseteq E$, a pair $(F, A)$ is called an NSS over $U$, where $F: A \rightarrow N S(U)$ is a mapping.

This concept has been redefined by Deli and Broumi [22] as given below.

### 2.5 Definition [22]

Let $U$ be an initial universe set and $E$ be a set of parameters. Let $N S(U)$ denote the set of all NSs of $U$. Then, a neutrosophic soft set $N$ over $U$ is a set defined by a set valued function $f_{N}$ representing a mapping $f_{N}: E \rightarrow N S(U)$ where $f_{N}$ is called approximate function of the neutrosophic soft set $N$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $N S(U)$ and therefore it can be written as a set of ordered pairs: $N=\left\{\left(e,\left\{<x, T_{f_{N}(e)}(x), I_{f_{N}(e)}(x), F_{f_{N}(e)}(x)>: x \in U\right\}\right): e \in E\right\}$ where $T_{f_{N}(e)}(x), I_{f_{N}(e)}(x), F_{f_{N}(e)}(x) \in[0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_{N}(e)$. Since supremum of each $T, I, F$ is 1 so the inequality $0 \leq T_{f_{N}(e)}(x)+I_{f_{N}(e)}(x)+$ $F_{f_{N}(e)}(x) \leq 3$ is obvious.

### 2.5.1 Example

Let $U=\left\{h_{1}, h_{2}, h_{3}\right\}$ be a set of houses and $E=\left\{e_{1}\right.$ (beautiful), $e_{2}$ (wooden), $e_{3}$ (costly) $\}$ be a set of parameters with respect to which the nature of houses are described. Let,

$$
\begin{aligned}
& \left.f_{N}\left(e_{1}\right)=\left\{<h_{1},(0.5,0.6,0.3)\right\rangle,\left\langle h_{2},(0.4,0.7,0.6)\right\rangle,\left\langle h_{3},(0.6,0.2,0.3)\right\rangle\right\} ; \\
& \left.\left.f_{N}\left(e_{2}\right)=\left\{<h_{1},(0.6,0.3,0.5)\right\rangle,\left\langle h_{2},(0.7,0.4,0.3)\right\rangle,<h_{3},(0.8,0.1,0.2)\right\rangle\right\} ; \\
& f_{N}\left(e_{3}\right)=\left\{\left\langle h_{1},(0.7,0.4,0.3)\right\rangle,\left\langle h_{2},(0.6,0.7,0.2)\right\rangle,\left\langle h_{3},(0.7,0.2,0.5)\right\rangle\right\} ;
\end{aligned}
$$

Then $N=\left\{\left[e_{1}, f_{N}\left(e_{1}\right)\right],\left[e_{2}, f_{N}\left(e_{2}\right)\right],\left[e_{3}, f_{N}\left(e_{3}\right)\right]\right\}$ is an NSS over $(U, E)$. The tabular representation of the NSS $N$ is given in Table 1 .

Table 1: Tabular form of NSS $N$

|  | $f_{N}\left(e_{1}\right)$ | $f_{N}\left(e_{2}\right)$ | $f_{N}\left(e_{3}\right)$ |
| :---: | :---: | :---: | :---: |
| $h_{1}$ | $(0.5,0.6,0.3)$ | $(0.6,0.3,0.5)$ | $(0.7,0.4,0.3)$ |
| $h_{2}$ | $(0.4,0.7,0.6)$ | $(0.7,0.4,0.3)$ | $(0.6,0.7,0.2)$ |
| $h_{3}$ | $(0.6,0.2,0.3)$ | $(0.8,0.1,0.2)$ | $(0.7,0.2,0.5)$ |

### 2.5.2 Definition [22]

The complement of a neutrosophic soft set $N$ is denoted by $N^{c}$ and is defined by :

$$
N^{c}=\left\{\left(e,\left\{<x, F_{f_{N}(e)}(x), 1-I_{f_{N}(e)}(x), T_{f_{N}(e)}(x)>: x \in U\right\}\right): e \in E\right\}
$$

### 2.5.3 Definition [22]

Let $N_{1}$ and $N_{2}$ be two NSSs over the common universe ( $U, E$ ). Then $N_{1}$ is said to be the neutrosophic soft subset of $N_{2}$ if $\forall e \in E$ and $\forall x \in U$,

$$
T_{f_{N_{1}}(e)}(x) \leq T_{f_{N_{2}}(e)}(x), \quad I_{f_{N_{1}}(e)}(x) \geq I_{f_{N_{2}}(e)}(x), \quad F_{f_{N_{1}}(e)}(x) \geq F_{f_{N_{2}}(e)}(x)
$$

We write $N_{1} \subseteq N_{2}$ and then $N_{2}$ is the neutrosophic soft superset of $N_{1}$.

### 2.5.4 Definition [22]

Let $N_{1}$ and $N_{2}$ be two NSSs over the common universe $(U, E)$. Then their union is denoted by $N_{1} \cup N_{2}=N_{3}$ and is defined as :

$$
N_{3}=\left\{\left(e,\left\{<x, T_{f_{N_{3}}(e)}(x), I_{\mathrm{f}_{N_{3}}(e)}(x), F_{f_{N_{3}}(e)}(x)>: x \in U\right\}\right): e \in E\right\}
$$

where $T_{f_{N_{3}}(e)}(x)=T_{f_{N_{1}}(e)}(x) \diamond T_{f_{N_{2}}(e)}(x), I_{f_{N_{3}}(e)}(x)=I_{f_{N_{1}}(e)}(x) * I_{f_{N_{2}}(e)}(x)$ and

$$
F_{f_{N_{3}}(e)}(x)=F_{f_{N_{1}}(e)}(x) * F_{f_{N_{2}}(e)}(x) ;
$$

Their intersection is denoted by $N_{1} \cap N_{2}=N_{4}$ and is defined as:

$$
N_{4}=\left\{\left(e,\left\{<x, T_{f_{N_{4}}(e)}(x), I_{f_{N_{4}}(e)}(x), F_{f_{N_{4}}(e)}(x)>: x \in U\right\}\right): e \in E\right\}
$$

where $T_{f_{N_{4}}(e)}(x)=T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x), I_{f_{N_{4}}(e)}(x)=I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x)$ and

$$
F_{f_{N_{4}}(e)}(x)=F_{f_{N_{1}}(e)}(x) \diamond F_{f_{N_{2}}(e)}(x) ;
$$

### 2.6 Definition [24]

Let $N_{1}$ and $N_{2}$ be two NSSs over the common universe ( $U, E$ ). Then their 'AND' operation is denoted by $N_{1} \wedge N_{2}=N_{5}$ and is defined as :

$$
N_{5}=\left\{\left[(a, b),\left\{<x, T_{f_{N_{5}}(a, b)}(x), I_{f_{N_{5}}(a, b)}(x), F_{f_{N_{5}}(a, b)}(x)>: x \in U\right\}\right]:(a, b) \in E \times E\right\}
$$

where $T_{f_{N_{5}}(a, b)}(x)=T_{f_{N_{1}}(a)}(x) * T_{f_{N_{2}}(b)}(x), I_{f_{N_{5}}(a, b)}(x)=I_{f_{N_{1}}(a)}(x) \diamond I_{f_{N_{2}}(b)}(x)$ and

$$
F_{f_{N_{5}}(a, b)}(x)=F_{f_{N_{1}}(a)}(x) \diamond F_{f_{N_{2}}(b)}(x) ;
$$

Their 'OR' operation is denoted by $\mathrm{N}_{1} \vee N_{2}=N_{6}$ and is defined as :

$$
N_{6}=\left\{\left[(a, b),\left\{<x, T_{f_{N_{6}}(a, b)}(x), I_{f_{N_{6}}(a, b)}(x), F_{f_{N_{6}}(a, b)}(x)>: x \in U\right\}\right]:(a, b) \in E \times E\right\}
$$

where $\quad T_{f_{N_{6}}(a, b)}(x)=T_{f_{N_{1}}(a)}(x) \circ T_{f_{N_{2}}(b)}(x), I_{f_{N_{6}}(a, b)}(x)=I_{f_{N_{1}}(a)}(x) * I_{f_{N_{2}}(b)}(x)$ and

$$
F_{f_{N_{6}}(a, b)}(x)=F_{f_{N_{1}}(a)}(x) * F_{f_{N_{2}}(b)}(x) ;
$$

### 2.7 Definition [24]

Let $g$ be a mapping from a set $X$ to a set $Y$. If $M$ and $N$ are two neutrosophic soft sets over $X$ and $Y$, respectively, then the image of $M$ under $g$ is defined as a neutrosophic soft set $g(M)=\left\{\left[e, f_{g(M)}(e)\right]: e \in E\right\}$ over Y where $T_{f_{g(M)}(e)}(y)=T_{f_{M}(e)}\left[g^{-1}(y)\right]$, $I_{f_{g(M)}(e)}(y)=I_{f_{M}(e)}\left[g^{-1}(y)\right], \quad F_{f_{g(M)}(e)}(y)=F_{f_{M}(e)}\left[g^{-1}(y)\right] ; \forall y \in Y$.

The pre-image of $N$ under $g$ is defined as a neutrosophic soft set $g^{-1}(N)=\left\{\left[e, f_{g^{-1}(N)}(e)\right]: e \in E\right\}$ over X where $T_{f_{g^{-1}(N)}(e)}(x)=T_{f_{N}(e)}[g(x)], \quad I_{g_{g^{-1}(N)}(e)}(x)=I_{f_{N}(e)}[g(x)], F_{f_{g^{-1}(N)}(e)}(x)=F_{f_{N}(e)}[g(x)] ; \forall x \in X$.

### 2.8 Definition [29]

A neutrosophic set $B=\left\{\left\langle x, T_{B}(x), I_{B}(x), F_{B}(x)\right\rangle: x \in K\right\}$ over a field $(K,+, \cdot)$ is called a neutrosophic subfield of $(K,+, \cdot)$ if the followings hold.

$$
\text { (i) }\left\{\begin{array}{l}
T_{B}(x+y) \geq T_{B}(x) * T_{B}(y) \\
I_{B}(x+y) \leq I_{B}(x) \diamond I_{B}(y) \\
F_{B}(x+y) \leq F_{B}(x) \diamond F_{B}(y) ; \quad \forall x, y \in K .
\end{array}\right.
$$

(ii) $\left\{\begin{array}{l}T_{B}(-x) \geq T_{B}(x) \\ I_{B}(-x) \leq I_{B}(x) \\ F_{B}(-x) \leq F_{B}(x) ; \quad \forall x \in K .\end{array}\right.$
(iii) $\left\{\begin{array}{l}T_{B}(x . y) \geq T_{B}(x) * T_{B}(y) \\ I_{B}(x . y) \leq I_{B}(x) \bullet I_{B}(y) \\ F_{B}(x . y) \leq F_{B}(x) \bullet F_{B}(y) ; \quad \forall x, y \in K .\end{array}\right.$
(iv) $\left\{\begin{array}{l}T_{B}\left(x^{-1}\right) \geq T_{B}(x) \\ I_{B}\left(x^{-1}\right) \leq I_{B}(x) \\ F_{B}\left(x^{-1}\right) \leq F_{B}(x) ; \quad \forall x(\neq 0) \in K .\end{array}\right.$

An NSS $N$ is called a neutrosophic soft field over $[(K,+, \cdot), E]$ if $f_{N}(e)$ is a neutrosophic subfield of the field ( $\left.K,+, \cdot\right)$ for each $e \in E$.

### 2.8.1 Definition [29]

Each element $\left(e, f_{N}(e)\right)$ of the neutrosophic soft field $N$ over $[(K,+, \cdot), E]$ is called a neutrosophic soft scalar and is denoted by $\hat{e}_{N}$. A neutrosophic soft scalar $\hat{e}_{N} \in M, M$ being another neutrosophic soft field over $(K, E)$ if $f_{N}(e) \leq f_{M}(e)$ i.e.,

$$
T_{f_{N}(e)}(x) \leq T_{f_{M}(e)}(x), I_{f_{N}(e)}(x) \geq I_{f_{M}(e)}(x), F_{f_{N}(e)}(x) \geq F_{f_{M}(e)}(x) ; \quad \forall x \in K .
$$

### 2.8.2 Example

1. Let us consider the field $\boldsymbol{Z}_{3}=\{\overline{0}, \overline{1}, \overline{2}\}$ and $E=\left\{e_{1}, e_{2}, e_{3}, e_{4}\right\}$ be the set of parameters. We define $f_{N}\left(e_{1}\right), f_{N}\left(e_{2}\right), f_{N}\left(e_{3}\right), f_{N}\left(e_{4}\right)$ as given by the Table 2 .

Table 2: Tabular form of neutrosophic soft field $N$

|  | $f_{N}\left(e_{1}\right)$ | $f_{N}\left(e_{2}\right)$ | $f_{N}\left(e_{3}\right)$ | $f_{N}\left(e_{4}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\overline{0}$ | $(0.67,0.39,0.19)$ | $(0.85,0.29,0.27)$ | $(0.29,0.53,0.41)$ | $(0.31,0.21,0.19)$ |
| $\overline{1}$ | $(0.55,0.41,0.44)$ | $(0.41,0.78,0.32)$ | $(0.64,0.42,0.25)$ | $(0.72,0.19,0.16)$ |
| $\overline{2}$ | $(0.35,0.52,0.28)$ | $(0.63,0.52,0.41)$ | $(0.59,0.66,0.39)$ | $(0.48,0.31,0.27)$ |

Corresponding $t$-norm (*) and $s$-norm ( $\circ$ ) are defined as $a * b=\max \{a+b-1,0\}, a \diamond b=\min \{a+b, 1\}$. Then, $N$ forms a neutrosophic soft field over $\left[\left(\boldsymbol{Z}_{3},+, \cdot\right), E\right]$. Here, the neutrosophic soft field $N$ consists of four neutrosophic soft scalars viz., $\hat{e}_{1 N}, \hat{e}_{2 N}, \hat{e}_{3 N}, \hat{e}_{4 N} . S o$, it is a finite neutrosophic soft field over $\left[\left(Z_{3},+, \cdot\right), E\right]$.
2. Let $E=\boldsymbol{N}$ (the set of natural numbers) be the parametric set and $K=(\boldsymbol{R},+, \cdot)$ be the field of all real numbers. Define a mapping $f_{M}: N \rightarrow N S(R)$ where, for any $n \in N$ and $x \in \boldsymbol{R}$,

$$
\begin{aligned}
& T_{f_{M}(n)}(x)= \begin{cases}0 & \text { if } x \text { is rational } \\
\frac{1}{3 n} & \text { if } x \text { is irrational } .\end{cases} \\
& I_{f_{M}(n)}(x)= \begin{cases}1-\frac{1}{n} & \text { if } x \text { is rational } \\
0 & \text { if } x \text { is irrational } .\end{cases} \\
& F_{f_{M}(n)}(x)= \begin{cases}\frac{1}{1+n} & \text { if } x \text { is rational } \\
0 & \text { if } x \text { is irrational } .\end{cases}
\end{aligned}
$$

The $t$-norm (*) and $s$-norm ( ${ }^{\circ}$ ) are defined as $a * b=\min \{a, b\}, a \diamond b=\max \{a, b\}$. Then, $M$ forms a neutrosophic soft field over $[(\boldsymbol{R},+, \cdot), \boldsymbol{N}]$. It is obviously an infinite neutrosophic soft field.

## 3 Neutrosophic soft field

Here, the characteristics of neutrosophic soft field have been investigated along with the development of some related theorems.

### 3.1 Proposition

Let $N$ be a neutrosophic soft field over $[(K,+, \cdot), E]$. Then for the additive identity $0_{k}$ and the multiplicative identity $1_{k}$ of the field $(K,+, \cdot)$, the followings hold if $a * b=\min \{a, b\}$ and $a \diamond b=\max \{a, b\}$.
(i) $\quad T_{f_{N}(e)}\left(0_{k}\right) \geq T_{f_{N}(e)}(x), I_{f_{N}(e)}\left(0_{k}\right) \leq I_{f_{N}(e)}(x), F_{f_{N}(e)}\left(0_{k}\right) \leq F_{f_{N}(e)}(x), \forall x \in K, \forall e \in E$.
(ii) $T_{f_{N}(e)}\left(1_{k}\right) \geq T_{f_{N}(e)}(x), I_{f_{N}(e)}\left(1_{k}\right) \leq I_{f_{N}(e)}(x), F_{f_{N}(e)}\left(1_{k}\right) \leq F_{f_{N}(e)}(x), \forall x\left(\neq 0_{k}\right) \in K, \forall e \in E$.
(iii) $T_{f_{N}(e)}\left(0_{k}\right) \geq T_{f_{N}(e)}\left(1_{k}\right), I_{f_{N}(e)}\left(0_{k}\right) \leq I_{f_{N}(e)}\left(1_{k}\right), F_{f_{N}(e)}\left(0_{k}\right) \leq F_{f_{N}(e)}\left(1_{k}\right)$.

Proof. (i) $\forall x \in K$ and $\forall e \in E$,

$$
\begin{aligned}
& T_{f_{N}(e)}\left(0_{k}\right)=T_{f_{N}(e)}(x-x) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(x)=T_{f_{N}(e)}(x), \\
& I_{f_{N}(e)}\left(0_{k}\right)=I_{f_{N}(e)}(x-x) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(x)=I_{f_{N}(e)}(x), \\
& F_{f_{N}(e)}\left(0_{k}\right)=F_{f_{N}(e)}(x-x) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(x)=F_{f_{N}(e)}(x) ;
\end{aligned}
$$

(ii) $\forall x\left(\neq 0_{k}\right) \in K$ and $\forall e \in E$,

$$
\begin{aligned}
& T_{f_{N}(e)}\left(1_{k}\right)=T_{f_{N}(e)}\left(x \cdot x^{-1}\right) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(x)=T_{f_{N}(e)}(x), \\
& I_{f_{N}(e)}\left(1_{k}\right)=I_{f_{N}(e)}\left(x \cdot x^{-1}\right) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(x)=I_{f_{N}(e)}(x), \\
& F_{f_{N}(e)}\left(1_{k}\right)=F_{f_{N}(e)}\left(x \cdot x^{-1}\right) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(x)=F_{f_{N}(e)}(x) ;
\end{aligned}
$$

(iii) By applying (i)

### 3.2 Proposition

An NSS $N$ over the field $[(K,+, \cdot), E]$ is called a neutrosophic soft field iff followings hold on the assumption that $a * b=$ $\min \{a, b\}$ and $a \diamond b=\max \{a, b\}$.
(i) $\left\{\begin{array}{l}T_{f_{N}(e)}(x-y) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y), \\ I_{f_{N}(e)}(x-y) \leq I_{f_{N}(e)}(x) \bullet I_{f_{N}(e)}(y), \\ \left.F_{f_{N}(e)}(x-y) \leq F_{f_{N}(e)}(x) \odot F_{f_{N}(e)}(y)\right) ; \quad \text { for } \quad x, y \in K .\end{array}\right.$
(ii) $\left\{\begin{array}{l}T_{f_{N}(e)}\left(x . y^{-1}\right) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y), \\ I_{f_{N}(e)}\left(x \cdot y^{-1}\right) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(y), \\ \left.F_{f_{N}(e)}\left(x \cdot y^{-1}\right) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(y)\right) ; \quad \text { for } x, y \in K .\end{array}\right.$

Proof. First suppose $N$ is a neutrosophic soft field over $[(K,+, \cdot), E]$. Then,
$T_{f_{N}(e)}(x-y) \geq T_{f_{N}(e)}(x+(-y)) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(-y) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y)$,
$I_{f_{N}(e)}(x-y) \leq I_{f_{N}(e)}(x+(-y)) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(-y) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(y)$,
$F_{f_{N}(e)}(x-y) \leq F_{f_{N}(e)}(x+(-y)) \leq F_{f_{N}(e)}(x) \odot F_{f_{N}(e)}(-y) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(y)$;
$T_{f_{N}(e)}\left(x . y^{-1}\right) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}\left(y^{-1}\right) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y)$,
$I_{f_{N}(e)}\left(x . y^{-1}\right) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}\left(y^{-1}\right) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(y)$,
$F_{f_{N}(e)}\left(x . y^{-1}\right) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}\left(y^{-1}\right) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(y) ;$
Conversely, for the additive identity $0_{k}$ and multiplicative identity $1_{k}$ in $(K,+, \cdot)$,

$$
\begin{aligned}
& T_{f_{N}(e)}(-x)=T_{f_{N}(e)}\left(0_{k}-x\right) \geq T_{f_{N}(e)}\left(0_{k}\right) * T_{f_{N}(e)}(x) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(x)=T_{f_{N}(e)}(x) \text {, } \\
& I_{f_{N}(e)}(-x)=I_{f_{N}(e)}\left(0_{k}-x\right) \leq I_{f_{N}(e)}\left(0_{k}\right) \diamond I_{f_{N}(e)}(x) \leq I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(x)=I_{f_{N}(e)}(x) \text {, } \\
& F_{f_{N}(e)}(-x)=F_{f_{N}(e)}\left(0_{k}-x\right) \leq F_{f_{N}(e)}\left(0_{k}\right) \circ F_{f_{N}(e)}(x) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(x)=F_{f_{N}(e)}(x) ; \\
& T_{f_{N}(e)}(x+y)=T_{f_{N}(e)}(x-(-y)) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(-y) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y), \\
& I_{f_{N}(e)}(x+y)=I_{f_{N}(e)}(x-(-y)) \leq I_{f_{N}(e)}(x) \odot I_{f_{N}(e)}(-y) \leq I_{f_{N}(e)}(x) \odot I_{f_{N}(e)}(y), \\
& F_{f_{N}(e)}(x+y)=F_{f_{N}(e)}(x-(-y)) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(-y) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(y) ; \\
& T_{f_{N}(e)}\left(x^{-1}\right)=T_{f_{N}(e)}\left(1_{k} \cdot x^{-1}\right) \geq T_{f_{N}(e)}\left(1_{k}\right) * T_{f_{N}(e)}(x) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(x)=T_{f_{N}(e)}(x), \\
& I_{f_{N}(e)}\left(x^{-1}\right)=I_{f_{N}(e)}\left(1_{k} \cdot x^{-1}\right) \leq I_{f_{N}(e)}\left(1_{k}\right) \diamond I_{f_{N}(e)}(x) \leq I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(x)=I_{f_{N}(e)}(x) \text {, } \\
& F_{f_{N}(e)}\left(x^{-1}\right)=F_{f_{N}(e)}\left(1_{k} \cdot x^{-1}\right) \leq F_{f_{N}(e)}\left(1_{k}\right) \circ F_{f_{N}(e)}(x) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(x)=F_{f_{N}(e)}(x) ; \\
& T_{f_{N}(e)}(x . y)=T_{f_{N}(e)}\left(x .\left(y^{-1}\right)^{-1}\right) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}\left(y^{-1}\right) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y),
\end{aligned}
$$

$I_{f_{N}(e)}(x . y)=I_{f_{N}(e)}\left(x .\left(y^{-1}\right)^{-1}\right) \leq I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}\left(y^{-1}\right) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(y)$,
$F_{f_{N}(e)}(x . y)=F_{f_{N}(e)}\left(x .\left(y^{-1}\right)^{-1}\right) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}\left(y^{-1}\right) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(y) ;$
This completes the proof.

### 3.3 Theorem

Let, $N_{1}$ and $N_{2}$ be two neutrosophic soft fields over $[(K,+, \cdot), E]$. Then, $N_{1} \cap N_{2}$ is also a neutrosophic soft field over $[(K,+, \cdot), E]$.
Proof. Let, $N_{1} \cap N_{2}=N_{3}$. Now, $\forall x, y \in K$ and $\forall e \in E$,

$$
\begin{aligned}
T_{f_{N_{3}}(e)}(x+y) & =T_{f_{N_{1}}(e)}(x+y) * T_{f_{N_{2}}(e)}(x+y) \\
& \geq\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(x) * T_{f_{N_{2}}(e)}(y)\right] \\
& =\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(y) * T_{f_{N_{2}}(e)}(x)\right] \text { (as } * \text { is commutative) } \\
& =T_{f_{N_{1}}(e)}(x) *\left[T_{f_{N_{1}}(e)}(y) * T_{f_{N_{2}}(e)}(y)\right] * T_{f_{N_{2}}(e)}(x)(\text { as } * \text { is associative) } \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{3}}(e)}(y) * T_{f_{N_{2}}(e)}(x) \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x) * T_{f_{N_{3}}(e)}(y)(\text { as } * \text { is commutative) } \\
& =T_{f_{N_{3}}(e)}(x) * T_{f_{N_{3}}(e)}(y)
\end{aligned}
$$

Hence, $\quad T_{f_{N_{3}}(e)}(x+y) \geq T_{f_{N_{3}}(e)}(x) * T_{f_{N_{3}}(e)}(y)$;

$$
\begin{aligned}
& I_{f_{N_{3}}(e)}(x+y)=I_{f_{N_{1}}(e)}(x+y) \diamond I_{f_{N_{2}}(e)}(x+y) \\
& \leq\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \circ\left[I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(y)\right] \\
& =\left[I_{f_{N_{1}}(e)}(x) \diamond \mathrm{I}_{f_{N_{1}}(e)}(y)\right] \circ\left[I_{f_{N_{2}}(e)}(y) \circ I_{f_{N_{2}}(e)}(x)\right] \text { (as } \circ \text { is commutative) } \\
& =I_{f_{N_{1}}(e)}(x) \circ\left[I_{f_{N_{1}}(e)}(y) \circ I_{f_{N_{2}}(e)}(y)\right] \circ I_{f_{N_{2}}(e)}(x) \text { (as } \circ \text { is associative) } \\
& =I_{f_{N_{1}}(e)}(x) \odot I_{f_{N_{3}}(e)}(y) \odot I_{f_{N_{2}}(e)}(x) \\
& =I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y) \text { (as } \circ \text { is commutative) } \\
& =I_{f_{N_{3}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y)
\end{aligned}
$$

Thus, $\quad I_{f_{N_{3}}(e)}(x+y) \leq I_{f_{N_{3}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y)$;
Similarly, $\quad F_{f_{N_{3}}(e)}(x+y) \leq F_{f_{N_{3}}(e)}(x) \odot F_{f_{N_{3}}(e)}(y) ; \quad$ Next,
$T_{f_{N_{3}}(e)}(-x)=T_{f_{N_{1}}(e)}(-x) * T_{f_{N_{2}}(e)}(-x) \geq T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x)=T_{f_{N_{3}}(e)}(x)$,
$I_{f_{N_{3}}(e)}(-x)=I_{f_{N_{1}}(e)}(-x) \diamond I_{f_{N_{2}}(e)}(-x) \leq I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x)=I_{f_{N_{3}}(e)}(x)$,
Similarly, $\quad F_{f_{N_{3}}(e)}(-x) \leq F_{f_{N_{3}}(e)}(x) ; \quad$ Next,

$$
\begin{aligned}
T_{f_{N_{3}}(e)}(x . y) & =T_{f_{N_{1}}(e)}(x . y) * T_{f_{N_{2}}(e)}(x . y) \\
& \geq\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(x) * T_{f_{N_{2}}(e)}(y)\right] \\
& =\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(y) * T_{T_{N_{2}}(e)}(x)\right] \quad \text { (as } * \text { is commutative) } \\
& =T_{f_{N_{1}}(e)}(x) *\left[T_{f_{N_{1}}(e)}(y) * T_{f_{N_{2}}(e)}(y)\right] * T_{f_{N_{2}}(e)}(x) \quad(\text { as } * \text { is associative) } \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{3}}(e)}(y) * T_{f_{N_{2}}(e)}(x) \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x) * T_{f_{N_{3}}(e)}(y)(\text { as } * \text { is commutative) } \\
& =T_{f_{N_{3}}(e)}(x) * T_{f_{N_{3}}(e)}(y)
\end{aligned}
$$

Hence, $\quad T_{f_{N_{3}}(e)}(x . y) \geq T_{f_{N_{3}}(e)}(x) * T_{f_{N_{3}}(e)}(y) ; \quad$ Next,

$$
\begin{aligned}
I_{f_{N_{3}}(e)}(x . y) & =I_{f_{N_{1}}(e)}(x . y) \diamond I_{f_{N_{2}}(e)}(x . y) \\
& \leq\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \diamond\left[I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(y)\right] \\
& =\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \diamond\left[I_{f_{N_{2}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(x)\right] \quad \text { (as } \diamond \text { is commutative) } \\
& =I_{f_{N_{1}}(e)}(x) \diamond\left[I_{f_{N_{1}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(y)\right] \diamond I_{f_{N_{2}}(e)}(x) \quad(\text { as } \diamond \text { is associative) } \\
& =I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(x) \\
& =I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y) \quad(\text { as } \diamond \text { is commutative) } \\
& =I_{f_{N_{3}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y)
\end{aligned}
$$

Hence, $\quad I_{f_{N_{3}}(e)}(x . y) \leq I_{f_{N_{3}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y)$;
Similarly, $\quad F_{f_{N_{3}}(e)}(x . y) \leq F_{f_{N_{3}}(e)}(x) \diamond I_{f_{N_{3}}(e)}(y) ; \quad$ Next,
$T_{f_{N_{3}}(e)}\left(x^{-1}\right)=T_{f_{N_{1}(e)}}\left(x^{-1}\right) * T_{f_{N_{2}}(e)}\left(x^{-1}\right) \geq T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x)=T_{f_{N_{3}}(e)}(x)$,
$I_{f_{N_{3}}(e)}\left(x^{-1}\right)=I_{f_{N_{1}}(e)}\left(x^{-1}\right) \diamond I_{f_{N_{2}}(e)}\left(x^{-1}\right) \leq I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x)=I_{f_{N_{3}}(e)}(x)$,
Similarly, $\quad F_{f_{N_{3}}(e)}\left(x^{-1}\right) \leq F_{f_{N_{3}}(e)}(x)$;
This completes the theorem.
The theorem is also true for a family of neutrosophic soft fields over a field.

### 3.3.1 Remark

For two neutrosophic soft fields $N_{1}$ and $N_{2}$ over $[(K,+, \cdot), E], N_{1} \cup N_{2}$ is not generally a neutrosophic soft field over $[(K,+, \cdot), E]$. It is possible if any one is contained in other.

For example, let, $K=(\boldsymbol{Q},+, \cdot), E=2 \boldsymbol{Z}$. Consider two neutrosophic soft fields $N_{1}$ and $N_{2}$ over $[(\boldsymbol{Q},+, \cdot), 2 \boldsymbol{Z}]$ as following. For $x \in \boldsymbol{Q}, n \in \boldsymbol{Z}$,

$$
\begin{aligned}
& T_{f_{N_{1}}(2 n)}(x)= \begin{cases}\frac{1}{2} & \text { if } x=2 k n, \exists k \in \boldsymbol{Z} \\
0 & \text { others } .\end{cases} \\
& I_{f_{N_{1}}(2 n)}(x)= \begin{cases}0 & \text { if } x=2 k n, \exists k \in \boldsymbol{Z} \\
\frac{1}{4} & \text { others } .\end{cases} \\
& F_{f_{N_{1}(2 n)}}(x)= \begin{cases}\frac{2}{5} & \text { if } x=2 k n, \exists k \in \boldsymbol{Z} \\
1 & \text { others } .\end{cases}
\end{aligned}
$$

and

$$
\begin{aligned}
& T_{f_{N_{2}}(2 n)}(x)= \begin{cases}\frac{2}{3} & \text { if } x=3 k n, \exists k \in \boldsymbol{Z} \\
0 & \text { others } .\end{cases} \\
& I_{f_{N_{2}}(2 n)}(x)= \begin{cases}0 & \text { if } x=3 k n, \exists k \in \boldsymbol{Z} \\
\frac{1}{5} & \text { others } .\end{cases} \\
& F_{f_{N_{2}}(2 n)}(x)= \begin{cases}\frac{1}{6} & \text { if } x=3 k n, \exists k \in \boldsymbol{Z} \\
\frac{1}{3} & \text { others } .\end{cases}
\end{aligned}
$$

The $t$-norm (*) and $s$-norm ( $\odot$ ) are defined as $a * b=\min \{a, b\}, a \diamond b=\max \{a, b\}$. Let, $N_{1} \cup N_{2}=N_{3}$. Then, for $n=2, x=$ $4, y=6$ we have,

$$
\begin{aligned}
& T_{f_{N_{3}}(4)}(4-6)=T_{f_{N_{1}}(4)}(-2) \diamond T_{f_{N_{2}}(4)}(-2)=\max \{0,0\}=0 \quad \text { and } \\
& T_{f_{N_{3}}(4)}(4) * T_{f_{N_{3}}(4)}(6)=\left\{T_{f_{N_{1}(4)}}(4) \diamond T_{f_{N_{2}(4)}}(4)\right\} *\left\{T_{\left.f_{N_{1}(4)}(6) \diamond T_{f_{N_{2}(4)}}(6)\right\}}\right. \\
&=\min \left[\max \left\{\frac{1}{2}, 0\right\}, \max \left\{0, \frac{2}{3}\right\}\right]=\min \left(\frac{1}{2}, \frac{2}{3}\right)=\frac{1}{2}
\end{aligned}
$$

Hence, $T_{f_{N_{3}}(4)}(4-6)<T_{f_{N_{3}}(4)}(4) * T_{f_{N_{3}}(4)}(6)$ i.e., $\quad N_{1} \cup N_{2}$ is not a neutrosophic soft field, here.
Now, if we define $N_{2}$ over $[(\boldsymbol{Q},+, \cdot), 2 \boldsymbol{Z}]$ as follows :

$$
\begin{aligned}
& T_{f_{N_{2}}(2 n)}(x)= \begin{cases}\frac{1}{10} & \text { if } x=6 k n, \exists k \in \boldsymbol{Z} \\
0 & \text { others } .\end{cases} \\
& I_{f_{N_{2}}(2 n)}(x)= \begin{cases}0 & \text { if } x=6 k n, \exists k \in \boldsymbol{Z} \\
\frac{2}{3} & \text { others } .\end{cases} \\
& F_{f_{N_{2}}(2 n)}(x)= \begin{cases}\frac{3}{5} & \text { if } x=6 k n, \exists k \in \boldsymbol{Z} \\
1 & \text { others } .\end{cases}
\end{aligned}
$$

Then, it can be easily verified that $N_{2} \subseteq N_{1}$ and $N_{1} \cup N_{2}$ is a neutrosophic soft field over $[(\boldsymbol{Q},+, \cdot), 2 \boldsymbol{Z}]$.

### 3.4 Theorem

Let $N_{1}$ and $N_{2}$ be two neutrosophic soft fields over $[(K,+, \cdot), E]$. Then, $N_{1} \wedge N_{2}$ is also a neutrosophic soft field over $[(K,+, \cdot), E]$.
Proof. Let $N_{1} \wedge N_{2}=N_{3} \quad$ where $\quad f_{N_{3}}(a, b)=f_{N_{1}}(a) \cap f_{N_{2}}(b) \quad$ for $\quad(a, b) \in E \times E$.
Since intersection of two neutrosophic subfields is also so, hence $N_{1} \wedge N_{2}$ is a neutrosophic soft field.
The theorem is also true for a family of neutrosophic soft fields over a field.

### 3.5 Theorem

Let $g: K \rightarrow L$ be a field isomorphism in classical sense. If $M$ is a neutrosophic soft field over $K$ then $g(M)$ is a neutrosophic soft field over $L$.

Proof. Let $x_{1}, x_{2} \in K ; y_{1}, y_{2} \in L$ such that $y_{1}=g\left(x_{1}\right), y_{2}=g\left(x_{2}\right)$. Now,

$$
\begin{aligned}
T_{f_{g(M)}(e)}\left(y_{1}+y_{2}\right) & =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}+y_{2}\right)\right] \\
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)+g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =T_{f_{M}(e)}\left(x_{1}+x_{2}\right) \\
& \geq T_{f_{M}(e)}\left(\mathrm{x}_{1}\right) * T_{f_{M}(e)}\left(x_{2}\right) \\
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] * T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =T_{f_{g(M)}(e)}\left(y_{1}\right) * T_{f_{g(M)}(e)}\left(y_{2}\right)
\end{aligned}
$$

$T_{f_{g(M)}(e)}\left(-y_{1}\right)=T_{f_{M}(e)}\left[g^{-1}\left(-y_{1}\right)\right]=T_{f_{M}(e)}\left[-g^{-1}\left(y_{1}\right)\right]=T_{f_{M}(e)}\left(-x_{1}\right) \geq T_{f_{M}(e)}\left(x_{1}\right)=T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right]=T_{f_{g(M)}(e)}\left(y_{1}\right)$
i.e., $\quad T_{f_{g(M)}(e)}\left(-y_{1}\right) \geq T_{f_{g(M)}(e)}\left(y_{1}\right) ; \quad$ Next,

$$
\begin{aligned}
I_{f_{g(M)}(e)}\left(y_{1}+y_{2}\right) & =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}+y_{2}\right)\right] \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)+g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =I_{f_{M}(e)}\left(x_{1}+x_{2}\right) \\
& \leq I_{f_{M}(e)}\left(x_{1}\right) \diamond I_{f_{M}(e)}\left(x_{2}\right) \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] \diamond I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =I_{f_{g(M)}(e)}\left(y_{1}\right) \diamond I_{f_{g(M)}(e)}\left(y_{2}\right)
\end{aligned}
$$

$I_{f_{g(M)}(e)}\left(-y_{1}\right)=I_{f_{M}(e)}\left[g^{-1}\left(-y_{1}\right)\right]=I_{f_{M}(e)}\left[-g^{-1}\left(y_{1}\right)\right]=I_{f_{M}(e)}\left(-x_{1}\right) \leq I_{f_{M}(e)}\left(x_{1}\right)=I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right]=I_{f_{g(M)}(e)}\left(y_{1}\right)$
i.e., $\quad I_{f_{g(M)}(e)}\left(-y_{1}\right) \leq I_{f_{g(M)}(e)}\left(y_{1}\right)$;

Similarly, $\quad F_{f_{g(M)}(e)}\left(y_{1}+y_{2}\right) \leq F_{f_{g(M)}(e)}\left(y_{1}\right) \diamond F_{f_{g(M)}(e)}\left(y_{2}\right), \quad F_{f_{g(M)}(e)}\left(-y_{1}\right) \leq F_{f_{g(M)}(e)}\left(y_{1}\right) ;$
Further, $\quad T_{f_{g(M)}(e)}\left(y_{1} \cdot y_{2}\right)=T_{f_{M}(e)}\left[g^{-1}\left(y_{1} \cdot y_{2}\right)\right]$

$$
\begin{aligned}
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right) \cdot g^{-1}\left(y_{2}\right)\right], \quad \text { as } g^{-1} \text { is homomorphism } \\
& =T_{f_{M}(e)}\left(x_{1} \cdot x_{2}\right) \\
& \geq T_{f_{M}(e)}\left(x_{1}\right) * T_{f_{M}(e)}\left(x_{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] * T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =T_{f_{g(M)}(e)}\left(y_{1}\right) * T_{f_{g(M)}(e)}\left(y_{2}\right)
\end{aligned}
$$

$T_{f_{g(M)}(e)}\left(y_{2}^{-1}\right)=T_{f_{M}(e)}\left[g^{-1}\left(y_{2}^{-1}\right)\right]=T_{f_{M}(e)}\left[\left(g^{-1}\left(y_{2}\right)\right)^{-1}\right]=T_{f_{M}(e)}\left(x_{2}^{-1}\right) \geq T_{f_{M}(e)}\left(x_{2}\right)=T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right]=T_{f_{g(M)}(e)}\left(y_{2}\right)$ i.e., $\quad T_{f_{g(M)}(e)}\left(y_{2}^{-1}\right) \geq T_{f_{g(M)}(e)}\left(y_{2}\right) ; \quad$ Next,

$$
\begin{aligned}
I_{f_{g(M)}(e)}\left(y_{1} \cdot y_{2}\right) & =I_{f_{M}(e)}\left[g^{-1}\left(y_{1} \cdot y_{2}\right)\right] \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right) \cdot g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =I_{f_{M}(e)}\left(x_{1} \cdot x_{2}\right) \\
& \leq I_{f_{M}(e)}\left(x_{1}\right) \diamond I_{f_{M}(e)}\left(x_{2}\right) \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right) \circ \circ I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right]\right. \\
& =I_{f_{g(M)}(e)}\left(y_{1}\right) \diamond I_{f_{g(M)}(e)}\left(y_{2}\right)
\end{aligned}
$$

$I_{f_{g(M)}^{(e)}}\left(y_{2}^{-1}\right)=I_{f_{M}(e)}\left[g^{-1}\left(y_{2}^{-1}\right)\right]=I_{f_{M}(e)}\left[\left(g^{-1}\left(y_{2}\right)\right)^{-1}\right]=I_{f_{M}(e)}\left(x_{2}^{-1}\right) \leq I_{f_{M}(e)}\left(x_{2}\right)=I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right]=I_{f_{g(M)}(e)}\left(y_{2}\right)$ i.e., $\quad I_{f_{g(M)}(e)}\left(y_{2}^{-1}\right) \leq I_{f_{g(M)}(e)}\left(y_{2}\right)$;

In a similar fashion, $\quad F_{f_{g(M)}(e)}\left(y_{1} \cdot y_{2}\right) \leq F_{f_{g(M)}(e)}\left(y_{1}\right) \diamond F_{f_{g(M)}(e)}\left(y_{2}\right), \quad F_{f_{g(M)}(e)}\left(y_{2}^{-1}\right) \leq F_{f_{g(M)}(e)}\left(y_{2}\right) ;$
This proves the theorem.

### 3.6 Theorem

Let $g: K \rightarrow L$ be a field homomorphism in classical sense. If $N$ is a neutrosophic soft field over $L$, then $g^{-1}(N)$ is a neutrosophic soft field over $K$. [ Note that $g^{-1}(N)$ is the inverse image of $N$ under the mapping $g$. Here $g^{-1}$ may not be a mapping.]

Proof. Let $y_{1}, y_{2} \in L ; x_{1}, x_{2} \in K$ so that $y_{1}=g\left(x_{1}\right), y_{2}=g\left(x_{2}\right)$. Now,

$$
\begin{aligned}
T_{f_{g^{-1}(N)}(e)}\left(x_{1}+x_{2}\right) & =T_{f_{N}(e)}\left[g\left(x_{1}+x_{2}\right)\right] \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right)+g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =T_{f_{N}(e)}\left(y_{1}+y_{2}\right) \\
& \geq T_{f_{N}(e)}\left(y_{1}\right) * T_{f_{N}(e)}\left(y_{2}\right) \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right)\right] * T_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =T_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) * T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)
\end{aligned}
$$

Also, $T_{f_{g^{-1}(N)}(e)}\left(-x_{1}\right)=T_{f_{N}(e)}\left[g\left(-x_{1}\right)\right]=T_{f_{N}(e)}\left[-g\left(x_{1}\right)\right]=T_{f_{N}(e)}\left(-y_{1}\right) \geq T_{f_{N}(e)}\left(y_{1}\right)=T_{f_{N}(e)}\left[g\left(x_{1}\right)\right]=T_{f_{g^{-1}(N)}(e)}\left(x_{1}\right)$ i.e., $\quad T_{f_{g^{-1}(N)}(e)}\left(-x_{1}\right) \geq T_{f_{g^{-1}(N)}(e)}\left(x_{1}\right)$; Next,

$$
\begin{aligned}
I_{f_{g^{-1}(N)}(e)}\left(x_{1}+x_{2}\right) & =I_{f_{N}(e)}\left[g\left(x_{1}+x_{2}\right)\right] \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right)+g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =I_{f_{N}(e)}\left(y_{1}+y_{2}\right) \\
& \leq I_{f_{N}(e)}\left(y_{1}\right) \circ I_{f_{N}(e)}\left(y_{2}\right) \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right)\right] \circ I_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =I_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond I_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)
\end{aligned}
$$

Also, $\quad I_{f_{g^{-1}(N)}(e)}\left(-x_{1}\right)=I_{f_{N}(e)}\left[g\left(-x_{1}\right)\right]=I_{f_{N}(e)}\left[-g\left(x_{1}\right)\right]=I_{f_{N}(e)}\left(-y_{1}\right) \leq I_{f_{N}(e)}\left(y_{1}\right)=I_{f_{N}(e)}\left[g\left(x_{1}\right)\right]=\mathrm{I}_{f_{g^{-1}(N)}(e)}\left(x_{1}\right)$ i.e., $\quad I_{f_{g^{-1}(N)}(e)}\left(-x_{1}\right) \leq I_{f_{g^{-1}(N)}}(e)\left(x_{1}\right)$;

Similarly, $\quad F_{f_{g^{-1}(N)}^{(e)}}\left(x_{1}+x_{2}\right) \leq F_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond F_{f_{g^{-1}(N)}(e)}\left(x_{2}\right), \quad F_{f_{g^{-1}(N)}(e)}\left(-x_{1}\right) \leq F_{f_{g^{-1}(N)}^{(e)}}\left(x_{1}\right)$;
Further, $\quad T_{f_{g^{-1}(N)}(e)}\left(x_{1} \cdot x_{2}\right)=T_{f_{N}(e)}\left[g\left(x_{1} \cdot x_{2}\right)\right]$

$$
\begin{aligned}
& =T_{f_{N}(e)}\left[g\left(x_{1}\right) \cdot g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =T_{f_{N}(e)}\left(y_{1} \cdot y_{2}\right) \\
& \geq T_{f_{N}(e)}\left(y_{1}\right) * T_{f_{N}(e)}\left(y_{2}\right) \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right)\right] * T_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =T_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) * T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)
\end{aligned}
$$

Also, $\quad T_{f_{g^{-1}(N)}(e)}\left(x_{2}^{-1}\right)=T_{f_{N}(e)}\left[g\left(x_{2}^{-1}\right)\right]=T_{f_{N}(e)}\left[\left(g\left(x_{2}\right)\right)^{-1}\right]=T_{f_{N}(e)}\left(y_{2}^{-1}\right) \geq T_{f_{N}(e)}\left(y_{2}\right)=T_{f_{N}(e)}\left[g\left(x_{2}\right)\right]=T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)$ i.e., $T_{f_{g^{-1}(N)}(e)}\left(x_{2}^{-1}\right) \geq T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right) ; \quad$ Next,

$$
\begin{aligned}
I_{f_{g^{-1}(N)}(e)}\left(x_{1} \cdot x_{2}\right) & =I_{f_{N}(e)}\left[g\left(x_{1} \cdot x_{2}\right)\right] \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right) \cdot g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =I_{f_{N}(e)}\left(y_{1} \cdot y_{2}\right) \\
& \leq I_{f_{N}(e)}\left(y_{1}\right) \otimes I_{f_{N}(e)}\left(y_{2}\right) \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right)\right] \diamond I_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =I_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond I_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)
\end{aligned}
$$

Also, $I_{f_{g^{-1}(N)}(e)}\left(x_{2}^{-1}\right)=I_{f_{N}(e)}\left[g\left(x_{2}^{-1}\right)\right]=I_{f_{N}(e)}\left[\left(g\left(x_{2}\right)\right)^{-1}\right]=I_{f_{N}(e)}\left(y_{2}^{-1}\right) \leq I_{f_{N}(e)}\left(y_{2}\right)=I_{f_{N}(e)}\left[g\left(x_{2}\right)\right]=I_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)$ i.e., $\quad I_{f_{g^{-1}(N)}(e)}\left(x_{2}^{-1}\right) \leq I_{f_{g^{-1}(N)}^{(e)}}\left(x_{2}\right)$;

Similarly, $\quad F_{f_{g^{-1}(N)}(e)}\left(x_{1} \cdot x_{2}\right) \leq F_{f_{g^{-1}(N)}^{(e)}}\left(x_{1}\right) \diamond F_{f_{g^{-1}(N)}^{(e)}}\left(x_{2}\right), \quad F_{f_{g^{-1}(N)}(e)}\left(x_{2}^{-1}\right) \leq F_{f_{g^{-1}(N)}(e)}\left(x_{2}\right) ;$
Hence, the theorem is proved.

## 4 Cartesian product of neutrosophic soft fields

In this section the concept of cartesian product of neutrosophic soft fields has been introduced along with a well-known theorem.

### 4.1 Definition

Let $M$ and $N$ be two neutrosophic soft fields over ( $K, E$ ) and ( $L, E$ ), respectively. Then their cartesian product is $M \times N=P$ where $f_{P}(a, b)=f_{M}(a) \times f_{N}(b)$ for $(a, b) \in E \times E$. Analytically,
$f_{P}(a, b)=\left\{<(x, y), T_{f_{p}(a, b)}(x, y), I_{f_{p}(a, b)}(x, y), F_{f_{p}(a, b)}(x, y)>:(x, y) \in K \times L\right\}$ with

$$
\left\{\begin{array}{l}
T_{f_{P}(a, b)}(x, y)=T_{f_{M}(a)}(x) * T_{f_{N}(b)}(y) \\
I_{f_{P}(a, b)}(x, y)=I_{f_{S^{\prime}}(a)}(x) \diamond I_{f_{N}(b)}(y) \\
F_{f_{P}(a, b)}(x, y)=F_{f_{M}(a)}(x) \diamond F_{f_{N}(b)}(y) .
\end{array}\right.
$$

This definition can be extended for more than two neutrosophic soft fields.

### 4.2 Theorem

Let $N_{1}$ and $N_{2}$ be two neutrosophic soft fields over $(K, E)$ and $(L, E)$, respectively. Then their cartesian product $N_{1} \times N_{2}$ is a neutrosophic soft field over ( $K \times L, E \times E$ ).

Proof. Let $N_{1} \times N_{2}=N_{3}$ where $f_{N_{3}}(a, b)=f_{N_{1}}(a) \times f_{N_{2}}(b)$ for $(a, b) \in E \times E$. Then for $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right) \in K \times L$,

$$
T_{f_{N_{3}}(a, b)}\left(\left(x_{1}, y_{1}\right)+\left(x_{2}, y_{2}\right)\right)=T_{f_{N_{3}}(a, b)}\left(x_{1}+x_{2}, y_{1}+y_{2}\right)
$$

$$
\begin{aligned}
& =T_{f_{N_{1}}(a)}\left(x_{1}+x_{2}\right) * T_{f_{N_{2}}(b)}\left(y_{1}+y_{2}\right) \\
& \geq\left[T_{f_{N_{1}}(a)}\left(x_{1}\right) * T_{f_{N_{1}}(a)}\left(x_{2}\right)\right] *\left[T_{f_{N_{2}}(b)}\left(y_{1}\right) * T_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =\left[T_{f_{N_{1}}(a)}\left(x_{1}\right) * T_{f_{N_{2}}(b)}\left(y_{1}\right)\right] *\left[T_{f_{N_{1}}(a)}\left(x_{2}\right) * T_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =\mathrm{T}_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) * T_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right) \\
I_{f_{N_{3}}(a, b)}\left(\left(x_{1}, y_{1}\right)+\left(x_{2}, y_{2}\right)\right) & =I_{f_{N_{3}}(a, b)}\left(x_{1}+x_{2}, y_{1}+y_{2}\right) \\
& =I_{f_{N_{1}}(a)}\left(x_{1}+x_{2}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{1}+y_{2}\right) \\
& \leq\left[I_{f_{N_{1}}(a)}\left(x_{1}\right) \diamond I_{f_{N_{1}}(a)}\left(x_{2}\right)\right] \odot\left[I_{f_{N_{2}}(b)}\left(y_{1}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =\left[I_{f_{N_{1}}(a)}\left(x_{1}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{1}\right)\right] \odot\left[I_{f_{N_{1}}(a)}\left(x_{2}\right) \diamond I_{f_{N_{2}( }(b)}\left(y_{2}\right)\right] \\
& =I_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) \diamond I_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)
\end{aligned}
$$

Similarly, $\quad F_{f_{N_{3}}(a, b)}\left(\left(x_{1}, y_{1}\right)+\left(x_{2}, y_{2}\right)\right) \leq F_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) \diamond F_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right) ; \quad$ Next,
$T_{f_{N_{3}}(a, b)}\left[-\left(x_{1}, y_{1}\right)\right]=T_{f_{N_{3}}(a, b)}\left(-x_{1},-y_{1}\right)=T_{f_{N_{1}}(a)}\left(-x_{1}\right) * T_{f_{N_{2}}(b)}\left(-y_{1}\right) \geq T_{f_{N_{1}}(a)}\left(x_{1}\right) * T_{f_{N_{2}}(b)}\left(y_{1}\right)=T_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right)$ i.e., $\quad T_{f_{N_{3}}(a, b)}\left[-\left(x_{1}, y_{1}\right)\right] \geq T_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right)$,
$I_{f_{N_{3}}(a, b)}\left[-\left(x_{1}, y_{1}\right)\right]=I_{f_{N_{3}}(a, b)}\left(-x_{1},-y_{1}\right)=I_{f_{N_{1}}(a)}\left(-x_{1}\right) \diamond I_{f_{N_{2}}(b)}\left(-y_{1}\right) \leq I_{\mathrm{I}_{N_{1}}(a)}\left(x_{1}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{1}\right)=I_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right)$
i.e., $I_{f_{N_{3}}(a, b)}\left[-\left(x_{1}, y_{1}\right)\right] \leq I_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right)$

Similarly, $\quad F_{f_{N_{3}}(a, b)}\left[-\left(x_{1}, y_{1}\right)\right] \leq F_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) ; \quad$ Next,

$$
\begin{aligned}
T_{f_{N_{3}}(a, b)}\left(\left(x_{1}, y_{1}\right) \cdot\left(x_{2}, y_{2}\right)\right) & =T_{f_{N_{3}}(a, b)}\left(x_{1} \cdot x_{2}, y_{1} \cdot y_{2}\right) \\
& =T_{f_{N_{1}}(a)}\left(x_{1} \cdot x_{2}\right) * T_{f_{N_{2}}(b)}\left(y_{1} \cdot y_{2}\right) \\
& \geq\left[T_{f_{N_{1}}(a)}\left(x_{1}\right) * T_{f_{N_{1}}(a)}\left(x_{2}\right)\right] *\left[T_{f_{N_{2}}(b)}\left(y_{1}\right) * T_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =\left[T_{f_{N_{1}}(a)}\left(x_{1}\right) * T_{f_{N_{2}}(b)}\left(y_{1}\right)\right] *\left[T_{f_{N_{1}}(a)}\left(x_{2}\right) * T_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =T_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) * T_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right) \\
I_{f_{N_{3}}(a, b)}\left(\left(x_{1}, y_{1}\right) \cdot\left(x_{2}, y_{2}\right)\right) & =I_{f_{N_{3}(a, b)}\left(x_{1} \cdot x_{2}, y_{1} \cdot y_{2}\right)} \\
& =I_{f_{N_{1}}(a)}\left(x_{1} \cdot x_{2}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{1} \cdot y_{2}\right) \\
& \leq\left[I_{f_{N_{1}}(a)}\left(x_{1}\right) \diamond I_{f_{N_{1}}(a)}\left(x_{2}\right)\right] \diamond\left[I_{f_{N_{2}}(b)}\left(y_{1}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =\left[I_{f_{N_{1}}(a)}\left(x_{1}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{1}\right)\right] \diamond\left[I_{f_{N_{1}}(a)}\left(x_{2}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{2}\right)\right] \\
& =I_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) \diamond I_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)
\end{aligned}
$$

Similarly, $\quad F_{f_{N_{3}}(a, b)}\left(\left(x_{1}, y_{1}\right) .\left(x_{2}, y_{2}\right)\right) \leq F_{f_{N_{3}}(a, b)}\left(x_{1}, y_{1}\right) \diamond F_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right) ; \quad$ Next,
$T_{f_{N_{3}}(a, b)}\left[\left(x_{2}, y_{2}\right)^{-1}\right]=T_{f_{N_{3}}(a, b)}\left(x_{2}^{-1}, y_{2}^{-1}\right)=T_{f_{N_{1}}(a)}\left(x_{2}^{-1}\right) * T_{f_{N_{2}}(b)}\left(y_{2}^{-1}\right) \geq T_{f_{N_{1}}(a)}\left(x_{2}\right) * T_{f_{N_{2}}(b)}\left(y_{2}\right)=T_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)$ i.e., $\quad T_{f_{N_{3}}(a, b)}\left[\left(x_{2}, y_{2}\right)^{-1}\right] \geq T_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)$;
$I_{f_{N_{3}}(a, b)}\left[\left(x_{2}, y_{2}\right)^{-1}\right]=I_{f_{N_{3}}(a, b)}\left(x_{2}^{-1}, y_{2}^{-1}\right)=I_{f_{N_{1}}(a)}\left(x_{2}^{-1}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{2}^{-1}\right) \leq I_{f_{N_{1}}(a)}\left(x_{2}\right) \diamond I_{f_{N_{2}}(b)}\left(y_{2}\right)=I_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)$
i.e., $\quad I_{f_{N_{3}}(a, b)}\left[\left(x_{2}, y_{2}\right)^{-1}\right] \leq I_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)$;

Similarly, $\quad F_{f_{N_{3}}(a, b)}\left[\left(x_{2}, y_{2}\right)^{-1}\right] \leq F_{f_{N_{3}}(a, b)}\left(x_{2}, y_{2}\right)$;
Hence, the theorem is proved.

## 5 Neutrosophic soft subfield

Here, the neutrosophic soft subfield has been defined and some related theorems have been developed.

### 5.1 Definition

Let $N_{1}$ and $N_{2}$ be two neutrosophic fields over $(K, E)$. Then $N_{1}$ is neutrosophic soft subfield of $N_{2}$ if $\forall x \in K, \forall e \in E$,

$$
T_{f_{N_{1}}(e)}(x) \leq T_{f_{N_{2}}(e)}(x), I_{f_{N_{1}}(e)}(x) \geq I_{f_{N_{2}}(e)}(x), F_{f_{N_{1}}(e)}(x) \geq F_{f_{N_{2}}(e)}(x)
$$

### 5.2 Theorem

Let $N$ be a neutrosophic soft field over $(K, E)$ and $N_{1}, N_{2}$ be two neutrosophic soft fields of $N$. If $a * b=\min \{a, b\}$ and $a \diamond b=$ $\max \{a, b\}$ then,
(i) $N_{1} \cap N_{2}$ is a neutrosophic soft subfield of $N$.
(ii) $N_{1} \wedge N_{2}$ is a neutrosophic soft subfield of $N \wedge N$.

Proof. The intersection $(\cap)$, $\operatorname{AND}(\Lambda)$ of two neutrosophic soft fields is also so by Theorems (3.3) and (3.4). Now to complete this theorem, we only verify the criteria of neutrosophic soft subfield in each case.
(i) Let $N_{3}=N_{1} \cap N_{2}$. For $x \in K$,

$$
\begin{aligned}
T_{f_{N_{3}}(e)} & (x) \\
I_{f_{N_{3}}(e)}(x) & =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(x)=I_{f_{N}(e)}(x) \\
F_{f_{N_{3}}(e)}(x) & =F_{f_{N_{1}}(e)}(x) \diamond F_{f_{N_{2}}(e)}(x) \geq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(x)=F_{f_{N}(e)}(x)
\end{aligned}
$$

(ii) Let $N_{3}=N_{1} \wedge N_{2}$ and $x \in K$. Then,

$$
\begin{aligned}
& T_{f_{N_{3}}(a, b)}(x)=T_{f_{N_{1}}(a)}(x) * T_{f_{N_{2}}(b)}(x) \leq T_{f_{N}(a)}(x) * T_{f_{N}(b)}(x)=T_{f_{N \wedge N}(a, b)}(x) \\
& I_{f_{N_{3}}(a, b)}(x)=I_{f_{N_{1}}(a)}(x) \diamond I_{f_{N_{2}}(b)}(x) \geq I_{f_{N}(a)}(x) \diamond I_{f_{N}(b)}(x)=I_{f_{N \Lambda N}(a, b)}(x) \\
& F_{f_{N_{3}}(a, b)}(x)=F_{f_{N_{1}}(a)}(x) \diamond F_{f_{N_{2}}(b)}(x) \geq F_{f_{N}(a)}(x) \diamond \mathrm{F}_{f_{N}(b)}(x)=F_{f_{N \wedge N}(a, b)}(x) ;
\end{aligned}
$$

The theorems are also true for a family of neutrosophic soft subfields of $N$.

### 5.3 Theorem

Let $N_{1}$ and $N_{2}$ be two neutrosophic soft fields over field $(K, E)$ such that $N_{1}$ is the neutrosophic soft subfield of $N_{2}$. Let $g: K \rightarrow L$ be a field isomorphism in classical sense. Then $g\left(N_{1}\right)$ and $g\left(N_{2}\right)$ are two neutrosophic soft fields over $(L, E)$. Moreover $g\left(N_{1}\right)$ is the neutrosophic soft subfield of $g\left(N_{2}\right)$.

Proof. The 1st part has been already proved in Theorem (3.5).
Let $x \in K, y \in L$ such that $y=g(x)$. Then,

$$
\begin{aligned}
& T_{f_{N_{1}}(e)}(x) \leq T_{f_{N_{2}}(e)}(x) \\
\Rightarrow & T_{f_{N_{1}}(e)}\left[g^{-1}(y)\right] \leq T_{f_{N_{2}}(e)}\left[g^{-1}(y)\right] \\
\Rightarrow & T_{f_{g\left(N_{1}\right)}(e)}(y) \leq T_{f_{g\left(N_{2}\right)}(e)}(y)
\end{aligned}
$$

Similarly, $I_{f_{g\left(N_{1}\right)}(e)}(y) \geq I_{f_{g\left(N_{2}\right)}(e)}(y) \quad$ and $\quad \mathrm{F}_{f_{g\left(N_{1}\right)}(e)}(y) \geq F_{f_{g\left(N_{2}\right)}(e)}(y) ;$
Hence, the theorem is proved.

## 6 Neutrosophic soft homomorphism

In this section, first we define a neutrosophic soft function, then define image and pre-image of an NSS under a neutrosophic soft function. In continuation, we introduce the notion of neutrosophic soft homomorphism along with some of it's properties.

### 6.1 Definition

Let $\varphi: K \rightarrow L$ and $\psi: E \rightarrow E^{\prime}$ be two crisp functions where $K, L$ are fields and $E, E^{\prime}$ are parametric sets. Then the pair $(\varphi, \psi)$ is
called a neutrosophic soft function from $(K, E)$ to $\left(L, E^{\prime}\right)$. We write, $\quad(\varphi, \psi):(K, E) \rightarrow\left(L, E^{\prime}\right)$.
Consider two NSSs $M, N$ defined over $(K, E),\left(L, E^{\prime}\right)$ respectively. Then,
(1) The image of $M$ under $(\varphi, \psi)$, denoted by $(\varphi, \psi)(M)$, is an NSS over $(L, E)$ and is defined as :

$$
(\varphi, \psi)(M)=\left\{\left\langle\psi(a), f_{\varphi(M)}(\psi(a))>: a \in E\right\} \text { where for } x \in K, y \in L, b \in E^{\prime},\right.
$$

$$
\begin{aligned}
& T_{f_{\varphi(M)}(b)}(y)=\left\{\begin{array}{l}
\max _{\varphi(x)=y} \max _{\psi(a)=b}\left[T_{f_{M}(a)}(x)\right], \text { if } x \in \varphi^{-1}(y) \\
0, \quad \text { otherwise } .
\end{array}\right. \\
& I_{f_{\varphi(M)}(b)}(y)=\left\{\begin{array}{l}
\min _{\varphi(x)=y} \min _{\psi(a)=b}\left[I_{f_{M}(a)}(x)\right], \text { if } x \in \varphi^{-1}(y) \\
1, \quad \text { otherwise } .
\end{array}\right. \\
& F_{f_{\varphi(M)}(b)}(y)=\left\{\begin{array}{l}
\min _{\varphi(x)=y} \min _{\psi(a)=b}\left[F_{f_{M}(a)}(x)\right], \text { if } x \in \varphi^{-1}(y) \\
1, \quad \text { otherwise } .
\end{array}\right.
\end{aligned}
$$

(2) The pre-image of $N$ under $(\varphi, \psi)$, denoted by $(\varphi, \psi)^{-1}(N)$, is an NSS over $(K, E)$ and is defined as, $\forall a \in \psi^{-1}\left(E^{\prime}\right), \forall x \in K$ :

$$
\begin{aligned}
& T_{f_{\varphi^{-1}(N)}(a)}(x)=T_{f_{N}[\psi(a)]}(\varphi(x)) \\
& I_{f_{\varphi^{-1}(N)}(a)}(x)=I_{f_{N}[\psi(a)]}(\varphi(x)) \\
& F_{f_{\varphi^{-1}(N)}(a)}(x)=F_{f_{N}[\psi(a)]}(\varphi(x))
\end{aligned}
$$

If $\psi$ and $\varphi$ is injective (surjective), then $(\varphi, \psi)$ is injective (surjective).

### 6.1.1 Example

Let $E=\boldsymbol{N}$ (the set of natural numbers) be the parametric set and $K=\left(\boldsymbol{Z}_{5},+, \cdot\right)$ be a field. Define a mapping $f_{M}: N \rightarrow N S\left(\boldsymbol{Z}_{5}\right)$ where, for any $n \in N$ and $x \in \boldsymbol{Z}_{5}$,

$$
\begin{aligned}
& T_{f_{M}(n)}(x)= \begin{cases}0 & \text { if } x \in\{\overline{1}, \overline{3}\} \\
\frac{1}{3 n} & \text { if } x \in\{\overline{0}, \overline{2}, \overline{4}\} .\end{cases} \\
& I_{f_{M}(n)}(x)= \begin{cases}1-\frac{1}{n} & \text { if } x \in\{\overline{1}, \overline{3}\} \\
0 & \text { if } x \in\{\overline{0}, \overline{,}, \overline{4}\} .\end{cases} \\
& F_{f_{M(n)}(x)}= \begin{cases}\frac{1}{n+1} & \text { if } x \in\{\overline{1}, \overline{3}\} \\
0 & \text { if } x \in\{\overline{0}, \overline{2}, \overline{4}\} .\end{cases}
\end{aligned}
$$

Now, let $\varphi: \boldsymbol{Z}_{5} \rightarrow \boldsymbol{Z}_{5}$ and $\psi: \boldsymbol{N} \rightarrow \boldsymbol{N}$ be given by $\varphi(x)=3 x+\overline{1}$ and $\psi(n)=n^{2}$. Then for $a \in \boldsymbol{N}^{2}, y \in 3 \boldsymbol{Z}_{5}+\overline{1}$, the image of $M$ under ( $\varphi, \psi$ ) as follows:

$$
\begin{aligned}
& T_{f_{\varphi(M)}(a)}(y)= \begin{cases}0 & \text { if } y \in\{\overline{0}, \overline{4}\} \\
\frac{1}{3 \sqrt{a}} & \text { if } y \in\{\overline{1}, \overline{2}, \overline{3}\} .\end{cases} \\
& I_{f_{\varphi(M)}(a)}(y)= \begin{cases}1-\frac{1}{\sqrt{a}} & \text { if } y \in\{\overline{0}, \overline{4}\} \\
0 & \text { if } y \in\{\overline{1}, \overline{2}, \overline{3}\} .\end{cases} \\
& F_{f_{\varphi(M)}(a)}(y)= \begin{cases}\frac{1}{1+\sqrt{a}} & \text { if } y \in\{\overline{0}, \overline{4}\} \\
0 & \text { if } y \in\{\overline{1}, \overline{2}, \overline{3}\} .\end{cases}
\end{aligned}
$$

### 6.2 Theorem

Let $N$ be a neutrosophic soft field over $(K, E)$ and $(\varphi, \psi):(K, E) \rightarrow\left(\mathrm{L}, E^{\prime}\right)$ be a neutrosophic soft homomorphism. Then $(\varphi, \psi)(N)$ is a neutrosophic soft field over $\left(L, E^{\prime}\right)$.

Proof. Let $b \in \psi(E)$ and $y_{1}, y_{2} \in L$.
If $\varphi^{-1}\left(y_{1}\right)=\phi$ or $\varphi^{-1}\left(y_{2}\right)=\phi$, the proof is straight forward.
So, we assume that there exists $x_{1}, x_{2} \in K$ such that $\varphi\left(x_{1}\right)=y_{1}, \varphi\left(x_{2}\right)=y_{2}$. Then,

$$
\begin{aligned}
T_{f_{\varphi(N)}(b)}\left(y_{1}+y_{2}\right) & =\max _{\varphi(x)==y_{1}+y_{2}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}(x)\right] \\
& \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}+x_{2}\right)\right] \\
& \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right) * T_{f_{N}(a)}\left(x_{2}\right)\right] \\
& =\max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right)\right] * \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}\right)\right] \\
T_{f_{\varphi(N)}(b)}\left(-y_{1}\right) & =\max _{\varphi(x)=-y_{1}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}(x)\right] \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(-x_{1}\right)\right] \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right)\right] \\
T_{f_{\varphi(N)}(b)}\left(y_{1} \cdot y_{2}\right) & =\max _{\varphi(x)=y_{1} y_{2}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}(x)\right] \\
& \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1} x_{2}\right)\right] \\
& \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right) * T_{f_{N}(a)}\left(x_{2}\right)\right] \\
& =\max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right)\right] * \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}\right)\right] \\
T_{f_{\varphi(N)}(b)}\left(y_{2}^{-1}\right) & =\max _{\varphi(x)=y^{-1}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}(x)\right] \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}^{-1}\right)\right] \geq \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}\right)\right]
\end{aligned}
$$

Since, this inequality is satisfied for each $x_{1}, x_{2} \in K$ satisfying $\varphi\left(x_{1}\right)=y_{1}, \varphi\left(x_{2}\right)=y_{2}$ so we have the followings.

$$
\begin{aligned}
T_{f_{\varphi(N)}(b)}\left(y_{1}+y_{2}\right) & \geq\left(\max _{\varphi\left(x_{1}\right)=y_{1}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right)\right]\right) *\left(\max _{\varphi\left(x_{2}\right)=y_{2}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}\right)\right]\right) \\
& =T_{f_{\varphi(N)}(b)}\left(y_{1}\right) * T_{f_{\varphi(N)}(b)}\left(y_{2}\right), \\
T_{f_{\varphi(N)}(b)}\left(y_{1} \cdot y_{2}\right) & \geq\left(\max _{\left.\varphi x_{1}\right)=y_{1}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right)\right]\right) *\left(\max _{\varphi\left(x_{2}\right)=y_{2}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}\right)\right]\right) \\
& =T_{f_{\varphi(N)}(b)}\left(y_{1}\right) * T_{f_{\varphi(N)}(b)}\left(y_{2}\right), \\
T_{f_{\varphi(N)}(b)}\left(-y_{1}\right) & \geq \max _{\varphi\left(x_{1)}=y_{1}\right.} \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{1}\right)\right]=T_{f_{\varphi(N)}(b)}\left(y_{1}\right), \\
T_{f_{\varphi(N)}(b)}\left(y_{2}^{-1}\right) & \geq \max _{\varphi\left(x_{2}\right)=y_{2}} \max _{\psi(a)=b}\left[T_{f_{N}(a)}\left(x_{2}\right)\right]=T_{f_{\varphi(N)}(b)}\left(y_{2}\right) ;
\end{aligned}
$$

Next, $\quad I_{f_{\varphi(N)}(b)}\left(y_{1}+y_{2}\right)=\min _{\varphi(x)=y_{1}+y_{2}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}(x)\right]$

$$
\leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}+x_{2}\right)\right]
$$

$$
\leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right) \diamond I_{f_{N}(a)}\left(x_{2}\right)\right]
$$

$$
=\min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right)\right] 。 \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}\right)\right]
$$

$$
I_{f_{\varphi(N)}(b)}\left(-y_{1}\right)=\min _{\varphi(x)=-y_{1}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}(x)\right] \leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(-x_{1}\right)\right] \leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right)\right]
$$

$$
I_{f_{\varphi(N)}(b)}\left(y_{1} \cdot y_{2}\right)=\min _{\varphi(x)=y_{1} y_{2}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}(x)\right]
$$

$$
\leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1} x_{2}\right)\right]
$$

$$
\leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right) \circ I_{f_{N}(a)}\left(x_{2}\right)\right]
$$

$$
=\min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right)\right] \stackrel{\left.\min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}\right)\right], 0\right] .}{ }
$$

$$
I_{f_{\varphi(N)}(b)}\left(y_{2}^{-1}\right)=\min _{\varphi(x)=y_{2}^{-1}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}(x)\right] \leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}^{-1}\right)\right] \leq \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}\right)\right]
$$

Since, this inequality is satisfied for each $x_{1}, x_{2} \in K$ satisfying $\varphi\left(x_{1}\right)=y_{1}, \varphi\left(x_{2}\right)=y_{2}$ so the followings hold.

$$
\begin{aligned}
I_{f_{\varphi(N)}(b)}\left(y_{1}+y_{2}\right) & \leq\left(\min _{\varphi\left(x_{1}\right)=y_{1}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right)\right]\right) \diamond\left(\min _{\varphi\left(x_{2}\right)=y_{2}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}\right)\right]\right) \\
& =I_{f_{\varphi(N)}(b)}\left(y_{1}\right) \diamond I_{f_{\varphi(N)}(b)}\left(y_{2}\right), \\
I_{f_{\varphi(N)}(b)}\left(y_{1} \cdot y_{2}\right) & \leq\left(\min _{\varphi\left(x_{1}\right)=y_{1}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right)\right]\right) \diamond\left(\min _{\varphi\left(x_{2}\right)=y_{2}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}\right)\right]\right) \\
& =I_{f_{\varphi(N)}(b)}\left(y_{1}\right) \diamond I_{f_{\varphi(N)}(b)}\left(y_{2}\right), \\
I_{f_{\varphi(N)}(b)}\left(-y_{1}\right) & \leq \min _{\varphi\left(x_{1}\right)=y_{1}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{1}\right)\right]=I_{f_{\varphi(N)}(b)}\left(y_{1}\right), \\
I_{f_{\varphi(N)}(b)}\left(y_{2}^{-1}\right) & \leq \min _{\varphi\left(x_{2}\right)=y_{2}} \min _{\psi(a)=b}\left[I_{f_{N}(a)}\left(x_{2}\right)\right]=I_{f_{\varphi(N)}(b)}\left(y_{2}\right) ;
\end{aligned}
$$

Similarly, we can show that

$$
\begin{aligned}
& F_{f_{\varphi(N)}(b)}\left(y_{1}+y_{2}\right) \leq F_{f_{\varphi(N)}(b)}\left(y_{1}\right) \diamond F_{f_{\varphi(N)}(b)}\left(y_{2}\right), \quad F_{f_{\varphi(N)}(b)}\left(-y_{1}\right) \leq F_{f_{\varphi(N)}(b)}\left(y_{1}\right) ; \\
& F_{f_{\varphi(N)}(b)}\left(y_{1} \cdot y_{2}\right) \leq F_{f_{\varphi(N)}(b)}\left(y_{1}\right) \diamond F_{f_{\varphi(N)}(b)}\left(y_{2}\right), \quad F_{f_{\varphi(N)}(b)}\left(y_{2}^{-1}\right) \leq F_{f_{\varphi(N)}(b)}\left(y_{2}\right) ;
\end{aligned}
$$

This completes the proof.

### 6.3 Theorem

Let $M$ be a neutrosophic soft field over $\left(L, E^{\prime}\right)$ and $(\varphi, \psi):(K, E) \rightarrow\left(L, E^{\prime}\right)$ be a neutrosophic soft homomorphism. Then $(\varphi, \psi)^{-1}(M)$ is a neutrosophic soft field over $(K, E)$.

Proof. For $a \in \psi^{-1}\left(E^{\prime}\right)$ and $x_{1}, x_{2} \in K$, we have,

$$
\begin{aligned}
& T_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}+x_{2}\right)=T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}+x_{2}\right)\right) \\
&=T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)+\varphi\left(x_{2}\right)\right) \\
& \geq T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)\right) * T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right) \\
&=T_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right) * T_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}\right) \\
& T_{f_{\varphi^{-1}(M)}(a)}\left(-x_{1}\right)=T_{f_{M}[\psi(a)]}\left(\varphi\left(-x_{1}\right)\right)=T_{f_{M}[\psi(a)]}\left(-\varphi\left(x_{1}\right)\right) \geq T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)\right)=T_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right) \\
& T_{f_{\varphi^{-1}(M)}(a)}\left(x_{1} \cdot x_{2}\right)=T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1} x_{2}\right)\right) \\
&=T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right) \varphi\left(x_{2}\right)\right) \\
& \geq T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)\right) * T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right) \\
&=T_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right) * T_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}\right) \\
& T_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}^{-1}\right)=T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}^{-1}\right)\right)=T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right)^{-1} \geq T_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right)=T_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}\right) \\
& N_{\text {Next }} \quad \begin{aligned}
I_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}+x_{2}\right) & =I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}+x_{2}\right)\right) \\
& =I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)+\varphi\left(x_{2}\right)\right) \\
& \leq I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)\right) \diamond I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right) \\
& =I_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right) \diamond T_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}\right)
\end{aligned} \\
&
\end{aligned}
$$

$$
\begin{aligned}
&=I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right) \varphi\left(x_{2}\right)\right) \\
& \leq I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{1}\right)\right) \circ I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right) \\
&=I_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right) \circ I_{f_{\varphi^{-}}(M)}(a) \\
&\left.I_{2}\right)
\end{aligned} \quad \begin{aligned}
& I_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}^{-1}\right)=I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}^{-1}\right)=I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right)^{-1} \leq I_{f_{M}[\psi(a)]}\left(\varphi\left(x_{2}\right)\right)=I_{f_{\varphi^{-1}(M)}}(a)\right. \\
& \left(x_{2}\right)
\end{aligned}
$$

Similarly, $\quad F_{f_{\varphi^{-1}(M)}}(a)\left(x_{1}+x_{2}\right) \leq F_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right) \diamond F_{\mathrm{f}_{\varphi^{-1}(M)}(a)}\left(x_{2}\right), F_{f_{\varphi^{-1}(M)}(a)}\left(-x_{1}\right) \leq F_{f_{\varphi^{-1}(M)}(a)}\left(x_{1}\right)$; $F_{f_{\varphi^{-1}(M)}(a)}\left(x_{1} \cdot x_{2}\right) \leq F_{f_{\varphi^{-1}(M)}^{(a)}}\left(x_{1}\right) \diamond F_{f_{\varphi^{-1}(M)}}(a)\left(x_{2}\right), F_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}^{-1}\right) \leq F_{f_{\varphi^{-1}(M)}(a)}\left(x_{2}\right) ;$

Thus, the theorem is completed.

## 7 Neutrosophic soft algebra over a neutrosophic soft field

The concept of neutrosophic soft algebra over a neutrosophic soft field has been brought here. The structural characteristics of it have been investigated along with the development of some related theorems.

### 7.1 Definition

Let $M$ be a neutrosophic soft field over $(K, E)$ and $U$ be an algebra over $K$ where $K$ is a field and $E$ is a set of parameters. Then an NSS $N$ over $(U, E)$ is called a neutrosophic soft algebra if $\forall x, y \in U, \forall e \in E$ and $\lambda \in K$, the followings hold.

$$
\begin{aligned}
& \text { (i) }\left\{\begin{array}{l}
T_{f_{N}(e)}(x+y) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y) \\
I_{f_{N}(e)}(x+y) \leq I_{f_{N}(e)}(x) \bullet I_{f_{N}(e)}(y) \\
F_{f_{N}(e)}(x+y) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(y) .
\end{array}\right. \\
& \text { (ii) }\left\{\begin{array}{l}
T_{f_{N}(e)}(\lambda x) \geq T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x) \\
I_{f_{N}(e)}(\lambda x) \leq I_{f_{M}(e)}(\lambda) \odot I_{f_{N}(e)}(x) \\
F_{f_{N}(e)}(\lambda x) \leq F_{f_{M}(e)}(\lambda) \odot F_{f_{N}(e)}(x) .
\end{array}\right. \\
& \text { (iii) }\left\{\begin{array}{l}
T_{f_{N}(e)}(x . y) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y) \\
I_{f_{N}(e)}(x . y) \leq I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(y) \\
F_{f_{N}(e)}(x . y) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(y) .
\end{array}\right. \\
& \text { (iv) }\left\{\begin{array}{l}
T_{f_{M}(e)}\left(1_{k}\right) \geq T_{f_{N}(e)}(x) \\
I_{f_{M}(e)}\left(1_{k}\right) \leq I_{f_{N}(e)}(x) \\
F_{f_{M}(e)}\left(1_{k}\right) \leq F_{f_{N}(e)}(x) .
\end{array}\right.
\end{aligned}
$$

We write, the triplet $(N, U, E)$ is a neutrosophic soft algebra over the triplet $(M, K, E)$, a neutrosophic soft field.

### 7.1.1 Corollary

If ( $N, U, E$ ) is a neutrosophic soft algebra over the neutrosophic soft field $(M, K, E)$, then $\forall x \in U, \forall e \in E$ and for the additive identity $0_{k} \in K$,

$$
T_{f_{M}(e)}\left(0_{k}\right) \geq T_{f_{N}(e)}(x), \quad I_{f_{M}(e)}\left(0_{k}\right) \leq I_{f_{N}(e)}(x), \quad F_{f_{M}(e)}\left(0_{k}\right) \leq F_{f_{N}(e)}(x) ;
$$

Proof. It directly follows from the Proposition [3.1](iii) and from the Definition [7.1](iv);

### 7.1.2 Corollary

Let $a * b=\min \{a, b\}$ and $a \diamond \mathrm{~b}=\max \{a, b\}$. Then $(N, U, E)$ is a neutrosophic soft algebra over the neutrosophic soft field ( $M, K, E$ ) where $M, N, K, U, E$ are defined in [7.1] iff for any $\lambda, \mu \in K$ and $x, y \in U$ followings hold.

$$
\begin{aligned}
& \left\{\begin{array}{l}
T_{f_{N}(e)}(\lambda x+\mu y) \geq\left(T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x)\right) *\left(T_{f_{M}(e)}(\mu) * T_{f_{N}(e)}(y)\right),
\end{array}\right. \\
& \text { (i) }\left\{\begin{array}{l}
I_{f_{N}(e)}(\lambda x+\mu y) \leq\left(I_{f_{M}(e)}(\lambda) \diamond I_{f_{N}(e)}(x)\right) \circ\left(I_{f_{M}(e)}(\mu) \diamond I_{f_{N}(e)}(y)\right)
\end{array}\right. \\
& F_{F_{f_{N}(e)}}(\lambda x+\mu y) \leq\left(F_{f_{M}(e)}(\lambda) \circ F_{f_{N}(e)}(x)\right) \circ\left(F_{f_{M}(e)}(\mu) \circ F_{f_{N}(e)}(y)\right) \text {. } \\
& \text { (ii) }\left\{\begin{array}{l}
T_{f_{N}(e)}(x . y) \geq T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y) \\
I_{f_{N}(e)}(x . y) \leq I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(y) \\
F_{f_{N}(e)}(x . y) \leq F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(y) .
\end{array}\right. \\
& \text { (iii) }\left\{\begin{array}{l}
T_{f_{M}(e)}\left(1_{k}\right) \geq T_{f_{N}(e)}(x) \\
I_{f_{M}(e)}\left(1_{k}\right) \leq I_{f_{N}(e)}(x) \\
F_{f_{M}(e)}\left(1_{k}\right) \leq F_{f_{N}(e)}(x) .
\end{array}\right.
\end{aligned}
$$

Proof. First let $(N, U, E)$ be a neutrosophic soft algebra over the neutrosophic soft field $(M, K, E)$. Then,

$$
\begin{align*}
& T_{f_{N}(e)}(\lambda x+\mu y) \geq T_{f_{N}(e)}(\lambda x) * T_{f_{N}(e)}(\mu y) \geq\left(T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x)\right) *\left(T_{f_{M}(e)}(\mu) * T_{f_{N}(e)}(y)\right)  \tag{i}\\
& I_{f_{N}(e)}(\lambda x+\mu y) \leq I_{f_{N}(e)}(\lambda x) \circ I_{f_{N}(e)}\left(\mu y \leq\left(I_{f_{M}(e)}(\lambda) \diamond I_{f_{N}(e)}(x)\right) \circ\left(I_{f_{M}(e)}(\mu) \circ I_{f_{N}(e)}(y)\right)\right. \\
& F_{f_{N}(e)}(\lambda x+\mu y) \leq F_{f_{N}(e)}(\lambda x) \circ F_{f_{N}(e)}(\mu y) \leq\left(F_{f_{M}(e)}(\lambda) \circ F_{f_{N}(e)}(x)\right) \circ\left(F_{f_{M}(e)}(\mu) \circ F_{f_{N}(e)}(y)\right)
\end{align*}
$$

(ii) and (iii) from Definition [7.1](iii),(iv);

Conversely, suppose the conditions hold.
(i) For $\lambda=\mu=1_{k}$ and $x, y \in U$,

$$
\begin{aligned}
& T_{f_{N}(e)}(x+y)=T_{f_{N}(e)}\left(1_{k} x+1_{k} y\right) \geq\left(T_{f_{M}(e)}\left(1_{k}\right) * T_{f_{N}(e)}(x)\right) *\left(T_{f_{M}(e)}\left(1_{k}\right) * T_{f_{N}(e)}(y)\right)=T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y) \\
& I_{f_{N}(e)}(x+y)=I_{f_{N}(e)}\left(1_{k} x+1_{k} y\right) \leq\left(I_{f_{M}(e)}\left(1_{k}\right) \diamond I_{f_{N}(e)}(x)\right) \circ\left(I_{f_{M}(e)}\left(1_{k}\right) \diamond I_{f_{N}(e)}(y)\right)=I_{f_{N}(e)}(x) \circ I_{f_{N}(e)}(y) \\
& F_{f_{N}(e)}(x+y)=F_{f_{N}(e)}\left(1_{k} x+1_{k} y\right) \leq\left(F_{f_{M}(e)}\left(1_{k}\right) \diamond F_{f_{N}(e)}(x)\right) \diamond\left(F_{f_{M}(e)}\left(1_{k}\right) \circ F_{f_{N}(e)}(y)\right)=F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(y)
\end{aligned}
$$

(ii) For $\mu=0_{k}$ and $x, y \in U$,

$$
\begin{aligned}
& T_{f_{N}(e)}(\lambda x)=T_{f_{N}(e)}\left(\lambda x+0_{k} x\right) \\
& \geq\left(T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x)\right) *\left(T_{f_{M}(e)}\left(0_{k}\right) * T_{f_{N}(e)}(x)\right) \\
& =T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x) * T_{f_{N}(e)}(x) \\
& =T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x) \\
& I_{f_{N}(e)}(\lambda x)=I_{f_{N}(e)}\left(\lambda \mathrm{x}+0_{k} x\right) \\
& \leq\left(I_{f_{M}(e)}(\lambda) \diamond I_{f_{N}(e)}(x)\right) \circ\left(I_{f_{M}(e)}\left(0_{k}\right) \diamond I_{f_{N}(e)}(x)\right) \\
& =I_{f_{M}(e)}(\lambda) \circ I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(x) \\
& =I_{f_{M}(e)}(\lambda) \diamond I_{f_{N}(e)}(x) \\
& F_{f_{N}(e)}(\lambda x)=F_{f_{N}(e)}\left(\lambda x+0_{k} x\right) \\
& \leq\left(F_{f_{M}(e)}(\lambda) \diamond F_{f_{N}(e)}(x)\right) \circ\left(F_{f_{M}(e)}\left(0_{k}\right) \circ F_{f_{N}(e)}(x)\right) \\
& =F_{f_{M}(e)}(\lambda) \bullet F_{f_{N}(e)}(x) \circ F_{f_{N}(e)}(x) \\
& =F_{f_{M}(e)}(\lambda) \diamond F_{f_{N}(e)}(x)
\end{aligned}
$$

(iii) and (iv) hold obviously.

This ends the proof.

### 7.2 Theorem

The intersection of two neutrosophic soft algebras over the same neutrosophic soft field is also a neutrosophic soft algebra on the assumption that $a * b=\min \{a, b\}$ and $a \diamond b=\max \{a, b\}$.

Proof. Let $\left(N_{1}, U, E\right)$ and $\left(N_{2}, U, E\right)$ be two neutrosophic soft algebras over the neutrosophic soft field $(M, K, E)$ and let $(N, U, E)=\left(N_{1}, U, E\right) \cap\left(N_{2}, U, E\right)$. Now for $x, y \in U, \lambda \in K$ and $\forall e \in E$,

$$
\begin{aligned}
T_{f_{N}(e)}(x+y) & =T_{f_{N_{1}}(e)}(x+y) * T_{f_{N_{2}}(e)}(x+y) \\
& \geq\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(x) * T_{f_{N_{2}}(e)}(y)\right] \\
& =\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(y) * T_{f_{N_{2}}(e)}(x)\right] \quad(\text { as } * \text { is commutative }) \\
& =T_{f_{N_{1}}(e)}(x) *\left[T_{f_{N_{1}}(e)}(y) * T_{f_{N_{2}}(e)}(y)\right] * T_{f_{N_{2}}(e)}(x) \quad(\text { as } * \text { is associative }) \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N}(e)}(y) * T_{f_{N_{2}}(e)}(x) \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x) * T_{f_{N}(e)}(y) \quad(\text { as } * \text { is commutative }) \\
& =T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y), \\
I_{f_{N}(e)}(x+y) & =I_{f_{N_{1}}(e)}(x+y) \diamond I_{f_{N_{2}}(e)}(x+y) \\
& \leq\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \diamond\left[I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(y)\right] \\
& =\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \diamond\left[I_{f_{N_{2}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(x)\right] \quad(\text { as } \diamond \text { is commutative }) \\
& =I_{f_{N_{1}}(e)}(x) \diamond\left[I_{f_{N_{1}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(y)\right] \diamond I_{f_{N_{2}}(e)}(x) \quad(\text { as } \diamond \text { is associative }) \\
& =I_{f_{N_{1}(e)}(x)}(x) \diamond I_{f_{N}(e)}(y) \diamond I_{f_{N_{2}}(e)}(x) \\
& =I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N}(e)}(y) \quad(\text { as } \diamond \text { is commutative }) \\
& =I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(y),
\end{aligned}
$$

Similarly, $\quad F_{f_{N}(e)}(x+y) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(y) ; \quad$ Next,

$$
\begin{aligned}
T_{f_{N}(e)}(\lambda x) & =T_{f_{N_{1}}(e)}(\lambda x) * T_{f_{N_{2}}(e)}(\lambda x) \\
& \geq\left[T_{f_{M}(e)}(\lambda) * T_{f_{N_{1}}(e)}(x)\right] *\left[T_{f_{M}(e)}(\lambda) * T_{f_{N_{2}}(e)}(x)\right. \\
& =\left[T_{f_{M}(e)}(\lambda) * T_{f_{M}(e)}(\lambda)\right] *\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x)\right] \quad(\text { as } * \text { is commutative }) \\
& =T_{f_{M}(e)}(\lambda) * T_{f_{N}(e)}(x), \\
I_{f_{N}(e)}(\lambda x) & =I_{f_{N_{1}(e)}}(\lambda x) \diamond I_{f_{N_{2}(e)}}(\lambda x) \\
& \leq\left[I_{f_{M}(e)}(\lambda) \diamond I_{f_{N_{1}}(e)}(x)\right] \diamond\left[I_{f_{M}(e)}(\lambda) \diamond I_{f_{N_{2}}(e)}(x)\right] \\
& =\left[I_{f_{M}(e)}(\lambda) \diamond I_{f_{M}(e)}(\lambda)\right] \diamond\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x)\right] \quad(\text { as } \diamond \text { is commutative }) \\
& =I_{f_{M}(e)}(\lambda) \diamond I_{f_{N}(e)}(x),
\end{aligned}
$$

Similarly, $\quad F_{f_{N}(e)}(\lambda x) \leq F_{f_{M}(e)}(\lambda) \diamond F_{f_{N}(e)}(x) ; \quad$ Next,

$$
\begin{aligned}
& T_{f_{N}(e)}(x . y)=T_{f_{N_{1}}(e)}(x . y) * T_{f_{N_{2}}(e)}(x . y) \\
& \geq\left[T_{f_{N_{1}(e)}}(x) * T_{f_{N_{1}(e)}}(y)\right] *\left[T_{f_{N_{2}(e)}}(x) * T_{f_{N_{2}}(e)}(y)\right] \\
& =\left[T_{f_{N_{1}}(e)}(x) * T_{f_{N_{1}}(e)}(y)\right] *\left[T_{f_{N_{2}}(e)}(y) * T_{f_{N_{2}}(e)}(x)\right] \quad \text { (as } \circ \text { is commutative) } \\
& =T_{f_{N_{1}}(e)}(x) *\left[T_{f_{N_{1}}(e)}(y) * T_{f_{N_{2}}(e)}(y)\right] * T_{f_{N_{2}}(e)}(x) \text { (as } \diamond \text { is associative) } \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N}(e)}(y) * T_{f_{N_{2}}(e)}(x) \\
& =T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x) * T_{f_{N}(e)}(y) \quad(\text { as } \diamond \text { is commutative) } \\
& =T_{f_{N}(e)}(x) * T_{f_{N}(e)}(y) \text {, }
\end{aligned}
$$

$$
\begin{aligned}
I_{f_{N}(e)}(x . y) & =I_{f_{N_{1}}(e)}(x . y) \diamond I_{f_{N_{2}}(e)}(x . y) \\
& \leq\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \diamond\left[I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(y)\right] \\
& =\left[I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{1}}(e)}(y)\right] \diamond\left[I_{f_{N_{2}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(x)\right] \quad(\text { as } \diamond \text { is commutative }) \\
& =I_{f_{N_{1}}(e)}(x) \diamond\left[I_{f_{N_{1}}(e)}(y) \diamond I_{f_{N_{2}}(e)}(y)\right] \diamond I_{f_{N_{2}}(e)}(x) \quad(\text { as } \diamond \text { is commutative }) \\
& =I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N}(e)}(y) \diamond I_{f_{N_{2}}(e)}(x) \\
& =I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}}(e)}(x) \diamond I_{f_{N}(e)}(y) \quad(\text { as } \diamond \text { is commutative }) \\
& =I_{f_{N}(e)}(x) \diamond I_{f_{N}(e)}(y)
\end{aligned}
$$

Similarly, $\quad F_{f_{N}(e)}(x . y) \leq F_{f_{N}(e)}(x) \diamond F_{f_{N}(e)}(y) ;$
Finally, for the multiplicative identity $1_{k}$ of the field $K$,

$$
\begin{aligned}
& T_{f_{M}(e)}\left(1_{k}\right) \geq T_{f_{N_{1}(e)}}(x) \quad \text { and } \quad T_{f_{M}(e)}\left(1_{k}\right) \geq T_{f_{N_{2}}(e)}(x) \\
\Rightarrow & T_{f_{M}(e)}\left(1_{k}\right) * T_{f_{M}(e)}\left(1_{k}\right) \geq T_{f_{N_{1}}(e)}(x) * T_{f_{N_{2}}(e)}(x) \\
\Rightarrow & T_{f_{M}(e)}\left(1_{k}\right) \geq T_{f_{N}(e)}(x), \\
& I_{f_{M}(e)}\left(1_{k}\right) \leq I_{f_{N_{1}}(e)}(x) \quad \text { and } \quad I_{f_{M}(e)}\left(1_{k}\right) \leq I_{f_{N_{2}}(e)}(x) \\
\Rightarrow & I_{f_{M}(e)}\left(1_{k}\right) \diamond I_{f_{M}(e)}\left(1_{k}\right) \leq I_{f_{N_{1}}(e)}(x) \diamond I_{f_{N_{2}(e)}(x)}(x) \\
\Rightarrow & I_{f_{M}(e)}\left(1_{k}\right) \leq I_{f_{N}(e)}(x),
\end{aligned}
$$

Similarly, $\quad F_{f_{M}(e)}\left(1_{k}\right) \leq F_{f_{N}(e)}(x) ;$
This follows the theorem.
The theorem is also true for a family of neutrosophic soft algebras over a neutrosophic soft field.

### 7.3 Theorem

Let $U, V$ be two algebras over the field $K$ and $(P, K, E)$ be a neutrosophic soft field. Suppose $g: U \rightarrow V$ be an algebraic isomorphism and $(M, U, E)$ be a neutrosophic soft algebra over $(P, K, E)$. Then $(g(M), V, E)$ is also a neutrosophic soft algebra over $(P, K, E)$.

Proof. Let $x_{1}, x_{2} \in U$ and $y_{1}, y_{2} \in V$ such that $y_{1}=g\left(x_{1}\right), y_{2}=g\left(x_{2}\right)$. Then $\forall e \in E$ and $\lambda \in K$,

$$
\begin{aligned}
T_{f_{g(M)}(e)}\left(y_{1}+y_{2}\right) & =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}+y_{2}\right)\right] \\
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)+g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =T_{f_{M}(e)}\left(x_{1}+x_{2}\right) \\
& \geq T_{f_{M}(e)}\left(x_{1}\right) * T_{f_{M}(e)}\left(x_{2}\right) \\
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] * T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =T_{f_{g(M)}(e)}\left(y_{1}\right) * T_{f_{g(M)}(e)}\left(y_{2}\right), \\
I_{f_{g(M)}(e)}\left(y_{1}+y_{2}\right) & =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}+y_{2}\right)\right] \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)+g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =I_{f_{M}(e)}\left(x_{1}+x_{2}\right) \\
& \leq I_{f_{M}(e)}\left(x_{1}\right) \diamond I_{f_{M}(e)}\left(x_{2}\right) \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] \diamond I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =I_{f_{g(M)}(e)}\left(y_{1}\right) \diamond I_{f_{g(M)}(e)}\left(y_{2}\right),
\end{aligned}
$$

Similarly, $\quad F_{f_{g(M)}(e)}\left(y_{1}+y_{2}\right) \leq F_{f_{g(M)}(e)}\left(y_{1}\right) \diamond F_{f_{g(M)}(e)}\left(y_{2}\right) ; \quad$ Next,

$$
\begin{aligned}
T_{f_{g(M)}(e)}\left(\lambda y_{2}\right) & =T_{f_{M}(e)}\left[g^{-1}\left(\lambda y_{2}\right)\right] \\
& =T_{f_{M}(e)}\left[\lambda g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =T_{f_{M}(e)}\left(\lambda x_{2}\right) \\
& \geq T_{f_{P}(e)}(\lambda) * T_{f_{M}(e)}\left(x_{2}\right) \\
& =T_{f_{P}(e)}(\lambda) * T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =T_{f_{P}(e)}(\lambda) * T_{f_{g(M)}(e)}\left(y_{2}\right), \\
I_{f_{g(M)}(e)}\left(\lambda y_{2}\right) & =I_{f_{M}(e)}\left[g^{-1}\left(\lambda y_{2}\right)\right] \\
& =I_{f_{M}(e)}\left[\lambda g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =I_{f_{M}(e)}\left(\lambda x_{2}\right) \\
& \leq I_{f_{P}(e)}(\lambda) \diamond I_{f_{M}(e)}\left(x_{2}\right) \\
& =I_{f_{P}(e)}(\lambda) \diamond I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =I_{f_{P}(e)}(\lambda) \diamond I_{f_{g(M)}(e)}\left(y_{2}\right),
\end{aligned}
$$

Similarly, $\quad F_{f_{g(M)}(e)}\left(\lambda y_{2}\right) \leq F_{f_{P}(e)}(\lambda) \diamond F_{f_{g(M)}(e)}\left(y_{2}\right) ; \quad$ Next,

$$
\begin{aligned}
T_{f_{g(M)}(e)}\left(y_{1} \cdot y_{2}\right) & =T_{f_{M}(e)}\left[g^{-1}\left(y_{1} \cdot y_{2}\right)\right] \\
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right) \cdot g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =T_{f_{M}(e)}\left(x_{1} \cdot x_{2}\right) \\
& \geq T_{f_{M}(e)}\left(x_{1}\right) * T_{f_{M}(e)}\left(x_{2}\right) \\
& =T_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] * T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =T_{f_{g(M)}(e)}\left(y_{1}\right) * T_{f_{g(M)}(e)}\left(y_{2}\right), \\
I_{f_{g(M)}(e)}\left(y_{1} \cdot y_{2}\right) & =I_{f_{M}(e)}\left[g^{-1}\left(y_{1} \cdot y_{2}\right)\right] \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right) \cdot g^{-1}\left(y_{2}\right)\right], \text { as } g^{-1} \text { is homomorphism. } \\
& =I_{f_{M}(e)}\left(x_{1} \cdot x_{2}\right) \\
& \leq I_{f_{M}(e)}\left(x_{1}\right) \diamond I_{f_{M}(e)}\left(x_{2}\right) \\
& =I_{f_{M}(e)}\left[g^{-1}\left(y_{1}\right)\right] \diamond I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right] \\
& =I_{f_{g(M)}(e)}\left(y_{1}\right) \diamond I_{f_{g(M)}(e)}\left(y_{2}\right)
\end{aligned}
$$

Similarly, $\quad F_{f_{g(M)}(e)}\left(y_{1} \cdot y_{2}\right) \leq F_{f_{g(M)}(e)}\left(y_{1}\right) \diamond F_{f_{g(M)}(e)}\left(y_{2}\right) ;$
Finally, for the multiplicative identity $1_{k}$ of the field $K$,

$$
\begin{gathered}
T_{f_{P}(e)}\left(1_{k}\right) \geq T_{f_{M}(e)}\left(x_{2}\right)=T_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right]=T_{f_{g(M)}(e)}\left(y_{2}\right), \\
I_{f_{P}(e)}\left(1_{k}\right) \leq I_{f_{M}(e)}\left(x_{2}\right)=I_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right]=I_{f_{g(M)}(e)}\left(y_{2}\right), \\
F_{f_{P}(e)}\left(1_{k}\right) \leq F_{f_{M}(e)}\left(x_{2}\right)=F_{f_{M}(e)}\left[g^{-1}\left(y_{2}\right)\right]=F_{f_{g(M)}(e)}\left(y_{2}\right)
\end{gathered}
$$

This completes the theorem.

### 7.4 Theorem

Let $U, V$ be two algebras over the field $K$ and $(P, K, E)$ be a neutrosophic soft field. Suppose $g: U \rightarrow V$ be an algebraic homomorphism and $(N, V, E)$ be a neutrosophic soft algebra over $(P, K, E)$. Then $\left(g^{-1}(N), U, E\right)$ is also a neutrosophic soft algebra over $(P, K, E)$.

Proof. Let $x_{1}, x_{2} \in U$ and $y_{1}, y_{2} \in V$ such that $y_{1}=g\left(x_{1}\right), y_{2}=g\left(x_{2}\right)$. Then $\forall e \in E$ and $\lambda \in K$,

$$
\begin{aligned}
T_{f_{g^{-1}(N)}(e)}\left(x_{1}+x_{2}\right) & =T_{f_{N}(e)}\left[g\left(x_{1}+x_{2}\right)\right] \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right)+g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =T_{f_{N}(e)}\left(y_{1}+y_{2}\right) \\
& \geq T_{f_{N}(e)}\left(y_{1}\right) * T_{f_{N}(e)}\left(y_{2}\right) \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right)\right] * T_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =T_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) * T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right), \\
I_{f_{g^{-1}(N)}(e)}\left(x_{1}+\mathrm{x}_{2}\right) & =I_{f_{N}(e)}\left[g\left(x_{1}+x_{2}\right)\right] \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right)+g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =I_{f_{N}(e)}\left(y_{1}+y_{2}\right) \\
& \leq I_{f_{N}(e)}\left(y_{1}\right) \diamond I_{f_{N}(e)}\left(y_{2}\right) \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right)\right] \diamond I_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =I_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond I_{f_{g^{-1}(N)}(e)}\left(x_{2}\right),
\end{aligned}
$$

Similarly, $\quad F_{f_{g^{-1}(N)}(e)}\left(x_{1}+x_{2}\right) \leq F_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond F_{f_{g^{-1}(N)}(e)}\left(x_{2}\right) ; \quad$ Next,

$$
\left.\begin{array}{rl}
T_{f_{g^{-1}(N)}(e)}\left(\lambda x_{2}\right) & =T_{f_{N}(e)}\left[g\left(\lambda x_{2}\right)\right] \\
& =T_{f_{N}(e)}\left[\lambda g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =T_{f_{N}(e)}\left(\lambda y_{2}\right) \\
& \geq T_{f_{P}(e)}(\lambda) * T_{f_{N}(e)}\left(y_{2}\right) \\
& =T_{f_{P}(e)}(\lambda) * T_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =T_{f_{P}(e)}(\lambda) * T_{f_{g-1}(N)}(e)\left(x_{2}\right), \\
I_{f_{g-1}(N)}(e) \\
\left(\lambda x_{2}\right) & =I_{f_{N}(e)}\left[g\left(\lambda x_{2}\right)\right] \\
& =I_{f_{N}(e)}\left[\lambda g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =I_{f_{N}(e)}\left(\lambda y_{2}\right) \\
& \leq I_{f_{P}(e)}(\lambda) \diamond I_{f_{N}(e)}\left(y_{2}\right) \\
& =I_{f_{p}(e)}(\lambda) \diamond I_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =I_{f_{P}(e)}(\lambda) \diamond I_{f_{g}-1}(())
\end{array} x_{2}\right),
$$

Similarly, $\quad F_{f_{g^{-1}(N)}(e)}\left(\lambda x_{2}\right) \leq F_{f_{P}(e)}(\lambda) \diamond F_{f_{g^{-1}(N)}(e)}\left(x_{2}\right) ; \quad$ Next,

$$
\begin{aligned}
T_{f_{g^{-1}(N)}(e)}\left(x_{1} \cdot x_{2}\right) & =T_{f_{N}(e)}\left[g\left(x_{1} \cdot x_{2}\right)\right] \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right) \cdot g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =T_{f_{N}(e)}\left(y_{1} \cdot y_{2}\right) \\
& \geq T_{f_{N}(e)}\left(y_{1}\right) * T_{f_{N}(e)}\left(y_{2}\right) \\
& =T_{f_{N}(e)}\left[g\left(x_{1}\right)\right] * T_{f_{N}(e)}\left[g\left(x_{2}\right)\right] \\
& =T_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) * T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right), \\
I_{f_{g^{-1}(N)}(e)}\left(x_{1} \cdot x_{2}\right) & =I_{f_{N}(e)}\left[g\left(x_{1} \cdot x_{2}\right)\right] \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right) \cdot g\left(x_{2}\right)\right], \text { as } g \text { is homomorphism. } \\
& =I_{f_{N}(e)}\left(y_{1} \cdot y_{2}\right) \\
& \leq I_{f_{N}(e)}\left(y_{1}\right) \diamond I_{f_{N}(e)}\left(y_{2}\right) \\
& =I_{f_{N}(e)}\left[g\left(x_{1}\right)\right] \diamond I_{f_{N}(e)}\left[g\left(x_{2}\right)\right]
\end{aligned}
$$

$$
=I_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond I_{f_{g^{-1}(N)}(e)}\left(x_{2}\right),
$$

Similarly, $\quad F_{f_{g^{-1}(N)}(e)}\left(x_{1} \cdot x_{2}\right) \leq F_{f_{g^{-1}(N)}(e)}\left(x_{1}\right) \diamond F_{f_{g^{-1}(N)}(e)}\left(x_{2}\right) ;$
Finally, for the multiplicative identity $1_{k}$ of the field $K$,

$$
\begin{gathered}
T_{f_{P}(e)}\left(1_{k}\right) \geq T_{f_{N}(e)}\left(y_{2}\right)=T_{f_{N}(e)}\left[g\left(x_{2}\right)\right]=T_{f_{g^{-1}(N)}(e)}\left(x_{2}\right) \\
I_{f_{P}(e)}\left(1_{k}\right) \leq I_{f_{N}(e)}\left(y_{2}\right)=I_{f_{N}(e)}\left[g\left(x_{2}\right)\right]=I_{f_{g^{-1}(N)}(e)}\left(x_{2}\right), \\
F_{f_{P}(e)}\left(1_{k}\right) \leq F_{f_{N}(e)}\left(y_{2}\right)=F_{f_{N}(e)}\left[g\left(x_{2}\right)\right]=F_{f_{g^{-1}(N)}(e)}\left(x_{2}\right)
\end{gathered}
$$

Hence, the theorem is proved.

## 8 Conclusion

The effort of the paper is to extend the concept 'Neutrosophic soft field' by investigating its structural characteristics. The Cartesian product of neutrosophic soft fields, neutrosophic soft subfield, neutrosophic soft algebra over neutrosophic soft field have been defined and some related theorems are established. Moreover the neutrosophic soft function over the crisp fields is defined and illustrated by suitable examples. The characteristics of neutrosophic soft homomorphic image and pre-image of a neutrosophic soft field are studied here. We expect the further work in this setting.

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