On #α-Regular Generalized Closed In Topological Spaces

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ABSTRACT: *In this paper we introduce a new class of sets called #α- regular generalized closed (briefly , #αrg-closed) sets in topological space. We prove that this class lies between #rg-closed sets and αg-closed sets. We discuss some basic properties of # regular generalized closed sets.*

Keywords: *rw-open sets,#rg-closed sets, #αrg-closed sets*

1.INTRODUCTION

Regular open sets and strong regular open sets have been introduced and investigated by Stone [18] and Tong [20] respectively. Levine [8,9]. Biswas[3] , Cameron[4], Sundaram and Sheik John [17], Bhattacharyya and Lahiri [2], Nagaveni [12], Pushpalatha [16], Gnanambal [6], gnanambal and balachandran [7], Palaniappan and Rao [13], Maki Devi and Balachandra [10], and Benchalli and Wali [1] ,Syed Ali Fathima[19] introduced and investigated semi open sets, generalized closed sets, regular semi open sets, weakly closed sets, semi generalized closed sets, weakly generalized closed sets, strongly generalized closed sets, generalized pre-regular closed sets, regular generalized closed sets, α-generalized closed sets ,Rw-closed sets and #regular generalized closed sets respectively.

We introduce a new class of sets called #α-regular generalized closed sets which is properly placed in between the class of #regular generalized closed sets and the class of αg-closed sets.

2.PRELIMINARIES

Throughout this paper (X,τ) represents a non-empty topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A denotes the complement of A in X. We recall the following definition and results.

Definition: 2.1. A Subset A of a space X is called

(1) a preopen set [11] if A ⊆ int(cl (A)) and a preclosed set if cl(int (A))⊆A

(2) a semiopen set [8] if A⊆ cl(int (A)) and a semiclosed set if int(cl (A) ⊆A

(3) an α-open set [21] if A ⊆ int(cl(int(A))) and an α-closed set [ ]if cl(int(cl(A))) ⊆A

(4) a regular open set [13] if A=int(cl (A))and a regular closed set if A=cl(int (A))

(5) a π-open set [1] if A is a finite union of regular open sets.

(6) regular semi open [15] if there is a regular open U such U ⊆A ⊆cl(U)

Definition 2.2. A subset A of a topological space (X,τ) is called.

(1) an *α-generalized* closed set (briefly *αg-closed*)[14] if αcl(A) ⊆U whenever A⊆U and U is open in (X,τ) .
(2) a generalized pre closed set (briefly gp-closed) [14] if pcl(A) ⊆U whenever A ⊆U and U is open (X,τ)
(3) a generalized semi pre closed set (briefly gsp-closed) [14] if spcl(A)⊆U whenever A⊆U and U is a open (X,τ)

(4) a generalized α--closed (i.e., g α--closed) set [21] if αcl(A) ⊆U whenever A⊆U and U is open set in X.

3. BASIC PROPERTIES OF # α--REGULAR GENERALIZED CLOSED SETS

We introduce the following definition

Definition 3.1

A subset A of a topological space X is called #α-regular generalized closed (briefly #αrg-closed)set if αcl(A) ⊆U whenever A⊆U and U is rw-open .We denote the set of all #αrg-closed sets in X is denoted by #αrgc(X).

First we prove that the class of # α--regular generalized closed sets properly lies between the class of #rg-closed sets and the class of αg-closed sets.

Theorem 3.2

Every #rg-closed sets are #αrg-closed sets but not conversely.

Proof.

Let A be #rg-closed set. To prove A is #αrg-closed. Let A⊆U and U is rw-open. To prove αcl(A) ⊆U. We know that αcl(A) ⊆cl(A). Since A is #rg-closed, cl(A) ⊆U. Therefore we have αcl(A) ⊆cl(A) ⊆U.This implies αcl(A) ⊆U. Hence A is #αrg-closed .

The converse of the theorem need not be true as seen from the following example.

Example 3.3

Let X={a,b,c,d} be with topology τ ={X,∅,{a,b}} then the set {c} is #αrg-closed but not #rg-closed set in X.

Corollary 3.4

Every regular closed set is #αrg-closed but not conversely.

Proof.

Follows from s.syed Ali Fathima [19] theorem 2.1

Corollary 3.5

Every δ - closed set is #αrg-closed but not conversely .

Proof.

Follows from s.syed Ali Fathima [19] theorem 2.1

Corollary 3.6

Every θ - closed set is #αrg-closed but not conversely .

Proof.

Follows from s.syed Ali Fathima [19] theorem 2.1

Corollary3.7

Every π - closed set is #αrg-closed but not conversely .

Proof.

Follows from s.syed Ali Fathima [19] theorem 2.1

Theorem 3.8

If a subset A of a topological space(X, τ) is closed then it is #αrg-closed but not conversely .

Proof:

Let A be a closed set in X. We assume that A⊆U where U is rw-open in X.. Then αcl(A) ⊆cl(A) ⊆U as A is closed. Hence A is #αrg-closed in (X, τ).

The converse of the theorem need not be true as seen in the following example.

Example 3.9

Let X={a,b,c,} be with topology τ ={X,∅, {a},{a,c}} then the set {c} is #αrg-closed but not closed set in X.

Theorem 3.10

If a subset A of a topological space(x, τ) is α- closed then it is #αrg-closed but not conversely .

Proof:

Let A be a α-closed set in X. Then αcl(A)=A.Therefore the result follows from the definition. The converse of the theorem need not be true as seen in the following example.

Example 3.11

Let X={a,b,c,d} be with topology τ ={X,∅, {a},{b},{a,b}} then the set {a,b,c} is #αrg-closed but not α- closed set in X.

Theorem 3.12

Every #αrg-closed set is gp-closed but not conversely.

Proof.

Let A be a #αrg-closed set in X. Suppose A⊆U and U is open in X. By the theorem " Every open set is rw-open set" A⊆U and U is rw- open in X. Since A is #αrg-closed, αcl(A) ⊆U. We have pcl(A)⊆αcl(A)⊆U. pcl(A)⊆U. Hence A is gp-closed in X.

The converse of the above theorem need not be true as seen in the following example.

Example 3.13

Let X= {a,b,c} be with topology τ ={X,∅,{a,b}} then the set {a,b} and {b,c} are gp-closed but not #αrg-closed.

Theorem 3.14

Every #αrg-closed set is αg-closed but not conversely.

Proof.

Let A be a #αrg-closed set in X.Suppose A⊆U and U is open in X.By the theorem " Every open set is rw-open set" .Since A is #αrg-closed, αcl(A) ⊆U. Hence A is αg--closed in X.

The converse of the above theorem need not be true as seen in the following example.

Example 3.15

Let X= {a,b,c,d} be with topology τ ={X,∅,{a,b}} then the set {b,c} is αg-- closed but not #αrg-closed.

Theorem 3.16

Every #αrg-closed set is gsp-closed but not conversely.

Proof.

Let A be a #αrg-closed set in X.Suppose A⊆U and U is open in X. Since every open set in X is pre open in X. We have U is pre open in X. Thus we get αcl(A)⊆U. This implies A is αg-closed in X. Hence A is gsp-closed set in X.

The converse of the above theorem need not be true as seen in the following example.

Example 3.17

Let X= {a,b,c,d} be with topology τ ={X,∅,{a,b}} then the set {b,c,d} is gsp-closed but not #αrg-closed.

Remark 3.18

g-closed and #αrg-closed sets are independent as seen in the following example.

Example 3.19

Let X= {a,b,c,} be with topology τ ={X, ∅,{c}{c,a} } then the set {a} is #αrg-closed but not g-closed. Also {b,c} is g-closed set but not #αrg-closed.

Theorem 3.20

The Union of two #αrg-closed subset of X is also #αrg-closed subset of X.

Proof.

Assume that A and B are #αrg-closed set in X. Let U be rw-open in X such that A∪B⊆U. Then A⊆ U and B⊆ U Since A and B are #αrg-closed, αcl(A)⊆ U and αcl(B)⊆ U. Since αcl(A∪B)= αcl(A)∪ αcl(B). αcl(A∪B) ⊆ U . Therefore A∪B is #αrg-closed set in X .

Remark 3.21

The intersection of two #αrg-closed set in X is not #αrg-closed set in X as seen in the following example.

Example 3.22

Let X= {a,b,c,d} be with topology τ ={X,∅, {c},{a,b}{a,b,c}} Here {b,c,d) and {a,b,d} are #αrg-closed but {b,d} is not #αrg-closed

Theorem 3.23

Let A be a #αrg-closed set in X then *αcl(A)\A* does not contain any non-empty rw-closed set.

Proof.

Let F be a non-empty rw-closed subset of *αcl(A)\A*. Then A ⊆ X\F Where A is #αrg-closed and X\F is rw-open. Then *αcl(A)*⊆ $X\backslash F$. For equivalently F⊆ *X\ αcl(A).* Since by assumption F⊆*αcl(A)*. We get a contradiction.

The Converse of the above theorem need not be true as seen from the following example.

Example 3.24

If *αcl(A)\A* contains no non-emply rw-closed subset in X then A need not be #αrg-closed set.

Let X= {a,b,c,d} be with topology τ ={X,∅,{a},{b},{a,b}}. Let A={b,c} then *αcl(A)\A* ={b,c,d}-{b,c}={d} does not contain non-empty rw-closed set in X but A is not #αrg-closed.

Corollary 3.25

If a subset A of X is #αrg-closed in X then *αcl(A)\A* does not contain any non-empty regular closed set in X but not conversely.

Proof.

Follows from theorem 3.23 and the fact that every regular closed set is rw-closed.

Theorem 3.26

Let A be a #αrg-closed in (X,τ) then A is *α-closed* if and only if *αcl(A)\A* is rw-closed.

Proof.

Necessity:- Let A be a #αrg-closed by hypothesis *αcl(A)=A* and so *αcl(A)\A*=∅ which is rw-closed.

Sufficiency:- Suppose *αcl(A)\A is* rw-closed then by Theorem 3.23 *αcl(A)\A*=∅. That is *αcl(A)=A*.. Hence A is *α-closed.*

Theorem 3.27

For every point X of a space X\{x} is #αrg-closed (or) rw-open.

Proof.

Suppose X\{x} is not rw-open. Then X is the only rw-open set containing X\{x} . This implies αcl(X\{x})⊆X. Hence X\{x} is #αrg-closed .

Theorem 3.28

If A is αrg-closed subset of X much that A⊆ ⊆ *αcl(A)* then B is also #αrg-closed subset of (X,τ).

Proof.

Let U be a rw-open set in X such that B⊆ U Then A⊆ U Since A is #αrg-closed then *αcl(A)*⊆ U**.** Now since B⊆*αcl(A). αcl(B)* ⊆ *αcl(αcl(A)=αcl(A)* ⊆*U.* Therefore *αcl(B)* ⊆ U. Hene B is also #αrg-closed .

Remark 3.29

The converse of the above theorem need not be true as seen in the following example.

Example 3.30

Let X={a,b,c,d} be with the topology τ ={X, ∅,{a},{b},{a,b}} let A={a,c} and B={a,b,c} Then A and B are #αrg-closed sets in (X,τ) but A⊆ B and B is not a subset of *αcl(A).*

Theorem 3.31

If a subset A of topological space X is both rw-open and #αrg-closed then it is *α-closed*..

Proof.

Suppose a subset A of a topological space X is both rw-open and $\#\,\alpha rg$-*closed* Now A$\subseteq A$. Then $\alpha cl(A) \subseteq A$. Thus A is α-closed.

Corollary 3.32

Let A be rw-open and #αrg-closed in X. Suppose that F ix α-closed in X. Then A∩F is a $\#\,\alpha rg$-*closed* set in X.

Proof.

Let A be rw-open and $\#\,\alpha rg$-*closed* in X and F be α-closed. Then by theorem 3.31 A is α-closed . So A∩F is α-closed and hence A∩F is $\#\,\alpha rg$-*closed* set in X.

Theorem 3.33

If A is open and αg-*closed* then A is $\#\,\alpha rg$-*closed.*

Proof.

Let A be an open and αg-*closed* set in X. Let A$\subseteq U$ and U be rw-open in X. Now A$\subseteq$ A, By hypothesis $\alpha cl(A) \subseteq A$. That is $\alpha cl(A) \subseteq U$. Thus A is #αg-*closed* .

Remark 3.34

(1) If A is α-*open* and gα-*closed* then A is $\#\,\alpha rg$-*closed.*

(2) If A is π-open and πg-closed then A $\#\,\alpha rg$-*closed* .

(3) If A is α-*open* and αg-*closed* then A is $\#\,\alpha rg$-*closed* .

Definition 3.35

A space X is called $T_{\#\alpha rg}$ – space if every $\#\,\alpha rg$-*closed* set in it is closed.

Example 3.36

Let X ={a,b,c} be with a topology τ ={ ϕ ,x{c},{c,a},{b,c}},Then #αrgc={X, $\emptyset$,{a},{b},{a,b}}. Thus (X,τ) is a $T_{\#\alpha rg}$ – space.

Example 3.37

Let X = {a,b,c} be with the topology τ ={ $\emptyset$,x{c},{c,a}} then # α rgc ={x, ϕ {a},{b},{a,b}}. Here the set {a} is $\#\,\alpha rg$-*closed* but not closed in X. Thus (X,τ) is not a $T_{\#\alpha rg}$ – space.

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Theorem 3.38

Every αT_b space is $T_{\#\alpha rg}$ – space.

Proof.

Let X be a αT_b space and A be $\#\,\alpha rg$-*closed* set in X. We hav e every $\#\,\alpha rg$-*closed* set is αg-*closed*. Hence A is i αg-*closed*. Since it is αT_b space , A is closed in X. Hence X is $T_{\#\alpha rg}$ – space.

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CONCLUSION

Difference forms of generalized closed sets have been introduced. In this paper #αrg-closed sets have been formulated and their basic properties, relationships with some generalized sets in topological space have also been discussed.

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