Sugeno Integral on Median Operations and Its Applications

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Abstract:

In this paper we generalize the definition of sugeno integrals by utilizing median operations which are a special kind of aggregation operations.some of the common properties of generalized integrals are also discussed.

1.Introduction:

The concept of fuzzy integral,which was introduced by sugeno(6) is now well developed.Recently various attempts have been made to generalize it.In this paper we propose to generalize sugeno integrals by replacing the two operations by the so called median operations which were introduced and investigated by Duboid and Prade(2)

Median operations possess all common properties that are satisfied by t-norms and t-conorms except boundry conditions.They are particular aggregation operations for which the max and min operations are the extreme cases.In sec 2 we discuss basic properties of median operations following the paper by Dubois and Prade(2) and Wu(10).In sec 3 we define median integrals and investigate their properties.In sec(4) we discuss the significance and prospective applicability of median Integral.

2.Median operations

In general norm operations possess all properties that are shared by t-norms all properties that are shared by t-norms and t-conorms but differ from them in boundary conditions .They form a special class of aggregation operations.

Definition:2.1

Let T be a t-norm $\perp$ be a t-conorm $\lambda \in (0,1)$.Then a binary operation * by the following formula is a norm operation that is neither a t-norm nor a t-conorm

$$a * b = \begin{cases} \min(a \perp b, \lambda) & \text{if } a,b \in [o,\lambda) \\ \max(a \top b, \lambda) & \text{if } a,b \in (\lambda,1] \\ \lambda & \text{otherwise} \end{cases}$$

Special norm operations which are called median operations are known to be the only continuous and idempotent norm operations for each $\lambda \in (0,1)$ h_λ is defined by

$$h = \begin{cases} \max(a,b) & \text{if } a,b \in [0,\lambda) \\ \min(a,b) & \text{if } a,b \in (\lambda,1] \\ \lambda & \text{otherwise} \end{cases}$$

It is easy to see that when λ=0,h_λ becomes the max operation when λ=1 ,h_λ becomes the min operation.

Proposition2.2

Let $a_i \in [0,1]$ iϵI

Then h { a_i(iϵI) }=max{ min $a_i l_i \in$ I}

Propositio:2.3

For any a,bϵ[0,1]

h_λ=max{min(a,b),min(a,λ),min(b,λ)}

=min{max(a,b),max(a,λ),max(b,λ)}

3.sugeno integral on median operations:

In this section we generalize the sugeno integral by replacing the max-min median operations.Let (X,$\mathfrak{F}$,μ) be a fuzzy measure space and let $\mathfrak{F}$ be the set of all non negative functions defined on (X,$\mathfrak{F}$) for our purpose we assume that μ(A)ϵ[0,1] for any Aϵ$\mathfrak{F}$ and moreover f(x)ϵ[0,1] for any fϵ$\mathfrak{F}$ and all xϵX using the median operations h_λ.

Definition: 3.1

Let E,f ∈ $\mathfrak{F}$,the median integral denoted by

(λ)∫fdμ=$h_{1-\lambda}$ [h_λ (α,μ(E∩f_α)/αϵ[0,1]]

Where λϵ[0,1]

Whenλ=0 the median integral becomes

(0)∫fdμ=max{min(α,μ(E∩f_α)/αϵ[0,1]}

Which is exactly the same as the sugeno integral when λ=1 the median integral becomes

(1)∫fdμ=min{max(α,μ(E∩f_α)/αϵ[0,1]}

Proposition3.2

If μ(E)=0 then (λ)∫fdμ=min(λ,1-λ)

Proof:

(λ)$\int_E f\,d\mu$ = h_{1-} {h_λ(α,μ(E∩f_α)/αϵ[0,1]}

= h_{1-}{h_λ(α,0)/αϵ[0,1]}

= h_{1-}{min(α,λ)/αϵ[0,1]}

=max{min{min(α,λ)/αϵ[0,1]}

min{1-λ,max {min(α,λ)/αϵ[0,1]}}}

=max{0, min(1-λ,λ)}

=min(1-λ,λ)

Proposition:3.3

If $f_1 \leq f_2$ then $(\lambda)\int_E f_1\,d\mu \leq (\lambda)\int_E f_2\,d\mu$

Proof:

Since $f_1 \leq f_2$ we have $(f_1) \leq (f_2)_{\alpha}$ for any $\alpha \in [0,1]$

Thus $\mu (E \cap (f_1)_{\alpha}) \leq \mu(E \cap (f_2)_{\alpha})$

Therefore,

$(\lambda)\int_E f_1 \, d\mu = h_{1-}\{h_{\lambda}(\alpha, \mu(E \cap (f_1)\alpha)/\alpha \epsilon [0,1]\}$

$= h_{1-}\{h_{\lambda}(\alpha, \mu(E \cap (f_2)\alpha)/\alpha \epsilon [0,1]\}$

$= (\lambda\int_E f_2 \, d\mu$

Proposition:3.3

If $A \subseteq B$ then $(\lambda)\int_A f d\mu \leq (\lambda)\int_B f d\mu$

Proof:

The proof of this proposition is similar to the above proof.

Proposition:3.4

Let $a \in [0,1]$ Then

$(\lambda)\int_E a \, d\mu = h_{\lambda} [1-\lambda, \min(a, \mu(E))]$

Proof:

$(\lambda)\int_E a \, d\mu = h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(E \cap a_{\alpha}))/\alpha \epsilon [0,1]\}$

$= h_{1-\lambda}\{ h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(E \cap a_{\alpha}))/\alpha \epsilon [0,a)\}$

$\{ h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(E \cap a_{\alpha}))/\alpha \epsilon (a,1]\}$

$= h_{1-\lambda}\{ h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(E)/\alpha \epsilon [0,a)\}$

$\{ h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(\phi)/\alpha \epsilon (a,1]\}$

$= h_{1-\lambda}\{ h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(E)/\alpha \epsilon [0,a)\}$

$\{ h_{1-\lambda}\{h_{\lambda}(\alpha, 0)/\alpha \epsilon (a,1]\}$

$= h_{1-\lambda}\{ h_{1-\lambda}\{h_{\lambda}(\alpha, \mu(E)/\alpha \epsilon [0,a)\}$

$\{ h_{1-\lambda}\{\min(\alpha, \lambda)/\alpha \epsilon (a,1]\}$

Since $h_{1-}\{\min(\alpha, \lambda)/\alpha \epsilon (a, 1]$

$= \max\{ \min(\min(\alpha, \lambda))/\alpha \epsilon (a,1]\} \min(1-\lambda, \max\{\min(\alpha, \lambda)/\alpha \epsilon (a,1]\})\}$

$= \max[\min(a, \lambda), \min(\lambda, 1-\lambda)]$ and $h_{1-\lambda}[h_{\lambda}(\alpha, \mu(E))/\alpha \epsilon [0,a)\}$

$= \max[\min(h_{\lambda}(\alpha, \mu(E))/\alpha \epsilon [0,a] \min(1-\lambda, \max\{h_{\lambda}(\alpha, \mu(E))/\alpha \epsilon [0,a)\})]$

$= \max[\min(\boldsymbol{\lambda}, \mu(E)), \min(1-\boldsymbol{\lambda}, h_{\boldsymbol{\lambda}}(a, \mu(E)))]$

$= \max[\min(\boldsymbol{\lambda}, \mu(E)), \min(1-\boldsymbol{\lambda}, \max(\min(a,\boldsymbol{\lambda}), \min(a,\mu(E)),$

$\min(\boldsymbol{\lambda},\mu(E)))]$

$= \max[\min(\boldsymbol{\lambda},\mu(E)), \min(a, \boldsymbol{\lambda}, 1-\boldsymbol{\lambda}), \min(a, 1-\boldsymbol{\lambda}, \mu(E))]$

We have

$(\boldsymbol{\lambda})\int_E a\, d\mu = h_{1-\boldsymbol{\lambda}}$ $\{\max[\min(\boldsymbol{\lambda}, \mu(E)), \min(a,\boldsymbol{\lambda},1-\boldsymbol{\lambda}), \min(a, 1-\boldsymbol{\lambda},\mu(E))],$

$\max[\min(a,\boldsymbol{\lambda}), \min(\boldsymbol{\lambda}, 1-\boldsymbol{\lambda}))]$

$=\max[\min(a,\boldsymbol{\lambda},\mu(E)), \min(\boldsymbol{\lambda}, 1-\boldsymbol{\lambda}), \min(a, 1-\boldsymbol{\lambda},\mu(E))]$

$=h_{\boldsymbol{\lambda}}(1-, \min(a, \mu(E))]$

Proposition:3.5

If f is measurable function defined by $f(x) = \begin{cases} a & \text{if } x \in E \\ 0 & \text{if } x \notin E \end{cases}$

for any $x \in X$ the$(\lambda)\int f\, d\mu = (\boldsymbol{\lambda})\int_E a\, d\mu$

proof:

$(\boldsymbol{\lambda})\int f\, d\mu = h_{1-\boldsymbol{\lambda}}\{h_{\boldsymbol{\lambda}}(\boldsymbol{\alpha}, \mu(f_{\boldsymbol{\alpha}}))/\boldsymbol{\alpha} \in [0,1]\}$

$= h_{1-\boldsymbol{\lambda}}\{h_{1-\boldsymbol{\lambda}}\{h_{\boldsymbol{\lambda}}(\boldsymbol{\alpha}, \mu(E))/\boldsymbol{\alpha} \in [0,a)\},$

$h_{\boldsymbol{\lambda}}(0, \mu(X)), \{h_{1-\boldsymbol{\lambda}}\{h_{\boldsymbol{\lambda}}(\boldsymbol{\alpha}, 0)/\boldsymbol{\alpha} \in (a,1]\}$

it follows the proof of proposition that

$(\boldsymbol{\lambda})\int f\, d\mu = h_{1-\boldsymbol{\lambda}}\{h_{\boldsymbol{\lambda}}[1-\boldsymbol{\lambda}, \min(a, \mu(E))]$

$\mathrm{Min}[\boldsymbol{\lambda},\mu(x)]\}$

$=\max[\min(a,\boldsymbol{\lambda},\mu(E)), \min(a,, 1-\boldsymbol{\lambda},\mu(E)),$

$\min[\boldsymbol{\lambda},1-\boldsymbol{\lambda}]$

$= h_{\boldsymbol{\lambda}}[1-, \min(a,\mu(E))]$

$=(\lambda)\int_E a\, d\mu$

Proposition:3.6

Let μ be null additive (8). If $f_1 = f_2$ a.e then

$(\boldsymbol{\lambda})\int f_1\, d\mu = (\boldsymbol{\lambda})\int f_2\, d\mu$

Proof:

It is easy to see that $\mu\{x / f_1(x) \neq f_2(x)\} = 0$ since $f_1 = f_2$ a.e however since μ is null additive

$\mu \{x/ f_2(x) \geq \boldsymbol{\alpha}\} \leq \mu \{ x/ f_1(x) \geq \boldsymbol{\alpha}\} \cup \{ x/ f_1(x) \neq f_2(x)\}$

$= \mu (\{ x / f_1(x) \geq \boldsymbol{\alpha} \})$

Similarly we have

$\mu (\{x / f_1(x) \geq \boldsymbol{\alpha} \}) \leq \mu (\{ x / f_2(x) \geq \boldsymbol{\alpha}\})$

and consequently

$\mu ((f_2)_{\alpha}) = \mu (\{ x / f_2(x) \geq \boldsymbol{\alpha}\}) = \mu (\{ x / f_2(x) \geq \boldsymbol{\alpha} \})$

$= \mu ((f_2)_{\alpha})$

Therefore

$(\boldsymbol{\lambda}) \int f_1 d\mu = (\boldsymbol{\lambda}) \int f_2 d\mu$

Proposition:.3.7

Let μ be null additive ,A ,B ∈ 𝕱 and μ (B)=0 then

$(\boldsymbol{\lambda})\int_{A\cup B} f\, d\mu = (\boldsymbol{\lambda})\int_A f\, d\mu$

Proof:

Since μ is null additive

$\mu [(A\cup B)\cap f_{\alpha}] = \mu [(A\cap f_{\alpha}) \cup (B\cap f_{\alpha})] = \mu (A \cap f_{\alpha})$

$(\boldsymbol{\lambda})\int_{A\cup B} f\, d\mu = (\boldsymbol{\lambda})\int_A f\, d\mu$.

Applications of median integrals:

Example:

We intend to evaluate five mobile sets for the sake f simplicity,we consider

Two equality factors picture and sound

These are denoted by x and y respectively

Mobile set no.	X (picture)	Y (SOUND)
1	1	0
2	0	1
3	0.45	0.45
4	0.2	0
5	0	0.2
6	1	1

$\mu[\{x_1\}] = 0.3$ $\mu[\{x_2\}] = 0.1$

$\mu(x) = 1$,$\mu(\boldsymbol{\phi}) = 0$

and using median integral $\boldsymbol{\lambda} \in [0,1]$

$E_1 = (\lambda) \int f_1 \, d\mu = h_{1-} (\lambda, 0.3)$

$E_2 = (\lambda) \int f_2 \, d\mu = h_{1-} (\lambda, 0.1)$

$E_3 = (\lambda) \int f_3 \, d\mu = h_{1-} (\lambda, 0.45)$

$E_4 = (\lambda) \int f_4 \, d\mu = h_{1-} (\lambda, 0.2)$

$E_5 = (\lambda) \int f_5 \, d\mu = h_{1-} (\lambda, 0.1)$

$E_6 = (\lambda) \int f_6 \, d\mu = h_{1-\lambda} (\lambda, 1-\lambda)$

λ =0 then the median integral becomes the original sugeno integral. In this case E_1 =0.3 ,E_2 =0.1, E_3 = 0.45 ,E_4 = 0.2 ,E_5 =0.1 ,E_6 =1.

If λ =1 we get the same results

If λ =0.3 we obtain E_1 =0.3 ,E_2 =0.1, E_3 = 0.45 ,E_4 = 0.3 ,E_5 =0.3 ,E_6 =0.7.

In this case no 1,2,4,5 have the same value of evalution.

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