Effect of Magnetic Field in the Presence of Radiation on Steady Flow Over a Heated Stretching Surface: A Quassi Linearisation and Finite Difference Study

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Abstract:
In the present paper Quassi linearisation and finite difference attempt is made to study the effect of magnetic field on steady flow of a viscous incompressible fluid past a heated stretching sheet in the presence thermal radition. A magnetic field is applied normal to the flow. Roseland approximation is used to describe the radiative heat flux in the energy equation With appropriate similarity transformations, the momentum and energy equations are reduced to ordinary differential equations, in which the equation of motion is a non-linear equation that is linearized by Quassi-linearization method. the governing linear differential equations with boundary conditions in the transformed form are solved numerically using finite difference method. Graphical results for velocity and temperature fields, are presented and discussed. It is noted that for the increasing values of Pr, the rate of decrement in the temperature profile is observed to be fast in the presence of radiation, than in the case of absence radiation.Further, it is concluded that a considerable decrement in the velocity of the fluid is observed in the presence of radiation and magnetic field.

Keywords: *Magnetic field, Thermal radiation, Steady flow, heated stretching sheet, Quassi-linearization and finite difference methods .*

I. INTRODUCTION

The boundary layer flow over a stretching sheet in a uniform stream of fluid has been considered widely in fluid mechanics. The flow over a heated stretching surface has received great interest during the last decades because of several applications in geophysics and energy related engineering problems that includes both metal and polymer sheets. For example, it happens in the aerodynamic extrusion of polymer sheets, thermal energy storage and recoverable systems, petroleum reservoirs, continuous filament extrusion from a dye, cooling of an infinite metallic plate in a cooling bath and during cooling reduction in both the thickness and width take place as these strips are sometimes elongated. The temperature distribution, thickness and width reduction are function of draw ratio and stretching distance. In all these technologies, the quality of the ultimate product depends on the rate of heat and mass transfer at the stretching surface. Sakiadis [1] studied first the boundary layer flow over a continuous solid surface moving in its own plane with constant speed. He has shown that the characteristics of the boundary layer in this case are quite different from that of the blassius flow owing to entrainment of the ambient fluid.

Erickson et al [2] investigated a similar problem in which the transverse velocity at the moving surface is non-zero, taking account of the heat and mass transfer in the boundary layer. Investigations of this type are important due their relevance to the problem of a polymer sheet extruded continuously from a dye. A tacit assumption is being made that the sheet is inextensible. In polymer industry it is necessary to tackle the boundary layer flow over a stretching sheet, McCormack and Crane [8]. Gupta and Gupta [5] carried out the analysis of momentum, heat and mass transfer in the boundary layer over a stretching sheet, subjected to suction or blowing. Radwan *et al* [9] examined the mass transfer over a stretching surface with variable concentration in a transverse magnetic field. In all the cases mentioned above the viscosity of the fluid was assumed uniform in the flow region. Jang *et al* [7] studied the rate of temperature dependent viscosity in the flow and vortex instability of a heated horizontal free convection boundary layer flow. Ioan Pop *et al* [6] analyzed the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate. Lai *et al* [3] studied the effect of variable viscosity on convective heat transfer along a vertical surface in saturated porous medium.

The flow of an electrically conducting fluid past stretching sheet under the effect of a magnetic field has attracted the attention of many researchers due to its wide applications in many engineering problems such as magneto-hydrodynamic (MHD) generator, plasma studies, nuclear rectors, oil exploration,and the boundary layer control in the field of aerodynamics.Consequently,Samad and Mohebujjaman [11] studied a steady-state two dimensional magneto hydrodynamic heat and mass transfer free convective flow along a vertical stretching sheet in the presence of a magnetic field with heat generation. Fadzilah et al. [12] discussed the steady magneto-hydrodynamic boundary-layer flow and heat transfer of a viscous and electrically conducting fluid over a stretching sheet with an induced magnetic field. Ishak [13,14] studied the steady MHD boundary-layer flow and heat transfer due to a stretching sheet. Mixed convection boundary layer in the stagnation point flow to-wards stretching sheet was studied by Ishak et al. [15]. Lahiri *et al* [10] analyzed the effects of transverse magnetic field on the momentum and heat transfer characteristics in the boundary layer of an incompressible fluid flow over a stretching sheet when viscosity of the fluid depends on temperature.

Actually, many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment, Nuclear power plants, gas turbines. In such cases one has to take into account the effects of radiation. Moreover, when the radiative heat transfer takes place, the fluid involved can be electrically conducting in the sense that it is ionized owing to high operating temperature. In such case one cannot neglect the effect of magnetic field on the flow field. So in the present article Quassi- linearization and finite difference attempt is made to study the effects of magnetic field on steady flow of a viscous incompressible fluid past a heated stretching sheet in the presence and absence of radiation. The governing equations in non-dimensional form are solved by using Quassi- linearization and finite difference methods.

II. Mathematical Formulation:

Steady two-dimensional flow of a viscous incompressible, electrically conducting fluid over a heated stretching sheet is considered. The motion of the fluid is being caused solely by the surface` which is moving horizontally with a speed proportional to the distance from the origin (x=0). Additionally, the viscosity of the fluid is assumed to be dependent on the temperature. A magnetic field of uniform strength is applied normal to the flow. The continuity, momentum and energy equations governing such a flow in the boundary layer when subjected to an magnetic field of strength β_0 (Ferraro *et al* [4]) are written, as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

The radiative flux q_r by using the Rosseland approximation [16], is given by

$$q_r=-\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y'} \tag{4}$$

It has been assumed that the temperature differences within the flow are sufficiently small. So T^4 may be expressed as a linear function of the temperature *T*. This can be accomplished by expanding T^4 in a Taylor series about T_∞ , as follows.

Let $f(T)=f(T_\infty)+(T-T_\infty)f'(T_\infty)+\frac{(T-T_\infty)^2}{2!}f''(T_\infty)+\ldots$

where, $f(T)=T^4$, then $f'(T)=4T^3,\ \ f''(T)=12T^2$ (5)

Simplification of (5), gives, $T^4=T_\infty^4+4(T-T_\infty)T_\infty^3+12\frac{(T-T_\infty)^2}{2!}T_\infty^2+\ldots$

In the above expansion, neglecting the higher order terms, we have

$$T^4\approx 4T_\infty^3T-3T_\infty^4 \tag{6}$$

Substituting (6) in (4) and then (4) in (3), we get

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho c_p a_R}\frac{\partial^2 T}{\partial y^2} \quad (7)$$

Where u and v are the components of velocity, respectively in the x and y directions, T is the temperature, α is the coefficient of thermal diffusivity, ρ is the fluid density, σ is the conductivity of the fluid and μ is the coefficient of fluid viscosity. The boundary conditions are given by

$$u=cx, \quad v=0, \quad T=T_{wt} \quad at \quad y=0 \quad (8)$$

$$u\to 0, \quad T\to T_\infty \quad as \quad y\to\infty \quad (9)$$

Hence $c(>0)$ is a constant, T_w is the uniform wall temperature and T_∞ is the free-stream Temperature. We now introduce the following relations for u, v and θ as

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}, \quad (10)$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \quad (11)$$

Where ψ is the stream function. The temperature dependent– viscosity is given by (Bird et al 1960)

$$\mu=\mu^* e^{a(T_w-T)}, \quad (12)$$

Where μ^* is the reference viscosity and a is a constant.

Using the relations (10), (11) and (12) in the equations (2) to (7), we obtain the following

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=-Av^* e^{A(T_w-T)}\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2}+v^* e^{A(1-\theta)}\frac{\partial^3\psi}{\partial y^3}-M\frac{\partial\psi}{\partial y} \quad (13)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2\theta}{\partial y^2}+\frac{16\sigma T_\infty^3}{3\rho c_p a_R}\frac{\partial^2\theta}{\partial y^2} \quad (14)$$

Where $\alpha=\frac{k}{\rho C_p}, v^*=\frac{\mu^*}{\rho}, M=\frac{\sigma B_0^2}{\rho c}$ and c has a dimension (1/ time).

Using the following similarity transformations

$$\psi=(c v^*)^{\frac{1}{2}} x f(\eta), \ \eta=\left(\frac{c}{v^*}\right)^{\frac{1}{2}} y \quad (15)$$

in to the equations (13) and (14), we get following differential equations with boundary conditions in dimensional less form, they are given by in two different cases.

Case I: Presence of radiation

$$\left(\frac{df}{d\eta}\right)^2-f\frac{d^2 f}{d\eta^2}+Ae^{A(1-\theta)}\frac{d\theta}{d\eta}\frac{d^2 f}{d\eta^2}=e^{A(1-\theta)}\frac{d^3 f}{d\eta^3}-M\frac{df}{d\eta} \quad (16)$$

$$\frac{d^2\theta}{d\eta^2}+\left(1+\frac{4}{3R}\right)^{-1}\Pr f\frac{d\theta}{d\eta}=0 \quad (17)$$

The corresponding boundary conditions reduced to

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1 \quad (18)$$

$$f'(\infty)=0, \quad \theta(\infty)=0 \tag{19}$$

Case II: Absence of radiation

$$\left(\frac{df}{d\eta}\right)^2 - f\frac{d^2 f}{d\eta^2} + Ae^{A(1-\theta)}\frac{d\theta}{d\eta}\frac{d^2 f}{d\eta^2} = e^{A(1-\theta)}\frac{d^3 f}{d\eta^3} - M\frac{df}{d\eta} \tag{20}$$

$$\frac{d^2\theta}{d\eta^2} + \Pr f\frac{d\theta}{d\eta} = 0 \tag{21}$$

Where $P_r = \frac{v^*}{\alpha}, \quad R = \frac{k\, a_R}{4\sigma T_\infty^3}$

The boundary conditions are reduced to

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1 \tag{22}$$

$$f'(\infty)=0, \quad \theta(\infty)=0$$

III. Method of solution

Quasi- linearization of all non-linear terms in equations (16) and (20), we get the following

$$B_2\frac{d^3 f}{d\eta^3} + (F - B_1)\frac{d^2 f}{d\eta^2} - (M + 2F')\frac{df}{d\eta} + F'' f = F F'' - (F')^2 \tag{23}$$

Where $F' = \frac{F(i+1) - F(i)}{h}$, $F'' = \frac{F(i+1) - 2F(i) + F(i-1)}{h^2}$, they are called finite difference approximations of F' and F''. Here F is assumed to be a known function.

In order to facilitate the application of finite difference scheme in the range $(0,1)$, we transform the set of equations (17), (23) and (21),(23) to a new system of co-ordinates. So applying the transformation, $z = 1 - e^{-b\eta}$ on the above set of equations we get the following set of equations in the presence and absence of radiation respectively

$$B_2 Q^2\frac{d^3 f}{dz^3} + F_1\frac{d^2 f}{dz^2} + F_2\frac{df}{dz} + F_3 f = F_4 \tag{24}$$

$$Q\frac{d^2\theta}{dz^2} + \left\{\left(1 + \frac{4}{3R}\right)^{-1}\Pr f - b\right\}\frac{d\theta}{dz} = 0 \tag{25}$$

and

$$B_2 Q^2\frac{d^3 f}{dz^3} + F_1\frac{d^2 f}{dz^2} + F_2\frac{df}{dz} + F_3 f = F_4 \tag{26}$$

$$Q\frac{d^2\theta}{dz^2} + \{\Pr f - b\}\frac{d\theta}{dz} = 0 \tag{27}$$

with corresponding boundary conditions for both the cases given by:

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1 \tag{28}$$

$$f'(1)=0 \quad \theta(1)=0$$

Where b = Constant, $B_1 = Ae^{A(1-\theta)}\theta'$, $B_2 = e^{A(1-\theta)}$, $Q = b(1-z)$

$F_1 = (F - B_1) - 3b B_2 Q$, $F_2 = b^2 B_2 - b(F - B_1) - 2QF' - M$

$F_3 = QF'' - bF'$, $F_4 = F(QF'' - bF') - Q(F')^2$

Let $\frac{d^2v}{dz^2} = \frac{v(i+1)-v(i)}{h}$, $\frac{d^2v}{dz^2} = \frac{v(i+1)-2v(i)+v(i-1)}{h^2}$ (29)

$\frac{d^3v}{dz^3} = \frac{v(i+2)-3v(i+1)+3v(i)-v(i-1)}{h^3}$, where v stands f and **θ**

Applying the finite difference formulae (29) on the set of equations (24),(25) and (26),(27), the following set of system of equations are obtained in both presence and absence of radiation respectively

$$A_1[i]\ f[i+2]+A_2[i]\,f[i+1]+A_3[i]\,f[i]+A_4[i]\,f[i-1]=A_5[i] \quad (30)$$

$$H_1[i]\theta[i+1]-H_2[i]\,\theta[i]+Q\theta[i-1]=0 \quad (31)$$

and

$$A_1[i]\ f[i+2]+A_2[i]\,f[i+1]+A_3[i]\,f[i]+A_4[i]\,f[i-1]=A_5[i] \quad (32)$$

$$H_3[i]\theta[i+1]-H_4[i]\,\theta[i]+Q\theta[i-1]=0 \quad (33)$$

The boundary conditions in finite difference form are reduced to

$$f(i)=0,\quad \frac{f(i+1)-f(i)}{h}=1,\ \theta(i)=1,\quad for\ i=0 \quad (34)$$

$$\frac{f(i)-f(i-1)}{h}=0,\ \theta(i)=0,\qquad for\ i=10$$

Where

$$A_1[i]=B_2[i]Q^2[i],\quad A_2[i]=F_1[i]h+F_2[i]h^2-3A_1[i]$$

$$A_3[i]=3A_1[i]-2F_1[i]h-F_2[i]h^2+F_3[i]\,h^3$$

$$A_4[i]=F_1[i]h-A_1[i],\quad A_5[i]=F_4[i]h^3,\quad H_1[i]=Q(i)+\left\{\left(1+\frac{4}{3R}\right)^{-1}\Pr f(i)-b\right\}h,$$

$$H_2[i]=2Q(i)+\left\{\left(1+\frac{4}{3R}\right)^{-1}\Pr f(i)-b\right\}h,\quad H_3[i]=Q(i)+\left(P_r f(i)-b\right)h,$$

$H_4[i]=2Q[i]+\left(P_r f(i)-b\right)h$, where the h is a mesh size in z-direction**.**

Set of equations (30)**,**(31) and (32),(33) with corresponding boundary conditions have been solved by using Gauss-seidel iteration method**,** for which numerical code is executed using C-Program. In the above system f is considered as the $(n)^{th}$ order iterative solutions and F is the $(n-1)^{th}$ order solutions. After each cycle of iteration the convergence check is performed, i.e the tolerance set at 10^{-6}, i.e., $|F-f|<10^{-6}$ is satisfied at all points, then f is considered as convergent solution. Otherwise f becomes the new F and another cycle of iteration is carried out.

IV. RESULTS AND DISCUSSION:

In order to get the physical understanding of the problem and to discuss the significance of the various parameters, a parametric study is conducted. To be realistic, the values of Prandtl number (Pr) are chosen to be Pr = 0.71 and Pr = 7.0, which represent air and water at temperature 20◦C and one atmosphere pressure, respectively.

Magnetic parameter M describes the ratio of electromagnetic force to the viscous force. Figures (1) and (2) show the effect of magnetic parameter **M** on velocity field u in the absence and presence of radiation respectively while figures (6) reveals the effect of magnetic parameter **M** on temperature field. It is observed from the figure that as the value of M increases the velocity of the fluid flow decreases due to the Lorentz type of resistive force. This is due to fact that the interaction of the magnetic field with an electrically conducting fluid produces a body force known as Lorentz force, which plays the role of a resistive type force on the velocity and this force acts against to the fluid flow when the magnetic field is applied perpendicular to it. Therefore it is likely to suppress the flow thereby declining the primary velocity. On the other hand, an opposed effect is noted in the case of temperature field as the value of M increases. Further, it is noted that a comparative study of the graphs show that the presence of heavier radiative number fluid flow is found to decelerate velocity profile. This due to the fact that an increase in the thermal radiation leads to decline in the rate of radiative heat, transferred to the fluid. Further from figure (3) it is seen that the velocity decreases as a non-zero value of A (temperature dependent viscosity) increases.

Figures (7) and (8) show the effect of radiation parameter R on temperature and velocity and fields respectively. It is observed that the temperature and velocity reduce as the radiation parameter increases. It is observed from a non-dimensional radiation parameter that a **decrease** in the radiation parameter $R = k\, a_R / 4\sigma T_\infty^3$ (forgiven k and T_∞) gives a decline in the Roseland radiation absorbptivity, a_R. From the equations 3) and (4), it is also noted that as Roseland radiation absorbptivity a_R decreases, the divergence of the radiation heat flux ($\partial q_r / \partial y^*$) enhances, it indicates that an increase in the thermal radiation leads to decline in the rate of radiative heat, transferred to the fluid. So it causes a reduce in kinetic energy of the fluid particles. This consequence leads to diminish in the velocity and temperature of the fluid.

Figures (4) and (5) demonstrate the effect of Pr in the absence and presence of radiation on temperature profile. It is observed that the presence of heavier Prandtl number fluid is found to slow down temperature profile. This is owing to the truth that a fluid with high Prandtl number has a relatively low thermal conductivity which consequences in the decline of the thermal boundary layer. Further, it is interesting to note that temperature of the fluid decreases in the presence of radiation and also it is noted that in the presence of radiation as the value of Pr increases the rate of decrement in the temperature profile is observed to be more in comparison to the case of absence radiation.

V. Conclusions:

- The temperature and velocity of the fluid decrease in the presence of thermal radiation. Magnetic parameter reduces the velocity of the flow due to the magnetic pull of Lorenz force.
- In the presence of radiation, as the value of Pr increases the rate of decrement in the temperature field is observed more in comparison to the case of absence radiation.

VI. Figures

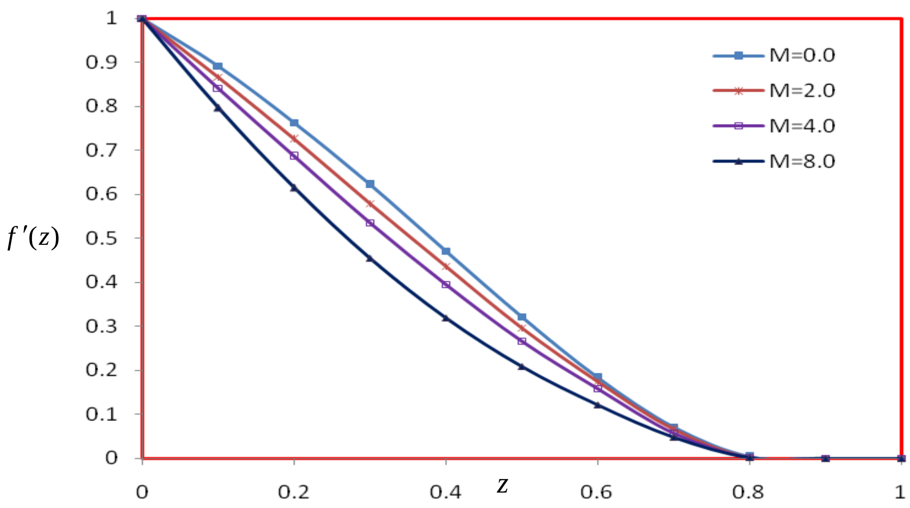

Fig 1: Effect of Magnetic parameter M on velocity field in the absence of radiation A=1.0 and Pr =0.71

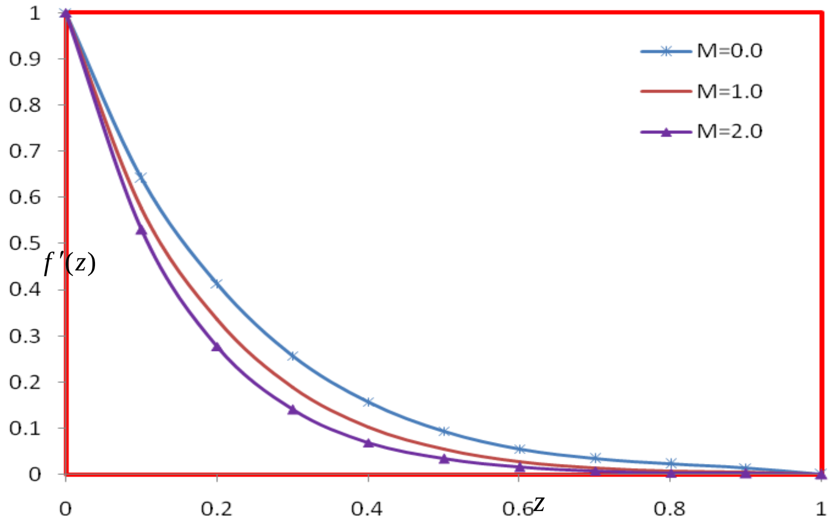

Fig 2: Effect of Magnetic parameter M on velocity field in the presence of radiation (R=2.0) when A=1.0 and Pr =0.71

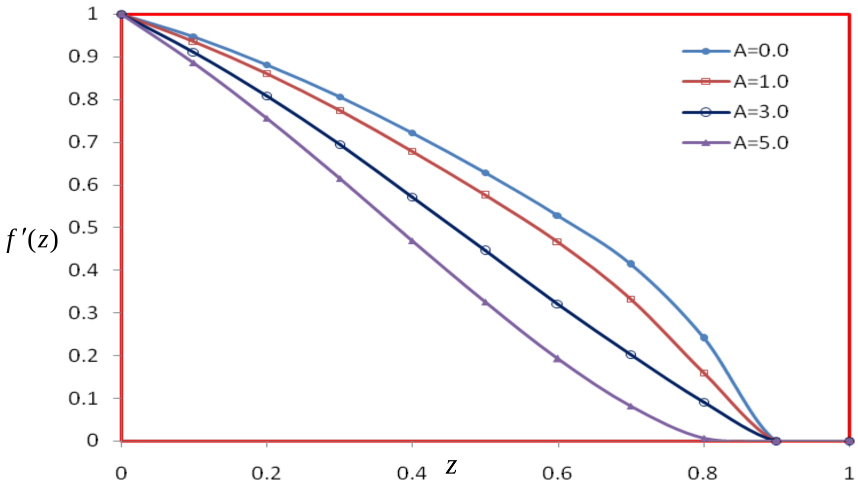

Fig 3: Effect of temperature dependent viscosity A on velocity field in the absence of radiation when M=1.0 and Pr =0.71

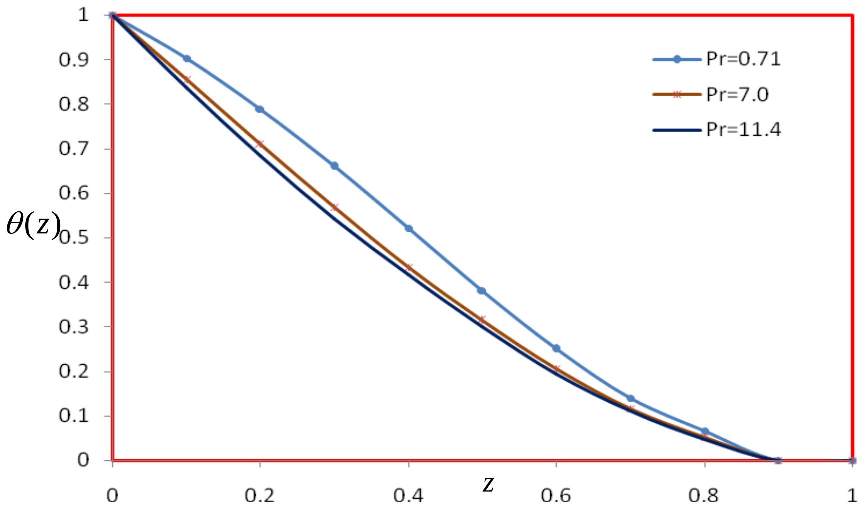

Fig 4: Effect of Pr on temperature field in the absence of radiation when A=1.0, M=1.0 and Pr =0.71

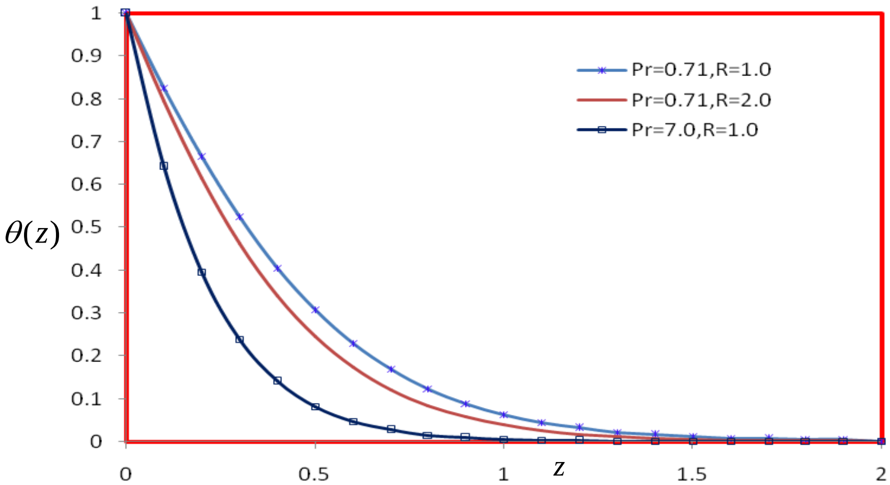

Fig 5: Effect of Prandtl number Pr on temperature field in the presence of radiation when A=1.0, M=1.0 and Pr =0.71

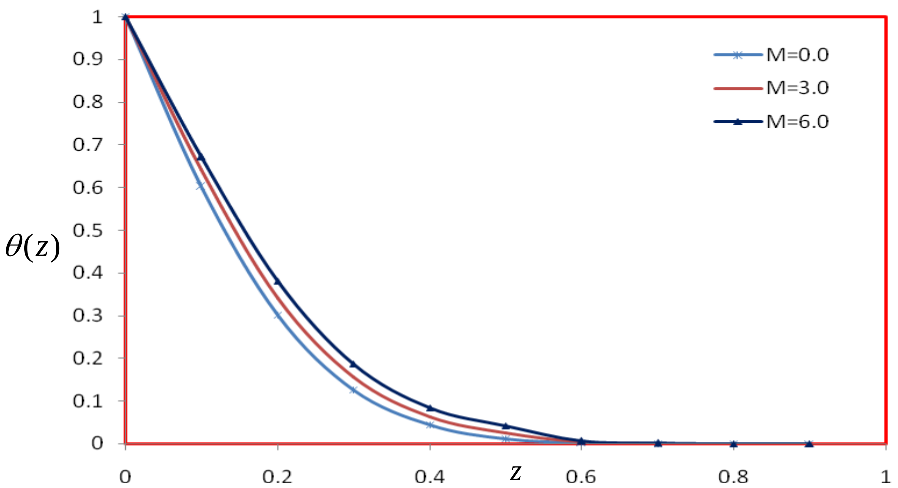

Fig 6: Effect of Magnetic parameter M on temperature field in the absence of radiation A=1.0, R=2.0, R=1.0 and Pr =0.71)

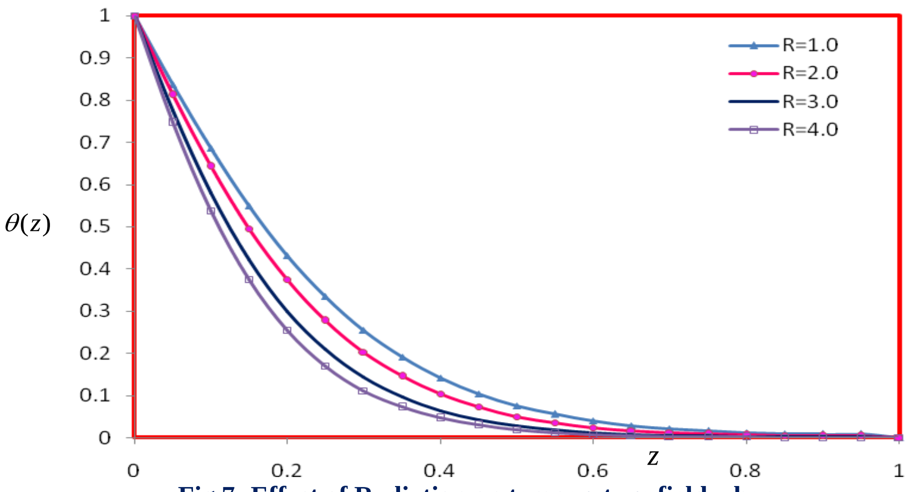

Fig 7: Effect of Radiation on temperature field when
A=1.0. M=1.0 and Pr =0.71

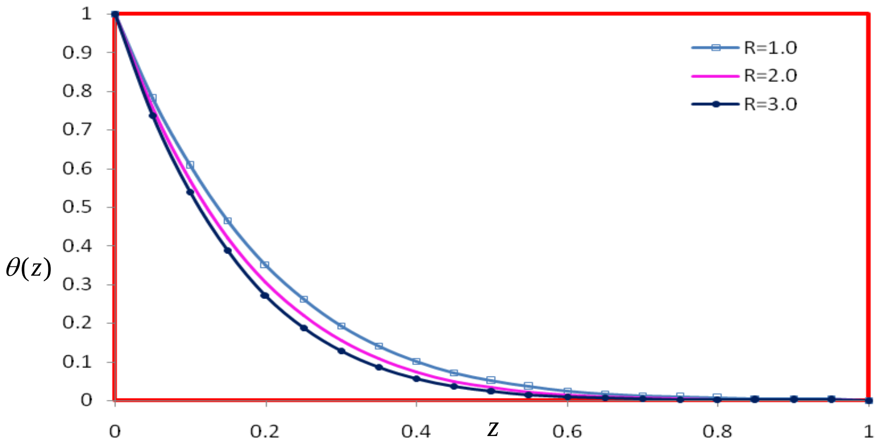

Fig 8: Effect of Radiation on velocity field when A=1.0,
M=1.0 and Pr =0.71

A. References

1) B.C.Sakiadi, "Boundary layer behaviour on continuous solid surfaces," Engng.Journal.7., pp.26-28, 1961.
2) L.E.Erickson, "Heat and mass transfer on a moving continuous flat plate with suction or injection," *Ind.Engng.Chem.Fun.,* 5, 19, 1966.
3) F.C.Lai, F.A.Kulacki, "The effect of variable viscosity on convective heat transfer along a vertical surface in saturated porous medium," *Int. J.Heat Mass transfer.,* 33, 1028, (1990).
4) V.C.A.Ferraro and C.Plumton, "An introduction to Magneto fluid mechanics, Second edition," *Oxford Univ Press.*, 58, 1966.
5) P.S.Gupta and A.S.Gupta, "Heat and mass transfer on a stretching sheet with suction and blowing," *Can. J. Chem. Engng.*, 55, 744, 1977.
6) Ioan Pop, R.S.R.Gorla, M.Rashidi, "The effect of variable viscosity on flow and heat transfer to a continuous moving flat plate," *Int. J. Engng.Sci.,* 30 (1), 1-6, (1992).
7) J.Y.Jang. and J.S.Leu, "Variable viscosity effects on the vortex instability of free convection boundary layer over a horizontal surface," *Numerical heat transfer.*, 25,495, 1994.
8) P.D.McCoromack and I.J.Crane, "Physical Fluid dynamics," *Academic press, New York.*, 1973.
9) A.E.Rawdan and E.M.A. Elbashbeshy, " Mass transfer over a stretching surface with variable concentration in a transverse magnetic field," *Lnuovo Cemento.,* 105b, 615,1990.
10) J.Lahiri, S. Pramanic, G.C Layec, A.K.Chakraborty and H.P. Muzundar, "Hydro magnetic flow over a heated stretching surface with temperature-Dependent Viscosity," *Bull. Cal. Math.Soc.*, 97 (3), 207-21, 2005.
11) M.A.Samad and M.Mohebujjaman, "MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of Magnetic field with heat generation," *Res J Appl Sci, Eng Technol*, 1(3), 98–106,(2009).
12) M.A.Fadzilah,R.Nazar, M.Arifin and I.Pop, "MHD boundary-layer flow and heat transfer over a stretching sheet with induced magnetic field," *J Heat MassTransfer.,* 47, 155–62, 2011.
13) A.Ishak, R.Naza, I.Pop, "MHD boundary-layer flow due to a moving extensible surface," *J Eng Math,*)., 62, 23–33,2008.
14) A.Ishak, R.Naza and I.Pop, "Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet, " *Heat Mass Transfer,*).,44, 921–27, 2008.
15) A.Ishak, K. Jafar, R.Naza and I.Pop, "MHD stagnation point flow towards a stretching sheet," *Phys A.*, 388, 3377–83,2009.
16) R.Siegel and J.R Howell, "Thermal Radiation Heat Transfer, Student addition," *Mac Graw-Hill.*, (1972).