On βwg-Continuous and βwg-Irresolute Functions in Topological Spaces

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Abstract: *The purpose of this paper is to introduce a new type of functions called the βwg - continuous functions. Here, also βwg - irresolute maps, βwg-closed and ßwg-open functions are defined and studied. Further some of their fundamental properties are investigated.*

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I. INTRODUCTION

In 1982, A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb [21] introduced the concept of pre-continuity in topological spaces. In 1983, M. E. Abd El - Monsef, S.N. El-Deeb and R.A.Mahmoud [1] introduced the concept of β-open sets and β-continuous mappings in topological spaces Later, K. Balachandran, P. Sundram and H. Maki [6] introduced and studied the concept of generalized continuous functions. I. Arokirani, et. al., [4] defined gp-irresolute and gp-continuous functions and investigated their properties. M.K.R.S.Veerakumar [36] introduced g*p-closed sets, g*p-continuous maps, g*p-irresolute maps and their properties. C.Sekar and J.Rajakumari [31] introduced αg*p - closed sets and their properties. Recently, the authors [24] have introduced ßwg - closed sets and some of their properties. In this paper we study a new class of functions, namely, βwg-continuous functions and βwg-irresolute functions. Also, we study some of the characterization and basic properties of these functions.

II. PRELIMINARIES

In this paper, the spaces X, Y and Z always mean topological spaces (X, τ), (Y, σ) and (Z, η) respectively. For a subset A of X, the closure of A and the interior of A will be denoted by cl (A) and int (A) respectively. The union of all β-open sets of X contained in A is called β-interior of A and it is denoted by βint(A). The intersection of all β-closed sets of X containing A is called β-closure of A and it is denoted by βcl (A).

We recall the following definitions which are useful in the sequel.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) preopen [21] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
(ii) semi-open [15] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.
(iii) α-open [13] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.
(iv) semi-preopen[3] (β-open[1]) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed[3] (β-closed [2]) if $int(cl(int(A))) \subseteq A$.
(v) regular open [32] if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$.

Definition 2.2: A subset A of a topological space (X, τ) is called a

(i) g-closed [16] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
(ii) sg-closed [7] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
(iii) gs-closed [10] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
(iv) gα-closed [17] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in X.
(v) αg-closed [19] if αcl(A) ⊆ U whenever A ⊆ U and U is open in X.
(vi) gp-closed [20] if pcl(A) ⊆ U whenever A ⊆ U and U is open in X.
(vii) gsp-closed [10] if spcl(A) ⊆ U whenever A ⊆ U and U is open in X .
(viii) gpr-closed [12] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
(ix) rg-closed [25] if cl(A) ⊆ U whenever A ⊆ U and U is regular-open in X.
(x) wg-closed [23] if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in X.
(xi) rwg-closed [23] if cl(int(A)) ⊆ U whenever A ⊆ U and U is regular open in X.
(xii) g*-closed [35] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in X.
(xiii) mg-closed [26] if cl(int(A)) ⊆ U whenever A ⊆ U and U is gopen in X.
(xiv) g*p-closed set [36] if pcl (A) ⊆ U whenever A ⊆ U and U is gopen in X.
(xv) (gsp)* -closed [28] if ßcl(A) ⊆ U whenever A ⊆ U and U is g-open in X.
(xvi) αg*p-closed set [31] if pcl (A) ⊆ U whenever A ⊆ U and U is αg-open in X.
(xvii) gp*-closed set [14] if cl (A) ⊆ U whenever A ⊆ U and U is gpopen in X.
(xviii) αg*-closed set [31] if αcl (A) ⊆ U whenever A ⊆ U and U is αg-open in X.

Definition 2.3[24]: A subset A of a topological space (X, τ) is called beta w generalized closed set (briefly ßwg-closed) if ßcl(A) ⊆ U whenever A ⊆ U and U is αg-open in X.
Definition 2.4: For a subset A of (X, τ), the intersection of all βwg-closed sets containing A is called the βwg-closure of A and is denoted by βwg-cl(A). That is, βwg-cl(A) = ∩{F : F is βwg -closed in X , A ⊆ F}.

Definition 2.5: A function f: (X, τ) → (Y, σ) is said to be
(i) precontinuous [21] if $f^{-1}(V)$ is preclosed in X for every closed subset V of Y.
(ii) semi-continuous [15] if $f^{-1}(V)$ is semi-closed in X for every closed subset V of Y.
(iii) α-continuous [22] if $f^{-1}(V)$ is $_{\alpha}$-closed in X for every closed subset V of Y.
(iv) regular continuous [5] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
(v) semi-precontinuous [3] if $f^{-1}(V)$is semipre-closed in X for every closed subset V of Y.
(vi) g-continuous [6] if $f^{-1}(V)$ is g-closed in X for every closed subset V of Y.
(vii) g*-continuous [35] if $f^{-1}(V)$ is g*-closed in X for every closed subset V of Y
(viii) αg-continuous [19] if $f^{-1}(V)$ is αg-closed in X for every closed subset V of Y.
(ix) gα-continuous [17] if $f^{-1}(V)$ is gα-closed in X for every closed subset V of Y.
(x) gs-continuous [9] if $f^{-1}(V)$ is gs-closed in X for every closed subset V of Y.
(xi) sg-continuous [33] if $f^{-1}(V)$ is sg-closed in X for every closed subset V of Y.
(xii) gp-continuous [4] if $f^{-1}(V)$ is gp-closed in X for every closed subset V of Y.
(xiii) gsp-continuous [10] if $f^{-1}(V)$ is gsp-closed in X for every closed subset V of Y.
(xiv) gpr-continuous [12] if $f^{-1}(V)$ is gpr-closed in X for every closed subset V of Y.
(xv) gp*-continuous if $f^{-1}(V)$ is gp*-closed in X for every closed subset V of Y.
(xvi) g*p-continuous [36] if $f^{-1}(V)$ is g*p-closed in X for every closed subset V of Y.
(xvii) αg*-continuous [29] if $f^{-1}(V)$ is αg*-closed in X for every closed subset V of Y.
(xviii)αg*p-continuous [29] if $f^{-1}(V)$ is αg*p-closed in X for every closed subset V of Y.
(xix) (gsp)*-continuous [28] if $f^{-1}(V)$ is (gsp)*-closed in X for every closed subset V of Y.
(xx) rg-continuous [25 if $f^{-1}(V)$ is rg-closed in X for every closed subset V of Y.
(xxi) wg-continuous [23] if $f^{-1}(V)$ is wg-closed in X for every closed subset V of Y.
(xxii) rwg-continuous [23] if $f^{-1}(V)$ is rwg-closed in X for every closed subset V of Y.
(xxiii) mg-continuous [26] if $f^{-1}(V)$ is mg-closed in X for every closed subset V of Y.

Definition 2.6: A function f: (X, τ) → (Y, σ) is said to be

(i) β-irresolute [18] if $f^{-1}(V)$ is β-closed in X for every β-closed subset V of Y.
(ii) pre-irresolute [30] if $f^{-1}(V)$ is preclosed in X for every preclosed subset V of Y.
(iii) α-irresolute [34] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
(iv) αg-irresolute [8] if $f^{-1}(V)$ is αg-closed in X for every αg-closed subset V of Y.
(v) preclosed [11] if f (V) is preclosed in Y for every closed subset V of X.

Definition 2.7[32]: A function f: (X, τ) → (Y, σ) is called almost continuous if $f^{-1}(V)$ is closed set in (X, τ) for each regular closed set V in (Y, σ).

III. βwg CONTINUOUS FUNCTIONS AND βwg IRRESOLUTE FUNCTIONS

In this section, we introduce βwg-continuous functions and study some of their properties in the following

Definition 3.1: A function f: X → Y is called βwg-continuous if $f^{-1}(V)$ is βwg-closed set in X for every closed set V in Y.

Theorem 3.2 (i) Every continuous function is βwg-continuous function and thus every precontinuous function, every α-continuous function and every regular continuous function is βwg-continuous function.

(ii) Every βwg-continuous function is β-continuous function

Proof: Obvious

The converse of the above theorem need not be true as seen in the following example.

Example 3.3: Let X = Y= {a, b, c, d}, τ = {φ, {a}, {b}, {a,b}, {a,b,c}, X} and σ = {φ, {d},{c,d}, {a,c,d}, {b,c,d}, Y}. Let f: X→Y be defined f (a) = c, f (b) = d, f(c) = a, f (d) = b. Then the function f is βwg -continuous but not continuous. Since, {a} is a closed set of Y, $f^{-1}(\{a\})$ = {c} is βwg-closed in X but not closed in X.

Example 3.4: Let X = Y= {a, b, c, d}, τ ={φ,{a},{b},{a,b},{a,b,c}, X} and σ = {φ, {a}, {b},{a,b},{b,c},{a,b,c}, Y}. Let f: X→Y be defined by f (a) = a, f (b) = b, f(c) = c and f (d) = d. Then the function f is βwg-continuous but not pre-continuous. Since, {a, d} is a closed set of Y, $f^{-1}(\{a, d\})$ = {a, d} is βwg-closed in X but not pre-closed in X.

Example 3.5: Let X = Y= {a,b,c,d}, τ = {φ, {a,b}, X} and σ = {φ,{a},{b},{a,b},{a,b,c}, Y}. Now βwgC (X) = {φ,{a},{b},{c}, {d},{a,c},{a,d},{b,c}, b,d},{c,d},{a,c,d},{b,c,d}, X}. Let f: X→Y be a function defined by f (a) = a, f (b) = b, f(c) = c, f (d) = d. Then the function f is βwg-continuous but not α-continuous. Since, {a,c, d} is a closed set of Y, $f^{-1}(\{a,c,d\})$ = {a, c, d} is βwg-closed in X but not α-closed set in X.

Example 3.6: Let X = Y= {a,b,c}, τ = {φ, {a,b}, X} and σ = {φ,{a},{b},{a,b}, Y}.Define a function f: X→Y by f(a) = a, f(b) = b and f(c) = c. The function f is βwg-continuous but not regular continuous. Since, {a, c} is a closed set of Y, $f^{-1}(\{a, c\})$ = {a, c} is βwg-closed in X but not regular-closed set in X.

Example 3.7: Let X = {a, b, c, d} = Y, τ = {φ,{a, b}, X} and σ ={φ,{c},{d },{c, d}, Y} be topologies on X. Let f: X→Y be a function defined by f(a) = a , f(b) = b , f(c) =c, f(d) = d. The function f is β-continuous but not βwg-continuous. Since, {a, c} is closed set in Y, $f^{-1}(\{a, c\})$ = {a, c} is not βwg-closed but it is β-closed in X.

Remark 3.8: βwg-continuity is independent of semi-continuity as seen from the following example.

Example 3.9: Let X = Y= {a, b, c, d} with topologies, τ = {X, φ, {a}, {b}, {a,b}, {a,c}, {a,b,c}} and σ = {Y, φ,{c},{b,c},{a,c,d}}. Let f: X→Y be a function defined by f(a) = c, f(b) = b, f(c) = d, f(d) = a. Then the function f is semi-continuous and β-continuous but not βwg-continuous, since $f^{-1}(\{b\}) = \{b\}$ is both semi-closed and β-closed but not βwg-closed in X.

Example 3.10: Let X = Y= {a, b, c, d} with topologies, τ = {X, φ, {a}, {a, b}} and σ = {Y, φ, {c},{b, c},{a, c, d}}. Let f: X→Y be a function defined by f (a) = c, f (b) = a, f (c) = b and f(d) = d. Then the function f is βwg - continuous but not semi-continuous and β-continuous, since $f^{-1}(\{b\}) = \{c\}$ is not semi-closed and semi-preclosed but it is βwg-closed in X.

Theorem 3.11: If a function f: X→Y is continuous, then the following holds.

(i) If f is βwg-continuous, then f is g*p-continuous,

(ii) If f is βwg-continuous, then f is gs-continuous (resp. gp-continuous, gsp-continuous, gpr-continuous, αg-continuous).

(iii) If f is βwg-continuous, then f is mg-continuous and thus rg-continuous, g-continuous, wg-continuous, rwg-continuous.

Proof: (i) Let V be a closed set in Y. Since f is βwg-continuous, then $f^{-1}(V)$ is βwg-closed in X. Since every βwg-closed set is

g*p-closed then $f^{-1}(V)$ is g*p-closed in X. Hence f is g*p-continuous.

Similarly we can prove (ii).

The converse of the above theorem need not be true as seen from the following example.

Example 3.12: Let X = Y= {a, b, c, d}, τ = {φ, {a}, {b}, {a,b},X} and σ = {φ,{c}, Y}. Let f: X→Y be an Identity function, defined by f (a) = a, f(b) = b, f(c) = c, f(d) = d. Then the function f is gpr-continuous but not βwg-continuous. Since, {a, b} is a closed set of Y, $f^{-1}(\{a, b\}) = \{a, b\}$ is not βwg-closed in X but it is gpr-closed set in X.

Example 3.13: Let X = Y= {a, b, c, d}, τ = {φ, {a}, {b}, {a, b}, {a, b, c}, X} and σ = {φ, {c}, {a, b, d}, Y}. Let f: X→Y be a function defined by f (a) = b, f (b) = a, f (c) = c and f (d) = d. Then the function f is g*p-continuous but not ßwg-continuous. Since, {a, b, d} is a closed set of Y, $f^{-1}(\{a, b, d\}) = \{a, b, d\}$ is not ßwg-closed in X but it is g*p-closed set in X.

Example 3.14: Let X = Y= {a,b,c}, τ = {φ, {a}, X} and σ = {Y, φ, {c}, {a,c}, {b,c}}. Now GpC(X) = {X, φ, {b}, {c}, {a,b}, {a,c}, {b, c}} = GsC(X) = GspC(X) = αGC(X) = GC(X) and βwgC(X) = {φ, {b}, {c}, {b, c}, X}. Define a function f: X→Y by f(a) = a, f(b) = c and f(c) = b. The f is gp-continuous (resp. gsp-continuous, gs-continuous, αg-continuous, g-continuous) function but not βwg-continuous. Since, {a, b} is a closed set of Y, $f^{-1}(\{a, b\}) = \{a, c\}$ is not βwg-closed in X.

Example 3.15: Let X = {a, b, c} = Y, τ = {φ, {a}, X} and σ = {Y, φ, {b}, {a, b}}.

Now rgC(X) = P(X) and βwg C(X) = {X, φ, {b}, {c}, {b, c}}. Define a function f: X→Y by f(a) = c, f(b) = b and f(c) = a. Then f is rg-continuous function but not βwg-continuous. Since, for the closed set {a, c} in Y, $f^{-1}(\{a, c\}) = \{a, c\}$ is not βwg-closed but it is rg-closed in X.

Example 3.16: Let X = {a, b, c, d} = Y, τ = {φ, {a}, {b}, {a,b}, {a,b,c}, X} and σ = {Y, φ, {c}, {a,c,d}. Define a function f: X→Y by f(a) = d, f(b) = a, f(c) = c and f(d) = b. Then f is mg-continuous function but not ßwg-continuous. Since, for the closed set {a,b,d} in Y, f^{-1}({a, b, d}) = {a, b, d} is not ßwg-closed but it is mg-closed in X.

Example 3.17: Let X = {a, b, c, d} = Y, τ = {φ, {a}, {b},{a,b}, X} and σ = {Y, φ, {c}, {a,b}, {a,b,c}. Define a function f: X→Y by f(a) = b, f(b) = a, f(c) = c and f(d) = d. Then the function f is wg-continuous but not βwg-continuous. Since, for the closed set {a,b,d} in Y, f^{-1}({a,b,d}) = {a, b, d} is mg-closed but not βwg-closed X.

Example 3.18: Let X = {a,b,c,d} = Y, τ = {φ, {a}, {b},{ b},{d}, {a,b},{a,d}, {b,d}, {a,b,d},{a,b}, X} and

σ = {Y, φ, {c}, {b,c}, {c,d}, {a,c,d}, {b,c,d}}. Define a function f: X→Y by f(a) = b, f(b) = a, f(c) = c and f(d) = d. Then the function f is rwg-continuous but not βwg-continuous. Since, for the closed set {a, b, d} in Y, f^{-1}({a, b, d}) = {a, b, d} is rwg-closed but not βwg-closed X.

Theorem 3.19: Every (gsp)*-continuous function is ßwg-continuous function and thus every αg*p-continuous function is
βwg-continuous function.
(ii) Every gp*-continuous function and every αg*-continuous function is βwg-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 3.20: Let X = Y= {a, b, c} with topologies τ = {X, φ, {a}, {c}, {a, c}}} and σ = {Y, φ, {a, b}}. Let f: X→Y be a function defined by f(a) = a, f(b) = b and f(c) = c. Then the function f is βwg-continuous but not (gsp)*-continuous. Since, for the closed set {c} in Y, f^{-1}({c}) = {c} is βwg-closed but not (gsp)*-closed in X.

Example 3.21: Let X = Y = {a, b, c, d} be given the topologies τ = {X, φ,{a, b}} and σ ={Y, φ,{a, c},{a, c, d }}. Let f: X→Y be a function defined by f(a) = a, f(b) = b, f(c) = c and f(d) = d. Then the function f is ßwg-continuous but not αg*p-continuous. Since, the set {b, d} is closed in Y, f^{-1}({b, d}) = {b, d} is βwg-closed set but not αg*p-closed set in X.

Example 3.22: Let X = {a, b, c} = Y, τ = {X, φ, {a, b}} and σ = {Y, φ, {a}, {b}, {a, b}}.
Now gp*C(X) = {X, {c}} and ßwgC(X) = {X, φ, {a}, {b}, {c}, {a, c}, {b, c}}. Define a function f: X→Y be an Identity function. Then f is gp*-continuous function but not βwg-continuous. Since, for the closed set {b, c} in Y, f^{-1}({b, c}) = {b, c} is βwg-closed but not gp*-closed in X.

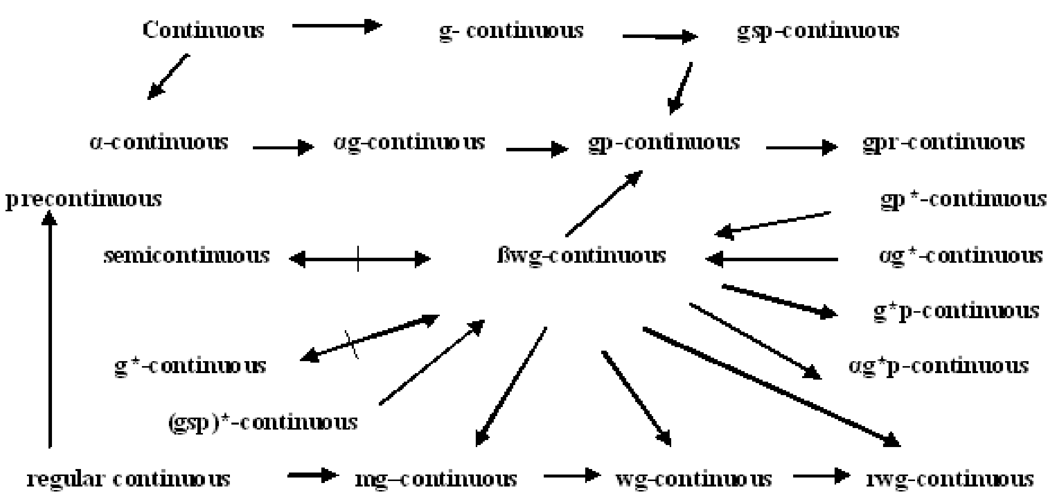
Example 3.23: Let X = {a, b, c} = Y, τ = {X, φ, {a,b}} and σ = {Y, φ, {a}, {a,b}}. Define a function f: X→Y by f(a) = a, f(b) = c, f(c) = b,. Then f is ßwg-continuous function but not αg*-continuous. Since, for the closed set {c} in Y, f^{-1}({c}) = {b} is ßwg-closed set but not αg*-closed set in X.

Remark 3.24: The following examples shows that ßwg-continuous functions are independent of g-continuous, g*continuous,
gs-continuous, sg-continuous and αg-continuous.

Example 3.25: Let X = {a, b, c} = Y with topologies τ = {X, φ, {a},{c},{a, c}}} and σ = {Y, φ, {a}, {a, c}}.
Let f: X→Y be an Identity function. Then the function f is βwg-continuous but not g, g*, sg, gs and αg-continuous. Since, for {b} is closed set in Y, f^{-1}({b}) = {b} is βwg-closed but not g-closed, g-closed, g*-closed, gs-closed, sg-closed, αg- closed sets in X.

Example 3.26: Let X = {a, b, c, d} = Y, with topologies τ = {φ, {a}, b},{a, b},{a, b, c}, X} and σ ={φ,{c},{b, c},{a, c},{a, b, c}, Y}. Let f: X→Y be defined by f(a) = b, f(b) = a, f(c) = c, f(d) = d. Then the function f is g, g*, sg, gs, αg-continuous, but not βwg-continuous, since f^{-1}({a,b,d}) = {a,b,d} is not ßwg-closed.

Remark 3.27: From the above discussions and known results we have the following implications.
A ⟶/⟶ B means A implies B but not conversely.
A ⟷ B means A and B are independent.



Diagram

IV. CHARACTERISTICS OF βwg-CONTINUITY

Theorem 4.1: Let $f: (X, \tau) \to (Y, \sigma)$ be a map .Then the following conditions are equivalent:

(i) f is βwg-continuous.

(ii) The inverse image of each open set in Y is βwg-open in X.

(iii) $f(\beta wg\text{-}cl(A)) \subseteq cl(f(A))$ for each subset A of X.

(iv) For each subset B of Y, $\beta wg\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$

Proof: (i) ⇒ (ii): Let U be an open set in Y. Then $Y \setminus U$ is closed in Y. By hypothesis, $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is βwg-closed in X. Hence $f^{-1}(G)$ is βwg-open in X.

(ii) ⇒ (i): Let U be a closed set in Y. Then $Y \setminus U$ is open in Y. By hypothesis, $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is βwg - open in X. Therefore $f^{-1}(U)$ is ßwg-closed in X. Hence f is βwg-continuous.

(i) ⇒ (iii): Let G be a subset of X. Since $G \subseteq f^{-1}(f(G))$ and $f(G) \subseteq cl(f(G))$, we have $G \subseteq f^{-1}(f(G)) \subseteq f^{-1}(cl(f(G)))$. Therefore by assumption $f^{-1}(cl(f(G)))$ is βwg-closed set of X. Hence $\beta wg\text{-}cl(G) \subseteq f^{-1}(cl(f(G)))$. Thus $f(\beta wg\text{-}cl(G)) \subseteq f(f^{-1}(cl(f(G)))) \subseteq cl(f(G))$.

(iii) ⇒ (iv): Let B be a subset of Y and $f(G) = B$. So by assumption, $f(\beta wg\text{-}cl(G)) = f(\beta wg\text{-}cl(f^{-1}(B)))$. Therefore $\beta wg\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(f(\beta wg\text{-}cl(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$.

(iv) ⇒ (i): Let B be a closed set in Y. Then by assumption, $\beta wg\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B)) = f^{-1}(B)$.Therefore $f^{-1}(B)$ is βwg-closed set in X. Hence f is βwg-continuous.

Theorem 4.2: Let A be a subset of a topological space X. Then $x \in \beta wg\text{-}cl(A)$ if and only if for any βwg-openset U containing x, $A \cap U \neq \varphi$.

Proof: Let x ∈ βwg-cl(A) and suppose that there is a ßwg-open set U in X such that x ∈ U and A ∩ U= φ implies that A ⊆ X \ U which is ßwg-closed in X implies ßwg-cl(A) ⊆ ßwg-cl(X \ U) = X \ U. Since x ∈ U implies that x ∉ X \ U implies that

x ∉ ßwgCl(A), this is a contradiction. Conversely, suppose that, for any ßwg - open set U containing x, A ∩ U ≠ φ. To prove that x ∈ ßwg-cl(A). Suppose that x ∉ ßwg-cl(A) then there is a ßwg-closed set F in X such that x ∉ F and A ⊆ F. Since x ∉ F implies that x ∈ X \ F which is ßwg-open in X. Since A⊆ F implies that A ∩ (X \ F) = φ, this is a contradiction. Thus x ∈ βwg-cl(A).

Theorem 4.3: Let f: X→Y be a function from a topological space X into a topological space Y. If f: X→Y is ßwg-continuous then f(ßwg-cl(A)) ⊆ cl(f(A)) for every subset A of X.

Proof: Since f (A) ⊆ cl(f(A)) then A ⊆ f^{-1}(cl(f(A))). Since cl(f (A)) is a closed set in Y and f is ßwg - continuous then by definition f^{-1}(cl(f(A))) is a ßwg-closed set in X containing A. Hence ßwg-cl(A) ⊆ f^{-1}(cl(f(A))). Therefore f(ßwg -cl(A)) ⊆ cl(f(A)).

The converse of the above theorem need not be true as shown in the following example

Example 4.4: Let X = Y = {a, b, c} with τ = {φ, {a, b}, X} and σ = {φ, {a}, {b}, {a, b}, Y}.

Define a function f: X→Y by, f(a) = a, f(b) = b and f(c) = c. For every subset A of X, f(ßwg-cl(A)) ⊆ cl(f(A)) holds. But f is not ßwg-continuous, since {c} is closed set in Y, f^{-1}({c}) = {c} which is not βwg-closed set in X.

Theorem 4.5: Let f: X→Y be a function. Then the following statements are equivalent:

(i) For each x ∈X and each open set V containing f(x) there exists a ßwg-open set U containing x such that f(U) ⊂ V.

(ii) f(ßwg-cl(A)) ⊂ cl(f(A)) for every subset A of X.

Proof: (i) ⇒ **(ii):** Let y ∈ f(ßwg-cl(A)) then there exists an x ∈ ßwg-cl(A) such that y = f(x). LetV be any open neighbourhood of y. Since x ∈ ßwg-cl(A) ,there exists a ßwg-open set U such that x ∈ U and U ∩ A ≠ φ, f(U) ⊂ V. Since U ∩ A ≠ φ, f (A) ∩ V ≠ φ. Therefore y = f(x) ∈ cl (f(A)).Hence f(ßwg-cl (A)) ⊂ cl (f(A)).

(ii) ⇒ **(i):** Let x ∈ X and V be any open set containing f (x). Let A = f^{-1} (Y \ V). Since f (ßwg-cl (A)) ⊂ cl (f (A)) ⊂ Y \ V then (βwg-cl(A)) ⊂ f^{-1} (Y \ V) = A. Hence βwg-cl(A) = A. Since f(x) ∈V ⇒ x ∈ f^{-1} (V) ⇒ x ∉A ⇒ x ∉ βwg-cl(A). Thus there exists an open set U containing x such that U ∩ A = φ. Therefore f (U) ⊂ V.

Definition 4.6: Let be a topological spaces. Then

(i) a space (X, τ) is called ${}_{ßwg}T_b$ -space if every ßwg-closed is closed.
(ii) a space (X, τ) is called ${}_{ßwg}T_d$ - space if every ßwg-closed is g-closed.
(iii) a space (X, τ) is called ßwg-$T_{1/2}$ space if every αg*p-closed is preclosed.
(iv) a space (X, τ) is called ${}_{ßwg}T_\alpha$ -space if every ßwg-closed set is α-closed set.

Theorem 4.7: Let f: X→Y be a function. Let (X, τ) and (Y, σ) be any two spaces such that $\tau_{ßwg}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, f(βwg-cl(A)) ⊆ cl(f(A)) holds.
(ii) f: (X, $\tau_{\beta wg}$) → (Y, σ) is continuous.

Proof: Suppose (i) holds. Let A be closed in Y. By hypothesis f(βwg-cl(f^{-1}(A))) ⊆ cl(f(f^{-1}(A))) ⊆ (A) = A.

Also f^{-1}(A) ⊆ βwg-cl(f^{-1}(A)). Hence βwg-cl(f^{-1}(A)) = f^{-1}(A). This implies f^{-1}(A) ∈ $\tau_{\beta wg}$. Thus f^{-1}(A) is closed in (X, $\tau_{\beta wg}$) and so f is continuous. This proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f: (X, $\tau_{ßwg}$) → (Y, σ) is continuous, f^{-1}(cl(A)) is closed in (X, $\tau_{ßwg}$) . By definition, ßwg-cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))).

Now we have, A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))) and by ßwg-closure, ßwg-cl(A) ⊆ ßwg-cl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A)). Therefore f(ßwg-cl(A)) ⊆ cl(f(A)). This proves (i).

Remark 4.8: The Composition of two ßwg-continuous functions need not be ßwg-continuous function and

but the following is valid.

Example 4.9: Let X = Y = {a,b,c,d} = Z, with topologies τ = { X, φ,{a},{b},{a,b},{a,b,c}}, σ = {Y, φ, {b,c}, {a,b,c}, Y} and η = {Z, φ,{a}}. Define g: Y→Z by g(a) = b, g(b) = c, g(c) = a, g(d) = d and define f: X→Y by f(a) = b, f(b) = d, f(c) = c, f(d) = a. Then both f and g are ßwg-continuous functions. But gof is not βwg-continuous function, since

$(gof)^{-1}$({b, c, d}) = f^{-1}[g^{-1}({b,c,d})] = f^{-1} ({a, b, d}) = {a, b, c} is not a ßwg-closed set in X.

Theorem 4.10: Let f: X→Y is ßwg-continuous function and g: Y→Z is continuous function then g ○ f: X→Z is ßwg-continuous.

Proof: Obvious.

Easy proofs of the following Theorems are omitted.

Theorem 4.11: Let f: X→Y is βwg - continuous function and g: Y→Z is βwg - continuous function and Y is βwg-T_b space then gof: X→Z is βwg-continuous.

Theorem 4.12: Let f: X→Y is βwg-continuous function and g: Y→Z is αg-continuous function and Y is ${}_{\alpha}T_b$ space then gof: X→Z is βwg-continuous.

Theorem 4.13: Let f: X→Y is ßwg-continuous function and g: Y→Z is α-continuous function and Y is α - space then gof: X→Z is ßwg-continuous function.

Definition 4.14: A function f: (X, τ) → (Y, σ) is called a βwg-irresolute if f^{-1}(V) is βwg-closed set in (X, τ) for every βwg-closed set V in (Y, σ).

Definition 4.15: A function f: (X,τ) → (Y,σ) is called βwg-closed if f (V) is βwg-closed set in (Y,σ) for every closed set V in (X,τ).

Definition 4.16: A function f: (X, τ) → (Y, σ) is called ßwg-open if f (V) is ßwg- open set in (Y, σ) for every open set V in (X, τ).

Theorem 4.17: (i) Every α-irresolute function is βwg-continuous.

(ii) Every ßwg-irresolute function is βwg-continuous.

(iii) Every ß-irresolute function is βwg-irresolute.

Proof: Obvious

The converse of the above theorem (ii) need not be true it can be seen from the following example.

Example 4.18: Let X=Y = {a,b,c,d}, τ = {X, φ,{a},{a,b}} and σ ={Y, φ,{a,b}}. Define a function f: X→Y by f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then f is ßwg -continuous but not βwg-irresolute, since for the closed set {b,c,d}in Y, f^{-1}({b,c,d}) = {a,c,d} is not a βwg-closed set in X.

Theorem 4.19: Let f: X→Y is βwg-irresolute function and g: Y→Z is βwg-irresolute function then gof: X→Z is βwg-irresolute.

Proof: Let g be βwg-irresolute function and V be any βwg - open set in Z then g^{-1}(V) is βwg-open in Y. Since f is βwg-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is βwg-open in X. Hence gof is βwg-irresolute.

Theorem 4.20: If a map f: (X, τ) → (Y, σ) is ßwg-irresolute, if and only if the inverse image f^{-1}(V) is ßwg-open set in X for every ßwg-open set V in Y.

Proof: Obvious.

Theorem 4.21: If a function f: (X, τ) → (Y, σ) is βwg-irresolute, then for every subset A of X, f(ßwg-cl(A) ⊆ αcl(f(A)).

Proof: If A ⊆ X then consider αcl(f(A)) which is ßwg-closed in Y. Since f is ßwg-irresolute, f^{-1}(αcl(f(A))) is ßwg-closed in X. Furthermore A ⊆ f^{-1} (f(A)) ⊆ f^{-1} (αcl(f(A))). Therefore by ßwg-closure, ßwg-cl(A) ⊆ f^{-1}(αcl(f(A))), consequently,
f(ßwg-cl(A) ⊆ f(f^{-1} (αcl(f(A)))) ⊆ αclf((A)).

Theorem 4.22: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be any two functions. Then

- **(i)** g o f : (X,τ) → (Z,η) is ßwg-continuous if g is regular-continuous and f is ßwg-irresolute.
- **(ii)** g o f : (X, τ) → (Z, η) is ßwg-irresolute if g is ßwg-irresolute and f is ßwg-irresolute.
- **(iii)** g o f : (X, τ) → (Z, η) is ßwg-continuous if g is ßwg-continuous and f is ßwg-rresolute.

Proof: (i) Let U be an open set in (Z,η). Since g is regular-continuous, $g^{-1}(U)$ is regular-open set in (Y, σ). Since every regular-open is ßwg-open then $g^{-1}(U)$ is ßwg-open in Y. Since f is ßwg-irresolute then $f^{-1}(g^{-1}(U))$ is a ßwg-open set in (X, τ). Thus
$(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is a ßwg-open set in (X, τ) and hence g ○ f is ßwg-continuous.

Similarly we can prove (ii) and (iii).

Theorem 4.23: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be any two functions. Then gof: (X,τ) → (Z,η) is ßwg-continuous

(i) if g is α-continuous and f is βwg-irresolute.
(ii) if g is gp*-continuous and f is βwg-irresolute.
(iii) if g is (gsp)*- continuous and f is βwg-irresolute.
(iv) if g is αg*p-continuous and f is βwg-rresolute.

Proof: Obvious.

Theorem 4.24: Let (X, τ) be topological space. Then

(i) Every βwg-T_b space is βwg-$T_{1/2}$ space

(ii) Every ßwg-T_b space is ßwg-T_d - space.

Proof: It follows from the definitions and the fact that every closed set is g-closed.

Theorem 4.25: Let f: (X, τ) → (Y, σ) be a function then,

(i) If f is βwg-irresolute and X is βwg-$T_{1/2}$ space, then f is pre-irresolute.
(ii) If is βwg-continuous and X is βwg-$T_{1/2}$ space, then f is pre-continuous.
(iii) If f is βwg-irresolute and X is βwg-T_α -space, then f is α-irresolute.

Proof: (i) Let V be pre-closed in Y, then V is βwg-closed in Y. Since f is ßwg-irresolute, $f^{-1}(V)$ is βwg-closed in X. Since X is βwg-$T_{1/2}$ space, $f^{-1}(V)$ is pre-closed in X. Hence f is pre-irresolute.
(ii) Let V be closed in Y. Since f is ßwg-continuous, $f^{-1}(V)$ is ßwg-closed in X. Since X is ßwg-$T_{1/2}$ space, $f^{-1}(V)$ is pre-closed. Therefore f is pre-continuous.

(iii) Let V be α-closed in Y, then V is βwg-closed in Y. Since f is ßwg-irresolute, $f^{-1}(V)$ is βwg-closed in X. Since X is ßwg-T_α -space, $f^{-1}(V)$ is α-closed in X. Hence f is α-irresolute.

Theorem 4.26: A function f: X→Y be a bijection. Then the following are equivalent:

(i) f is βwg-open,
(ii) f is βwg-closed,
(iii) f^{-1} is βwg-irresolute.

Proof: Suppose f is βwg-open. Let F be ßwg-closed in X. Then X \ F is βwg-open. By definition, f(X \ F) is ßwg-open. Since f is bijection, Y \ f(F) is ßwg-open in Y. Therefore f is ßwg-closed. This proves (i) ⇒ (ii).

Let g = f^{-1}. Suppose f is βwg-closed. Let V be ßwg-open in X. Then X \ V is βwg-closed in X. Since f is ßwg-closed, f(X \ V) is βwg - closed. Since f is a bijection, Y \ f(V) is ßwg-closed that implies f(V) is βwg-open in Y. Thus $g^{-1}(V)$ is ßwg-open in Y. Therefore f^{-1} is βwg-irresolute. This proves (ii) ⇒ (iii).

Let V be ßwg-open in X. Then X \ V is ßwg-closed in X. Since f^{-1} is ßwg - irresolute and $(f^{-1})^{-1}(X \setminus V) = f(X \setminus V) =$ Y \ f(V) is ßwg-closed in Y that implies f(V) is ßwg-open in Y. Therefore f is ßwg-open. This proves (iii) ⇒ (i).

Theorem 4.27: Let f: (X, τ) → (Y, σ) be a bijective, αg-irresolute and ß-closed function. Then f(A) is ßwg-closed in Y for every βwg-closed set A of X.

Proof: Let A be ßwg-closed in (X, τ). Let V be αg-open set of (Y, σ) containing f (A). Since f is αg-irresolute, $f^{-1}(V)$ is αg-open in X. Since A ⊆ $f^{-1}(V)$ and A is ßwg-closed, ßcl(A) ⊆ $f^{-1}(V)$. Since f is bijective and ß-closed function, f(ßcl(A)) = cl(f(ßcl(A)). Now ßcl(f(A)) ⊆ ßcl(f(ßcl(A)) = f(ßcl(A)) ⊆ V. Hence f (A) is ßwg-closed set in Y.

V. ON ALMOST ßwg-CONTINUOUS FUNCTION

We define and obtain some their properties the following

Definition 5.1: A function f: (X, τ) → (Y, σ) is called almost ßwg-continuous if $f^{-1}(V)$ is ßwg-closed set in (X, τ) for each regular closed set V in (Y, σ).

Theorem 5.2: Let f: (X, τ) → (Y, σ) is ßwg-continuous function. Then it is almost ßwg-continuous.

Proof: Let f: X→Y be a ßwg-continuous function. Let V be a regular closed set in Y. Since f is continuous, $f^{-1}(V)$ is closed in X. Since every regular closed set is a closed set and hence $f^{-1}(V)$ is ßwg-closed in X. Therefore f is almost βwg-continuous function.

The converse of the above theorem need not be true as seen in the following example.

Example 5.3: Let X = Y= {a,b,c}, τ = {φ,{a},X} and σ = {φ,{a},{a,b},Y}. Now RC(X,τ) = {Y,∅} and βwg C(X, τ) = {φ,{b}, {c},{b, c}, X}. Define function f: X→Y be defined by f (a) = c, f (b) = a, f(c) = b. Then the function f is almost ßwg-continuous but not ßwg-continuous. Since, {c} is a closed set of Y, $f^{-1}(\{c\})$ = {a} is not ßwg-closed in X.

Theorem 5.4: If X is a $_{\beta wg}T_b$ space and the function f: (X, τ) → (Y, σ) is almost βwg-continuous then f is almost continuous.

Proof: Obvious.

VI. CONCLUSIONS

In this article we have focused on βwg-closed sets, βwg-continuity and its characteristics and βwg-irresolute functions in topological spaces. Further with help these functions almost ßwg-continuous functions were studied.

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