

Common Fixed Point Theorems of Pair of Self Maps Satisfying Contractive Condition in G-Metric Space

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Abstract

In this paper we prove fixed point results of pair of self maps satisfying contractive condition involving maximum and minimum function in G-Metric space .

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1 Introduction

The metric fixed point theory is very important and useful in various areas such as variational inequalities, optimization and approximation theory .Many authors were introduced generalized structures of metric spaces.Gahler [1, 2] introduced generalized metric space called 2-metric space also B.C. Dhage [3, 4] obtained new structure known as D-Metric space.But in 2003,Z.Mustafa and B.Sims [5] found that there are some limitations in fundamental topological properties of D-metric space.And ,they [6] introduced a generalized Metric Space namely G-Metric space.For more detail information one can refer [7, 8, 9, 10].Ray (1976) [11] proved common fixed point theorem for the pair of self maps on complete Metric space X into itself.In 1977 B.Fisher [12] proved some results of common

fixed points for pair of continuous maps which satisfies a contractive condition in terms of Maximum function. Then Z. Mustafa et. al [7] proved some common fixed point theorems of pair of mapping satisfy contractive condition in terms of maximum function.

2 Preliminaries

Definition 2.1. Let X be a non empty set and $G : X^3 \rightarrow R^+$ which satisfies the following conditions

1. $G(a, b, c) = 0$ if $a = b = c$ i.e. every a, b, c in X coincides.
2. $G(a, a, b) > 0$ for every $a, b, c \in X$ s.t. $a \neq b$
3. $G(a, a, b) \leq G(a, b, c)$, $\forall a, b, c \in X$ s.t. $c \neq b$
4. $G(a, b, c) = G(b, a, c) = G(c, b, a) = \dots\dots\dots$
(symmetrical in all three variables)
5. $G(a, b, c) \leq G(a, x, x) + G(x, b, c)$, for all a, b, c, x in X
(rectangle inequality)

Then the function G is said to be generalized metric or simply G -metric on X and the pair (X, G) is said to be G -metric space.

Example 2.2. Let $G : X^3 \rightarrow R^+$ s.t. $G(a, b, c) =$ perimeter of the triangle with vertices at a, b, c in R^2 , also by taking p in the interior of the triangle then rectangle inequality is satisfied and the function G is a G -metric on X .

Remark 2.3. G -metric space is the generalization of the ordinary metric space that is every G -metric space is (X, G) defines ordinary metric space (X, d_G) by $d_G(a, b) = G(a, b, b) + G(a, a, b)$

Example 2.4. Let (X, d) be the usual metric space. Then the function $G : X^3 \rightarrow R^+$ defined by

$$G(a, b, c) = \max\{d(a, b), d(b, c), d(c, a)\}$$

for all $a, b, c \in X$ is a G -metric space.

Definition 2.5. A G -metric space (X, G) is said to be symmetric if $G(a, b, b) = G(a, a, b)$ for all $a, b \in X$ and if $G(a, b, b) \neq G(a, a, b)$ then G is said to be non symmetric G -metric space.

Example 2.6. Let $X = \{x, y\}$ and $G : X^3 \rightarrow R^+$ defined by $G(x, x, x) = G(y, y, y) = 0$, $G(x, x, y) = 1$, $G(x, y, y) = 2$ and extend G to all of X^3 by symmetry in the variables. Then X is a G -metric space but It is non symmetric. since $G(x, x, y) \neq G(x, y, y)$

Definition 2.7. Let (X, G) be a G -metric space, Let $\{a_n\}$ be a sequence of elements in X . The sequence $\{a_n\}$ is said to be G -convergent to a if

$$\lim_{m, n \rightarrow \infty} G(a, a_n, a_m) = 0$$

i.e for every $\epsilon > 0$ there is N s.t. $G(a, a_n, a_m) < \epsilon$ for all $m, n \geq N$ It is denoted as $a_n \rightarrow a$ or $\lim_{n \rightarrow \infty} a_n = a$

Proposition 2.8. If (X, G) be a G -metric space. Then the following are equivalent

1. $\{a_n\}$ is G -convergent to a .
2. $G(a_n, a_n, a) \rightarrow 0$ as $n \rightarrow \infty$
3. $G(a_n, a, a) \rightarrow 0$ as $n \rightarrow \infty$
4. $G(a_m, a_n, a) \rightarrow 0$ as $m, n \rightarrow \infty$

Definition 2.9. Let (X, G) be a G -metric space a sequence $\{a_n\}$ is called G -Cauchy if , for each $\epsilon > 0$ there is an $N \in I^+$ (set of positive integers) s.t.

$$G(a_n, a_m, a_l) < \epsilon \text{ for all } n, m, l \geq N$$

Proposition 2.10. Let (X, G) be a G -metric space then the function $G(a, b, c)$ is jointly continuous in all three of its variables.

Proposition 2.11. Let (X, G) be a G -metric space. Then, for any a, b, c, x in X it gives that

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1. if $G(a, b, c) = 0$ then $a = b = c$
2. $G(a, b, c) \leq G(a, a, b) + G(a, a, c)$
3. $G(a, b, b) \leq 2G(b, a, a)$
4. $G(a, b, c) \leq G(a, x, c) + G(x, b, c)$
5. $G(a, b, c) \leq \frac{2}{3}(G(a, x, x) + G(b, x, x) + G(c, x, x))$

B.Fisher [12] proved following common fixed point theorem for pair of continuous mappings.

Theorem 2.12. *If P and Q are two continuous mappings of the complete metric space X into itself s.t. $\rho(P^2x, Q^2y) \leq c \max\{\rho(Px, Qy), \rho(x, y)\}$ for all $x, y \in X$, where $0 \leq c < 1$. Then P and Q have a unique fixed point z .*

Z.Mustafa [14] proved a fixed point theorem in G-metric space which is a generalization of Banach contraction principle in G-metric space.

Theorem 2.13. *If (X, G) be a complete G-Metric space and F be a self mapping of X which satisfies the following condition for all $a, b, c \in X$ $G(Fa, Fb, Fc) \leq \alpha G(a, b, c)$ for $0 \leq \alpha < 1$. Then F has a unique fixed point.*

Z.Mustafa et. and all [7] proved following theorem.

Theorem 2.14. *Let (X, G) be a complete G-metric space, and F be a self map which satisfy the condition*

$$G(F(a), F(b), F(c)) \leq \alpha \max \left\{ \begin{array}{l} G(a, F(a), F(a)) \\ G(b, F(b), F(b)) \\ G(c, F(c), F(c)) \end{array} \right\}$$

or

$$G(F(a), F(b), F(c)) \leq \alpha \max \left\{ \begin{array}{l} G(a, a, F(a)) \\ G(b, b, F(b)) \\ G(c, c, F(c)) \end{array} \right\}$$

for all a, b, c in X , for $\alpha \in [0, 1)$. Then F has a unique fixed point.

Z. Mustafa and Hameed Obiedat [13] proved following common fixed point theorem.

Theorem 2.15. Let (X, G) be a complete G -metric space, and Let $F : X \rightarrow X$ be a map which satisfies

$$G(F(a), F(b), F(c)) \leq \beta G(a, b, c) + \alpha \max \left\{ \begin{array}{l} G(a, F(a), F(a)), G(b, F(b), F(b)) \\ G(c, F(c), F(c)) \end{array} \right\}$$

for all $a, b, c \in X$, where $0 \leq \beta + \alpha < 1$. Then F has a unique fixed point (say v), and F is G -continuous at v .

3 Main Result

Now we prove result of common fixed point theorem for pair of self maps for following contraction.

Theorem 3.1. If (X, G) be a complete G -Metric space. If the pair of self maps P, Q satisfy

$$\begin{aligned} \max \left\{ \begin{array}{l} G(P(a), Q(b), Q(b)), \\ G(Q(a), P(b), P(b)) \end{array} \right\} &\leq \alpha_1 G(a, b, b) \\ &+ \alpha_2 \min \left\{ \begin{array}{l} G(a, Q(b), Q(b)) + G(b, P(a), P(a)) \\ G(a, P(b), P(b)) + G(b, Q(a), Q(a)) \end{array} \right\} \\ &+ \alpha_3 \min \left\{ \begin{array}{l} G(a, P(a), P(a)) + G(b, Q(b), Q(b)), \\ G(a, Q(a), Q(a)) + G(b, P(b), P(b)) \end{array} \right\} \end{aligned} \quad (3.1)$$

for all a, b in X , where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ s.t. $\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$. Then P and Q have a unique common fixed point in X .

Proof. Let a_0 be an arbitrary element of X . We define a sequence $\{a_n\}$ by

$$a_n = \begin{cases} P(a_{n-1}), & \text{if } n \text{ is odd} \\ Q(a_{n-1}), & \text{if } n \text{ is even} \end{cases}$$

If $n \in I^+$ be an odd positive integer then by using (3.1), we have

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &= G(P(a_{n-1}), Q(a_n), Q(a_n)) \\ &\leq \max \left\{ \begin{aligned} &G(P(a_{n-1}), Q(a_n), Q(a_n)) \\ &G(Q(a_{n-1}), P(a_n), P(a_n)) \end{aligned} \right\} \\ &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \min \left\{ \begin{aligned} &G(a_{n-1}, Q(a_n), Q(a_n)) + G(a_n, P(a_{n-1}), P(a_{n-1})) \\ &G(a_{n-1}, P(a_n), P(a_n)) + G(a_n, Q(a_{n-1}), Q(a_{n-1})) \end{aligned} \right\} \\ &+ \alpha_3 \min \left\{ \begin{aligned} &G(a_{n-1}, P(a_{n-1}), P(a_{n-1})) + G(a_n, Q(a_n), Q(a_n)) \\ &G(a_{n-1}, Q(a_{n-1}), Q(a_{n-1})) + G(a_n, P(a_n), P(a_n)) \end{aligned} \right\} \end{aligned}$$

Thus,

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \{G(a_{n-1}, Q(a_n), Q(a_n)) + G(a_n, P(a_{n-1}), Q(a_{n-1}))\} \\ &+ \alpha_3 \{G(a_{n-1}, P(a_{n-1}), P(a_{n-1})) + G(a_n, Q(a_n), Q(a_n))\} \\ &= \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \{G(a_{n-1}, a_{n+1}, a_{n+1}) + G(a_n, a_n, a_n)\} \\ &+ \alpha_3 \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &+ \alpha_3 \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \end{aligned}$$

This gives, $G(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - \alpha_2 - \alpha_3} G(a_{n-1}, a_n, a_n)$

If $n \in N$ is an even then by using (3.1), we get

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &= G(Q(a_{n-1}), P(a_n), P(a_n)) \\ &\leq \max \left\{ \begin{aligned} &G(P(a_{n-1}), Q(a_n), Q(a_n)) \\ &G(Q(a_{n-1}), P(a_n), P(a_n)) \end{aligned} \right\} \\ &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \min \left\{ \begin{aligned} &G(a_{n-1}, Q(a_n), Q(a_n)) + G(a_n, P(a_{n-1}), P(a_{n-1})) \\ &G(a_{n-1}, P(a_n), P(a_n)) + G(a_n, Q(a_{n-1}), Q(a_{n-1})) \end{aligned} \right\} \\ &+ \alpha_3 \min \left\{ \begin{aligned} &G(a_{n-1}, P(a_{n-1}), P(a_{n-1})) + G(a_n, Q(a_n), Q(a_n)) \\ &G(a_{n-1}, Q(a_{n-1}), P(a_{n-1})) + G(a_n, P(a_n), P(a_n)) \end{aligned} \right\} \\ &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \{G(a_{n-1}, P(a_n), P(a_n)) + G(a_n, Q(a_{n-1}), Q(a_{n-1}))\} \\ &+ \alpha_3 \{G(a_{n-1}, Q(a_{n-1}), Q(a_{n-1})) + G(a_n, P(a_n), P(a_n))\} \end{aligned}$$

$$\begin{aligned} G(a_n, a_{n+1}, a_{n+1}) &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \{G(a_{n-1}, a_{n+1}, a_{n+1}) + G(a_n, a_n, a_n)\} \\ &+ \alpha_3 \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &\leq \alpha_1 G(a_{n-1}, a_n, a_n) \\ &+ \alpha_2 \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \\ &+ \alpha_3 \{G(a_{n-1}, a_n, a_n) + G(a_n, a_{n+1}, a_{n+1})\} \end{aligned}$$

which gives that, $G(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - \alpha_2 - \alpha_3} G(a_{n-1}, a_n, a_n)$ for any positive integer n , We have

$$(3.2) \quad G(a_n, a_{n+1}, a_{n+1}) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - \alpha_2 - \alpha_3} G(a_{n-1}, a_n, a_n)$$

Let $\beta = \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - \alpha_2 - \alpha_3}$, $\therefore 0 \leq \beta < 1$ as $\alpha_1, \alpha_2, \alpha_3 \geq 0$ and $\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$ with this (3.2) becomes

$$(3.3) \quad G(a_n, a_{n+1}, a_{n+1}) \leq \beta G(a_{n-1}, a_n, a_n)$$

by using relation (3.3) repeatedly, we get

$$(3.4) \quad G(a_n, a_{n+1}, a_{n+1}) \leq \beta^n G(a_0, a_1, a_1)$$

By using rectangular inequality repeatedly and using (3.4), we get, for all $n, m \in N$ s.t. $m > n$

$$\begin{aligned} G(a_n, a_m, a_m) &\leq G(a_n, a_{n+1}, a_{n+1}) + G(a_{n+1}, a_{n+2}, a_{n+2}) \\ &+ G(a_{n+2}, a_{n+3}, a_{n+3}) + \dots + G(a_{m-1}, a_m, a_m) \\ &\leq (\beta^n + \beta^{n+1} + \dots + \beta^{m-1}) G(a_0, a_1, a_1) \\ &\leq \frac{\beta^n}{1-\beta} G(a_0, a_1, a_1) \end{aligned}$$

taking limit as $n, m \rightarrow \infty$, we get $\lim G(a_n, a_m, a_m) = 0$, since $\lim_{n \rightarrow \infty} \frac{\beta^n}{1-\beta} = 0$. \therefore limit of R.H.S. is 0. $\therefore \{a_n\}$ is a G-Cauchy sequence, since X is G-complete. \therefore there exists some $v \in X$ s.t. $\{a_n\}$ converges to $v \in X$. Again by using rectangle inequality and by equation (3.1), we get

$$\begin{aligned} G(v, Q(v), Q(v)) &\leq G(v, a_{2n+1}, a_{2n+1}) + G(a_{2n+1}, Q(v), Q(v)) \\ &= G(v, a_{2n+1}, a_{2n+1}) + G(P(a_{2n}), Q(v), Q(v)) \\ &\leq G(v, a_{2n+1}, a_{2n+1}) + \max \left\{ \begin{aligned} &G(P(a_{2n}), Q(v), Q(v)), \\ &G(Q(a_{2n}), P(v), P(v)) \end{aligned} \right\} \\ &\leq G(v, a_{2n+1}, a_{2n+1}) + \alpha_1 G(a_{2n}, v, v) \\ &+ \alpha_2 \min \left\{ \begin{aligned} &G(a_{2n}, Q(v), Q(v)) + G(v, P(a_{2n}), P(a_{2n})), \\ &G(a_{2n}, P(v), P(v)) + G(v, Q(a_{2n}), Q(a_{2n})) \end{aligned} \right\} \\ &+ \alpha_3 \min \left\{ \begin{aligned} &G(a_{2n}, P(a_{2n}), P(a_{2n})) + G(v, Q(v), Q(v)), \\ &G(a_{2n}, Q(a_{2n}), Q(a_{2n})) + G(v, P(v), P(v)) \end{aligned} \right\} \\ &\leq G(v, a_{2n+1}, a_{2n+1}) + \alpha_1 G(a_{2n}, v, v) \\ &+ \alpha_2 \{G(a_{2n}, Q(v), Q(v)) + G(v, P(a_{2n}), P(a_{2n}))\} \\ &+ \alpha_3 \{G(a_{2n}, P(a_{2n}), P(a_{2n})) + G(v, Q(v), Q(v))\} \end{aligned}$$

Thus, we have

$$\begin{aligned} G(v, Q(v), Q(v)) &\leq G(v, a_{2n+1}, a_{2n+1}) + \alpha_1 G(a_{2n}, v, v) \\ &+ \alpha_2 \{G(a_{2n}, Q(v), Q(v)) + G(v, a_{2n+1}, a_{2n+1})\} \\ &+ \alpha_3 \{G(a_{2n}, a_{2n+1}, a_{2n+1}) + G(v, Q(v), Q(v))\} \end{aligned}$$

taking limit as $n \rightarrow \infty$, and given that the function G is continuous on its variables, \therefore we have $G(v, Q(v), Q(v)) \leq (\alpha_1 + \alpha_2)G(v, Q(v), Q(v))$ But, $0 \leq (\alpha_1 + \alpha_2) < 1$, $\therefore G(v, Q(v), Q(v)) \leq G(v, Q(v), Q(v))$, which is impossible. $\therefore G(v, Q(v), Q(v)) = 0$ \therefore This gives $Q(v) = v$. In the same way, we can prove that $P(v) = v$. $\therefore u$ is a common fixed point of P and Q . To prove uniqueness, if possible suppose that there exists some $u \in X$ s.t. $P(u) = Q(u) = u$. Then

$$\begin{aligned} G(v, u, u) &= G(P(v), Q(u), Q(u)) \\ &\leq \max \left\{ \begin{array}{l} G(P(v), Q(u), Q(u)), \\ G(Q(v), P(u), P(u)) \end{array} \right\} \\ &\leq \alpha_1 G(v, u, u) \\ &+ \alpha_2 \min \left\{ \begin{array}{l} G(v, Q(u), Q(u)) + G(u, P(v), P(v)), \\ G(v, P(u), P(u)) + G(u, Q(v), Q(v)) \end{array} \right\} \\ &\leq \alpha_3 \min \left\{ \begin{array}{l} G(v, P(v), P(v)) + G(u, Q(u), Q(u)), \\ G(v, Q(v), Q(v)) + G(u, P(u), P(u)) \end{array} \right\} \\ &= \alpha_1 G(v, u, u) \\ &+ \alpha_2 \{G(v, u, u) + G(u, v, v)\} \\ &+ \alpha_3 \{G(v, v, v) + G(u, u, u)\} \end{aligned}$$

Which gives,

$$G(v, u, u) \leq \frac{\alpha_2}{1 - \alpha_1 - \alpha_2} G(u, v, v)$$

Similarly, we get

$$G(u, v, v) \leq \frac{\alpha_2}{1 - \alpha_1 - \alpha_2} G(v, u, u)$$

\therefore from above two inequalities, we have

$$G(v, u, u) \leq \left(\frac{\alpha_2}{1 - \alpha_1 - \alpha_2} \right)^2 G(v, u, u)$$

Since $0 \leq \frac{\alpha_2}{1 - \alpha_1 - \alpha_2} < 1 \therefore$ this gives $u = v$. \square

Theorem 3.2. *If (X, G) be a complete G-Metric space. If the pair of self maps P, Q satisfy*

$$(3.5) \quad \begin{aligned} \max \left\{ \begin{array}{l} G(P(a), Q(b), Q(b)), \\ G(Q(a), P(b), P(b)) \end{array} \right\} &\leq \alpha_1 G(a, b, b) \\ &+ \alpha_2 \min \left\{ \begin{array}{l} G(a, a, Q(b)) + G(b, b, P(a)), \\ G(a, a, P(b)) + G(b, b, Q(a)) \end{array} \right\} \\ &+ \alpha_3 \min \left\{ \begin{array}{l} G(a, a, P(a)) + G(b, b, Q(b)), \\ G(a, a, Q(a)) + G(b, b, P(b)) \end{array} \right\} \end{aligned}$$

for all a, b in X , where $\alpha_1, \alpha_2, \alpha_3 \geq 0$ s.t. $\alpha_1 + 2\alpha_2 + 2\alpha_3 < 1$. Then P and Q have a unique common fixed point in X .

Proof. Let a_0 be an arbitrary element of X . We define a sequence $\{a_n\}$ by

$$a_n = \begin{cases} P(a_{n-1}), & \text{if } n \text{ is odd} \\ Q(a_{n-1}), & \text{if } n \text{ is even} \end{cases}$$

Then by similar procedure used in last theorem (3.1), we have for any $n \in I^+$

$$(3.6) \quad G(a_n, a_{n+1}, a_{n+1}) \leq \beta^n G(a_0, a_1, a_1)$$

then by using rectangle inequality and equation (3.6), we get, for all $n, m \in I^+, m > n$

$$\begin{aligned} G(a_n, a_m, a_m) &\leq G(a_m, a_{m-1}, a_{m-1}) + G(a_{m-1}, a_{m-2}, a_{m-2}) \\ &+ G(a_{m-2}, a_{m-3}, a_{m-3}) + \dots + G(a_{n+1}, a_n, a_n) \\ &\leq (\beta^n + \beta^{n+1} + \dots + \beta^{m-1}) G(a_0, a_1, a_1) \\ &\leq \frac{\beta^n}{1 - \beta} G(a_0, a_1, a_1) \end{aligned}$$

$\therefore \{a_n\}$ be a Cauchy sequence. And by completeness of G-metric space, there exists $v \in X$ s.t. $\{a_n\}$ is G-convergent to v . As in last proof we prove that,

$$G(v, v, Q(v)) \leq (\alpha_1 + \alpha_2)G(v, v, Q(v))$$

Thus required conclusion follows from same argument used in last theorem. \square

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