Ranking Generalized Intuitionistic Fuzzy Numbers

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Abstract

In this paper, Generalized Intuitionistic Pentagonal, hexagonal and Octagonal fuzzy numbers have been defined and a new ranking formula which includes the area of both membership and non membership parts of the fuzzy number have been proposed. The membership and the non membership area of the fuzzy numbers are splitted into plane figures and centroid of the centroids of these plane figures are calculated. The ranking formula is calculated by finding the area of this centroid from the origin. The advantage of this paper is that the ranking GIFN by this approach yields better solution when compared with other ranking methods. This approach is illustrated with numerical examples.

Key words: *Generalized fuzzy number, Intuitionistic fuzzy number, centroid.*

Introduction:

Atanassov[1,2] introduced the Intuitionistic fuzzy sets which is a generalization of the concept of fuzzy sets. Ranking of fuzzy numbers plays a vital role in fuzzy arithmetic and fuzzy decision making. An efficient method for ordering the fuzzy numbers is the ranking function which maps each fuzzy number into the real line, where a natural order exists.

Nagoor Gani and Mohamed[3] proposed a method for Ranking the Generalized Trapezoidal Intuitionistic Fuzzy Numbers. Annie Christi and Kasthuri[4] obtained a solution for Transportation Problem with Pentagonal Intuitionistic Fuzzy Numbers using Ranking Technique and Russell's Method. Helen and Uma[5] introduced a new arithmetic operation and ranking on Pentagonal Fuzzy Numbers.

Ponnivalavan and Pathinathan[6] introduced Intuitionistic Pentagonl fuzzy numbers with basic arithmetic operations and used the Accuracy function as a Ranking parameter. Siji and Selva Kumari[7] also developed an approach for solving Network problem with Pentagonal Institutionistic Fuzzy numbers using Accuracy function as Ranking technique. G.Menaka[8] developed new ranking formula for Octagonal Intuitionistic Fuzzy numbers. K.Selvakumari and S.Lavanya[11] have defined Generalized Intuitionistic Octagonal fuzzy numbers.

In this paper, Generalized Intuitionistic fuzzy numbers have been introduced with basic arithmetic operations and a new Ranking technique using the centroid concept is developed in which the result is more efficient when compared to the other ranking techniques.

Intuitionistic fuzzy sets:

Definition: Let X be the universal set. An Intuitionistic fuzzy set(IFS) A in X is given by A = $\{(x,(\mu_A(x),\gamma_A(x)):x\in X\}$ where the functions $\mu_A(x),\gamma_A(x)$ respectively, the degree of membership and degree of non-membership of the element x∈ X to the set A, which is a subset of X, and for every x∈ X, 0≤ $\mu_A(x)+\gamma_A(x)$ ≤1.

For each Intuitionistic fuzzy set A = $\{(x,(\mu_A(x),\gamma_A(x)):x\in X\}$ in X, $\pi_A(x)=1-\mu_A(x)-\gamma_A(x)$ is called the hesitancy degree of x to lie in A. If A is a fuzzy set, then $\pi_A(x)=0$ for all x∈ X.

Intuitionistic fuzzy Number:

Definition: An IFS A= $\{(x, (\mu_A(x), \gamma_A(x)) : x \in X\}$ of the real line R is called an intutionistic fuzzy number if

a) A is convex for the membership function $\mu_A(x)$.
b) A is concave for the non-member ship function $\gamma_A(x)$.
c) A is normal, that is there is some $x_0 \in R$ such that $\mu_A(x_0) = 1, \gamma_A(x_0) = 0$
.

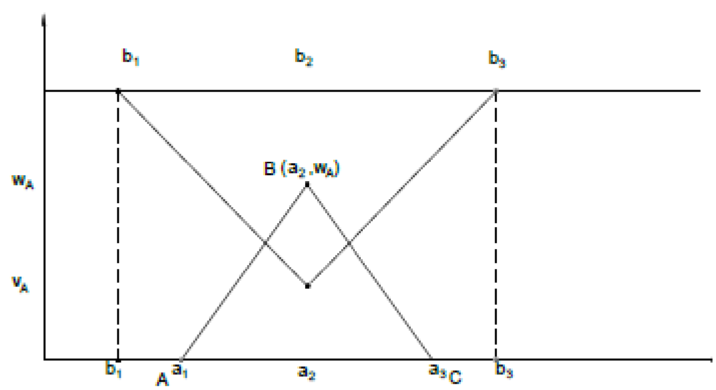
Generalized Intuitionistic Triangular Fuzzy Number:

An Intuitionistic triangular fuzzy number A is to be a generalized Intuitionistic triangular fuzzy number(GITFN) in the parameter $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3$ denoted by A=$\{(a_1, a_2, a_3), (b_1, b_2, b_3); W_A, V_A\}$, $0 \le W_A, V_A \le 1$ if its membership function and non membership function are as follows.

$$\mu(x) = \begin{cases} 0 & x < a_1 \\ \frac{W_A(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ w_A - \frac{(w_A)(x-a_3)}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

$$\gamma_A(X) = \begin{cases} 1 & x < b_1 \\ 1 + \frac{(v_A - 1)(x - b_1)}{b_2 - b_1}, & b_1 \le x \le b_2 \\ v_A + \frac{(1 - v_A)(x - b_2)}{b_3 - b_2}, & b_2 \le x \le b_3 \\ 1, & x > b_3 \end{cases}$$

Fig. 1 Graphical Representation of Generalized Intuitionistic Triangular Fuzzy Number



Consider the membership part of GITFN A=$\{(a_1, a_2, a_3), (b_1, b_2, b_3); W_A, V_A\}$, The centroid of a traingle is considered to be the balancing point of the triangle.

The Centroid of the triangle ABC is=$\left(\frac{a_1+a_2+a_3}{3}, \frac{W_A}{3}\right)$

Now we define S(μ_A) = $x_0.y_0$ = $\left(\frac{a_1+a_2+a_3}{3} * \frac{W_A}{3}\right)$

This is the area between the centroid of the centroid and the original point.

Similarly the triangle corresponding to the non membership function is considered. The centroid of this triangle is G_1=$\left(\frac{b_1+b_2+b_3}{3}, \frac{2+V_A}{3}\right)$;

Now we define S(γ_A) = $x_0.y_0$ = $\left(\frac{b_1+b_2+b_3}{3} * \frac{2+V_A}{3}\right)$

Using the above definitions, the rank of A is defined as follows:

$$R(A)= \frac{W_A S(\mu_A)+V_A S(\gamma_A)}{W_A+V_A}$$

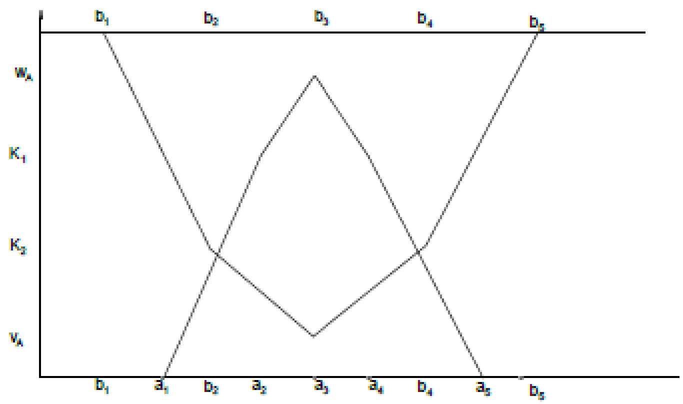
Proposed Definition and ranking method(Generalized Intuitionistic Pentagonal Fuzzy Number)

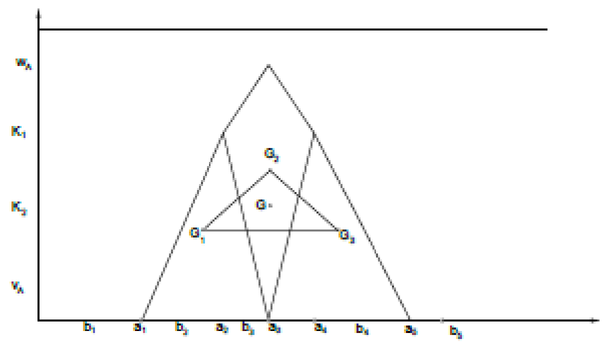
We define an Institutionistic pentagonal fuzzy number A to be a generalized Institutionistic pentagonal fuzzy number(GIPFN) in the parameter $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq a_4 \leq b_4 \leq a_5 \leq b_5$ denoted by A={$(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); W_A, V_A$}, 0≤ W_A, V_A ≤1, 0< $k_1 < w_A$, $V_A < K_2$ <1 if its membership function and non membership function are as follows.

$$\mu_A(x)=\begin{cases} 0 & x < a_1 \\ K_1 - \frac{K_1(x-a_2)}{a_1-a_2}, & a_1 \leq x \leq a_2 \\ W_A + \frac{(K_2-W_A)(x-a_3)}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ W_A + \frac{(K_1-W_A)(x-a_3)}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ K_1 - \frac{K_1(x-a_4)}{a_5-a_4}, & a_4 \leq x \leq a_5 \\ 0, & x > a_5 \end{cases}$$

$$\gamma_A(X) = \begin{cases} 1 & x < b_1 \\ 1 + \frac{(K_2-1)(x-b_1)}{b_2-b_1}, & b_1 \leq x \leq b_2 \\ K_2 + \frac{(v_A - K_2(x-b_2)}{b_3-b_2}, & b_2 \leq x \leq b_3 \\ V_A + \frac{(K_2-v_A)(x-a_3)}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ K_2 + \frac{(1-K_2)(x-b_4)}{b_5-b_4}, & b_4 \leq x \leq b_5 \\ 1, & x > b_5 \end{cases}$$

Fig. 2 Graphical Representation of Generalised Intuitionistic Pentagonal fuzzy number





Consider the membership part of GIPFN A={$(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); W_A, V_A$}. The centroid of a pentagon is considered to be the balancing point of the pentagon. Divide the membership part of pentagon into three plane figures. They are a triangle ABD, a quadrilateral BDEF(kite) and triangle BCF respectively. Let G_1, G_2, G_3 be the centroids of these three plane figures.

The Centroid of these centroids G_1, G_2, G_3 is considered as the point of reference to define the ranking of generalized pentagonal Institutionistic fuzzy numbers. As the centroid of these three plane figures are their balancing points , the centroid of these centroid points is a much better balancing point for a GIPFN.

The Centroids of these plane figures are

$G_1=\left(\frac{a_1+a_2+a_3}{3}, \frac{K_1}{3}\right)$; $G_2=\left(\frac{a_2+a_3+a_4}{3}, \frac{W_A+K_1}{3}\right)$ and $G_3=\left(\frac{a_3+a_4+a_5}{3}, \frac{K_1}{3}\right)$ respectively.

Thus G_1, G_2 and G_3 are not collinear and they form a triangle. Thus the centroid of these centroids is

$G(x_0,y_0) = \left(\frac{(a_1+2a_2+3a_3+2a_4+a_5)}{9}, \frac{3K_1+W_A}{9}\right)$

Now we define $S(\mu_A) = x_0.y_0 = \left(\frac{(a_1+2a_2+3a_3+2a_4+a_5)}{9}\right)\times \frac{3K_1+W_A}{9}$.

This is the area between the centroid of the centroids and the original point.

Similarly the pentagon corresponding to the non membership function is divided into three plane figures. In similar fashion, the centroid of the three plane figures and the centroid of these centroids are evaluated. The centroid of these plane figures are

$G_1=\left(\frac{b_1+b_2+b_3}{3}, \frac{2+K_2}{3}\right)$; $G_2=\left(\frac{b_2+b_3+b_4}{3}, \frac{V_A+1+K_2}{3}\right)$; $G_3=\left(\frac{b_3+b_4+b_5}{3}, \frac{2+K_2}{3}\right)$.

The centroid of these centroids is

$G^{'}(x_0,y_0) = \left(\frac{(b_1+2b_2+3b_3+2b_4+b_5)}{9}, \frac{3K_2+5+V_A}{9}\right)$.

Now we define $S(\gamma_A) = x_0.y_0 = \left(\frac{(b_1+2b_2+3b_3+2b_4+b_5)}{9}\right) \times \frac{3K_2+5+V_A}{9}$

Using the above definitions, the rank of A is defined as follows:

$$R(A) = \frac{W_A S(\mu_A) + V_A S(\gamma_A)}{W_A + V_A}$$

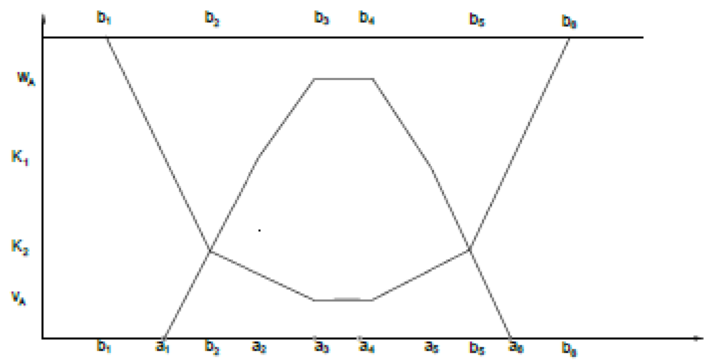
Proposed Definition and ranking method(Generalized Intuitionistic Hexagonal Fuzzy Number):

We define an Intuitionistic fuzzy number A to be a generalized Intuitionistic hexagonal fuzzy number(GIHFN) in the parameter $b_1 \le a_1 \le b_2 \le a_2 \le b_3 \le a_3 \le a_4 \le b_4 \le a_5 \le b_5 \le a_6 \le b_6$ denoted by $A=\{(a_1,a_2,a_3,a_4,a_5,a_6),(b_1,b_2,b_3,b_4,b_5,b_6); W_A, V_A\}$, $0 \le W_A, V_A \le 1$ $0 < k_1 < w_A$, $V_A < K_2 < 1$ if its membership function and non membership function are as follows.

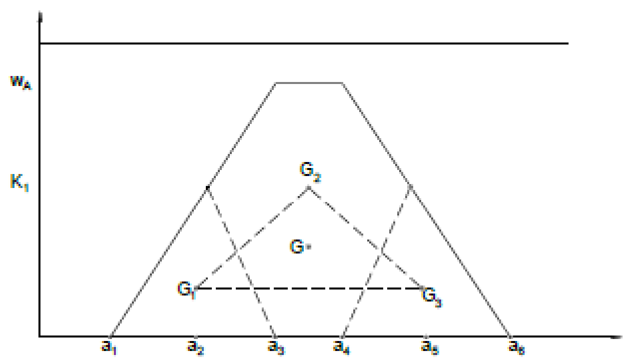
$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ \frac{K_1(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ K_1 + \frac{(W_A-K_1)(x-a_3)}{a_3-a_2}, & a_2 \le x \le a_3 \\ W_A & a_3 \le x \le a_4 \\ W_A + \frac{(K_1-W_A)(x-a_4)}{a_4-a_3}, & a_4 \le x \le a_5 \\ K_1 - \frac{K_1(x-a_5)}{a_6-a_5}, & a_5 \le x \le a_6 \\ 0, & x > a_6 \end{cases}$$

$$\gamma_A(X) = \begin{cases} 1 & x < b_1 \\ 1 + \frac{(K_2-1)(x-b_1)}{b_2-b_1}, & b_1 \le x \le b_2 \\ K_2 - \frac{(v_A-K_2)(x-b_2)}{b_3-b_2}, & b_2 \le x \le b_3 \\ V_A, & b_3 \le x \le b_4, \\ V_A + \frac{(K_2-v_A)(x-a_4)}{b_5-b_4}, & b_4 \le x \le b_5 \\ K_2 + \frac{(1-K_2)(x-b_5)}{b_6-b_5}, & b_5 \le x \le b_6 \\ 1, & x > b_6 \end{cases}$$

Fig. 3 Graphical Representation of Generalized Intuitionistic Hexagonal fuzzy number



Membership part of Generalized Intuitionistic Hexagonal Fuzzy number



Consider the membership part of Generalized Intuitionistic Hexagonall Fuzzy Number GIHFN A=$\{(a_1,a_2,a_3,a_4,a_5,a_6),(b_1,b_2,b_3,b_4,b_5,b_6);W_A,V_A\}$. The centroid of a Hexagon is considered to be the balancing point of the Hexagon. The centroid of Hexagon is the centroid of centroids G_1,G_2,G_3 which is considered as the point of reference to define the ranking of generalized hexagonal Intuitionistic fuzzy numbers. As the centroid of these three plane figures are their balancing points , the centroid of these centroid points is a much better balancing point for a GIHFN.

The Centroids of these plane figures are

$G_1=\left(\frac{a_1+a_2+a_3}{3},\frac{k_1}{3}\right)$, $G_3=\left(\frac{a_4+a_5+a_6}{3},\frac{k_1}{3}\right)$ and $G_2=\left(\frac{2a_1+7a_3+7a_4+2a_5}{18},\frac{7W_A+4k_1}{18}\right)$ respectively.

From the above figure, the centroid of hexagon is calculated as

G(x_0,y_0) = $\left(\frac{(6a_1+8a_2+13a_3+13a_4+8a_5+6a_6)}{54},\frac{7W_A+16k_1}{54}\right)$

Now we define S(μ_A) = $x_0.y_0$ = $\left(\frac{(6a_1+8a_2+13a_3+13a_4+8a_5+6a_6)}{54}\right)\times\frac{7W_A+16k_1}{54}$

This is the area between the centroid of the hexagon and the original point
Similarly for non membership function,
Now we define S(γ_A) = $x_0.y_0$ = $\left(\frac{(6b_1+8b_2+13b_3+13b_4+8b_5+6b_6)}{54}\right)\times\frac{7v_A+31+16k_2}{54}$.

Using the above definitions, the rank of A is defined as follows:

$$R(A)=\frac{W_A S(\mu_A)+V_A S(\gamma_A)}{W_{A+}V_A}$$

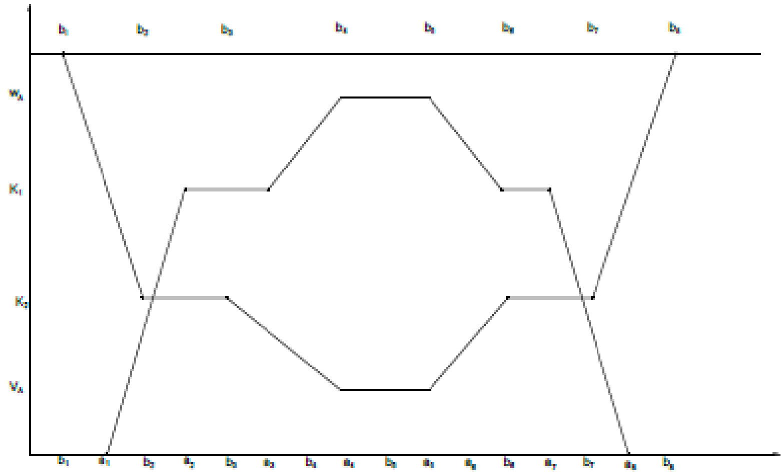
Generalized Intuitionistic Octagonal Fuzzy Number:

An Intuitionistic fuzzy number A is said to be a generalized Intuitionistic Octagonal fuzzy number(GIHFN) in the parameter $b_1 \le a_1 \le b_2 \le a_2 \le b_3 \le a_3 \le b_4 \le a_4 \le a_5 \le b_5 \le a_6 \le b_6 \le a_7 \le b_7 \le a_8 \le b_8$ denoted by A=$\{(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8),(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8);W_A,V_A\}$, $0\le W_A,V_A \le 1$, $0< k_1 < w_A$, $V_A < K_2 <1$ if its membership function and non membership function are as follows.

$$\mu_A(x)=\begin{cases} 0 & x < a_1 \\ \frac{k_1(x-a_1)}{a_2-a_1}, & a_1 \le x \le a_2 \\ k_1, & a_2 \le x \le a_3 \\ k_1+\frac{(w_A-K_1)(x-a_3)}{a_4-a_3}, & a_3 \le x \le a_4 \\ W_A & a_4 \le x \le a_5 \\ W_A+\frac{(K_1-W_A)(x-a_5)}{a_6-a_5}, & a_5 \le x \le a_6 \\ K_1, & a_6 \le x \le a_7 \\ K_1-\frac{K_1(x-a_7)}{a_8-a_7}, & a_7 \le x \le a_8 \\ 0, & x > a_8 \end{cases}$$

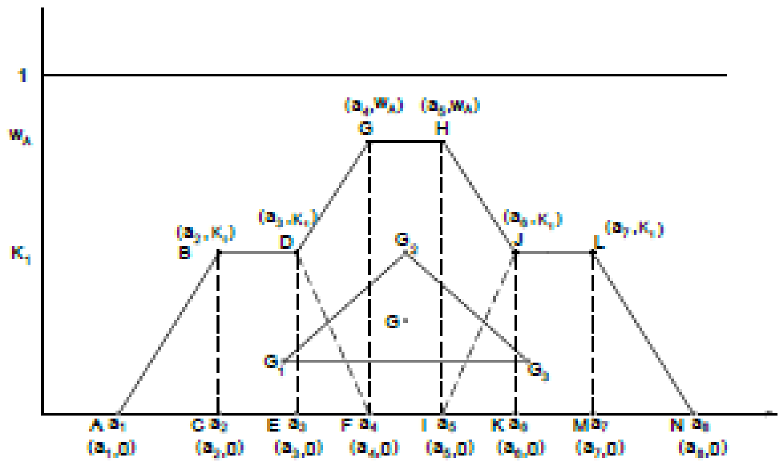
$$\gamma_A(X)=\begin{cases} 1 & x < b_1 \\ 1+\frac{(k_2-1)(x-b_1)}{b_2-b_1}, & b_1 \le x \le b_2 \\ k_2, & b_2 \le x \le b_3 \\ k_2-\frac{(v_A-k_2)(x-b_3)}{b_4-b_3}, & b_3 \le x \le b_4 \\ v_A & b_4 \le x \le b_5, \\ V_A+\frac{(K_2-v_A)(x-a_5)}{b_6-b_5} & ,b_5 \le x \le b_6 \\ K_2, & b_6 \le x \le b_7 \\ K_2+\frac{(1-K_2)(x-b_7)}{b_8-b_7}, & b_7 \le x \le b_8 \\ 1, & x > b_8 \end{cases}$$

Fig. 4 Graphical Representation of Generalized Intuitionistic Octagonal fuzzy number



Proposed Ranking method of GIOFN

Membership part of Generalized Intuitionistic Octagonal Fuzzy number



In similar fashion, The centroid of octagon is calculated as

$$G(x_0,y_0) = \left(\frac{(2a_1+7a_2+9a_3+9a_4+9a_5+9a_6+7a_7+2a_8)}{54}, \frac{18W_A+7}{54}\right)$$

Now we define $S(\mu_A) = x_0.y_0 = \left(\frac{(2a_1+7a_2+9a_3+9a_4+9a_5+9a_6+7a_7+2a_8)}{54}\right)\times \frac{7W_A+18k_1}{54}$.

This is the area between the centroid of the centroids and the original point.

Similarly,

Now we define $S(\gamma_A) = x_0.y_0 = \left(\frac{(2b_1+7b_2+9b_3+9b_4+9b_5+9b_6+7b_7+2b_8)}{54}\right)\times \frac{5V_A+29+18k_2}{54}$.

Using the above definitions, the rank of A is defined as follows:

R(A)= $\frac{W_A \mathrm{S}(\mu_A)+\mathrm{V_A}\,\mathrm{S}(\gamma_A)}{W_{A+}V_A}$

Arithmetic Operations:

If A=$\{(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8), (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8); W_A, V_A\}$, and
B=$\{(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8), (d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8); W_B, V_B\}$, are two GIOFN then

A+B = $\{(a_1 + c_1, a_2 + c_2, a_3 + c_2, a_4 + c_4, a_5 + c_5, a_6 + c_6, a_7 + c_7, a_8 + c_8), b_1 + d_1, b_2 + d_2, b_3 + d_3, b_4 + d_4, b_5 + d_5, b_6 + d_{6,}\ b_7 + d_7, b_8 + d_8)$; w, u} where w = min$\{w_A, w_B\}$,

u = max$\{V_A, V_B\}$,

A-B = $\{(a_1$ - $c_8, a_2 - c_7, a_3 - c_6, a_4 - c_5, a_5 - c_4, a_6 - c_3, a_7 - c_2, a_8 - c_{1),}(b_1 - d_8, b_2 - d_7, b_3 - d_6, b_4 - d_5, b_5 - d_4, b_6 - d_{3,}b_7 - d_{2,}b_8 - d_1); w, u\}$ where w = min$\{W_A, W_B\}$, u= max$\{V_A, V_B\}$
A*B = $\{(a_1c_{1,}a_2c_2, a_3c_3, a_4c_4, a_5c_5, a_6c_6, a_7c_7, a_8c_8)(b_1d_1, b_2d_2, b_3d_3, b_4d_4, b_5d_5, b_6d_6, b_7d_7, b_8d_8)$;
w ,u} where w = min$\{W_A, W_B\}$, u= max$\{V_A, V_B\}$if A, B > 0
A/B = $\{(\frac{a_1}{c_8}, \frac{a_2}{c_7}, \frac{a_3}{c_6}, \frac{a_4}{c_5}, \frac{a_5}{c_4}, \frac{a_6}{c_3}, \frac{a_7}{c_2}, \frac{a_8}{c_1})(\frac{b_1}{d_8}, \frac{b_2}{d_7}, \frac{b_3}{d_6}, \frac{b_4}{d_5}, \frac{b_5}{d_4}, \frac{b_6}{d_3}, \frac{b_7}{d_2}, \frac{b_8}{d_1})$;w,u} where w = min$\{W_A, W_B\}$, u= max$\{V_A, V_B\}$ if B>0
In Similar fashion the arithmetic operations can be defined for other fuzzy numbers.

Numerical example:

Let A ={(2,4,6,7,8,10),(1,3,6,7,9,11); 0.5,0.3} and

B={(1,3,5,6,7,9),(0,2,5,6,8,10);0.7,0.1}

Then S(μ_A) = 1.733 and S(γ_A)=4.136 and R(A) = 2.634

S(μ_B) = 1.766 ; S(γ_B)=3.163 then R(B) =1.940

Here R(A) > R(B) therefore A>B.

Conclusion:

This paper proposes generalized intuitionistic Pentagonal, hexagonal and Octagonal fuzzy numbers along with a new ranking technique which is simple and more efficient. This centroid based ranking method gives more efficient result when compared to ranking of intuitionistic fuzzy numbers by Accuracy function in [4] and ranking of fuzzy numbers in [5].

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