The Split Domination of a Jump Graph

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Abstract

A dominating set D of a jump graph J(G)=(V, E) is a split dominating set if the induced sub graph < V-D > is disconnected.A split dominating number $\surd_s$(J(G)) of jump graph J(G) is the minimum cardinality of a split dominating set. In this paper we study split dominating sets and investigate the relation of $\surd_s$(J(G)) with other known parameters of J(G).

1.INTRODUCTION

The jump graph considered here an finite, undirected without loop or multiple edge and have at least one component which is not complete or at least two components neither of which are isolated vertices, Unless otherwise stated, all graphs are assumed to have p vertices and q edges.

A set D of vertices in a jump graph J(G)= (V,E) is a dominatying setoff J(G) if every vertex in V-D is adjacent to some vertex in D. The domination number $\surd$(J(G)) of J(G) is minimum cardinality of a dominating set. A survey on $\surd$(J(G)) cn be found in[9]

And some recent results in[2]-[5]

The connectivity k(J(G)) is the minimum number of vertices whose removal results in a trivial or disconnected jump gaph

The purpose of this paper is to combine the above two concepts as follows,

The dominating set D of a jump graph J(G)=(V,E) is a split dominating set if the induced sub graph <V-D> is disconnected. The split domination number $\surd_s$(J(G)) of jump gapj J(G) is the minimum cardinality os a split dominating set.

A $\surd$-set is a minimum dominating set. A $\surd_s$-set can be defined similarly, we note that $\surd_s$-set exists if the graph is not complete and either it contains a non-split dominating set, Further we also note that in a disconnected jump graph containing a $\surd_s$-set is a $\surd_s$-set. Thus, if for the rest of this paper we will assume that G is a non-complete connected jump graph. Any term not defined in this paaper may be found in Harary [6]

2. RESULTS

We first obtain an upper bound for $\surd_s$(J(G)),

Theorem 1. For any jump graphJ(G)

$\surd_s(J(G)) \leq \alpha_0(J(G))$ …………….(1)

Where α_0(J(G)) is the vertex covering number of jump graph J(G).

Proof; Let S be a maximum independent set of vertices in J(G). Then S has at leat two vertices and every vertex in S is adjacent to some vertex in (V-S) This implies that (V-S) is a split dominating set of J(G) Thus (1) holds.

We state without proof a straight forward result the characterize dominating set of J(G) that are split dominating sets.

Theorem 2; a dominating set D of J(G) is a split dominating set if and only if there exists two vertices $w_1, w_2 \in$ V-D, such that every w_1-w_2 path contains a vertex of D.

Theorem 3; For any jump graph J(G)

(i) $\surd(J(G)) \leq \surd_s(J(G))$….(2)

(ii) $K(J(G)) \leq \surd_s(J(G))$….(3)

Proof; (2) and (3) follow from the definition of √(J(G) and $\surd_s$(J(G)).

Next we characterize split dominating sets of J(G) which are minimal

Theorem 4 ; A split dominating set D of J(G)I is minimal if and only if for each vertex v ∈ D one of the following is satisfied.

(i) There exists a vertex v∈ V-D such that N(u) ∩ D = {v}

(ii) V is isolated in <D>

(iii) < (v-D) ∪ {v} > is connected.

Proof; Suppose D is minimal and there exists a vertex v ∈ D such that v does not satisfy any of the above conditions. Then by condition (i) and (ii), D'= D – {v} is a dominating set of J(G) Also by (iii) <V-D> is disconnected. This implies that D' is a split dominating set of J(G). a contradiction.\

The converse is obvious.

Now we obtain a another upper bound on $\surd_s$(J(G))

Theorem 5; For any jump graph J(G)

$\surd_s$(J(G)) ≤ p. $\frac{\Delta(J(G))}{(\Delta(J(G))+1)}$ **…..(4)**

Where Δ (J(G)) is the maximum degree of jump graph J(G)

Proof; Let D be a $\surd_s$-set in J(G). Since D is minimalby theorem 4. It follows that for each vertex v ∈ D there exists a vertex

u ∈ V-D such that v is adjacent to u. Thios implies that V-D is a dominating set of J(G). Then √(J(G)) ≤ |V-D| ≤ p-$\surd_s$(J(G)).

Hence (4) follows from the fact that

√(J(G)) ≥ t$\frac{p}{\Delta(J(G))+1}$

In the next result we obtain a sufficient condition on J(G) such that √(J(G)) = $\surd_s$(J(G))

Theorem 6; For any jump gaph J(G) with an end vertex √(J(G)) = $\surd_s$(J(G)) ….(5)

Furthermore, there exists a $\surd_s$-set of J(G) containing all vertices adjacent to end -vertices.

Proof: Let v be an end –vertex of J(G0.then there exists a cut-vertex w adjacent to v. Let D be a √-set of J(G). suppose w ∈ D. Then d is a $\surd_s$-set of J(G). Suppose w ∈ V-D Then v∈ D and hencd D- {u} ∪ {w} is a $\surd_s$-set of J(G). repeating this process for all such cut-vertices adjacent to end-vertices, we obtain a $\surd_s$-set of J(G) containing all cut vertices adjacent to end-vertices.

We next consider the case where the diameter of J(G) is 2.

Theorem 7; If diam (J(G))=2 then

$\surd_s$(J(G)) ≤ δ(J(G))……..(6)

Where δ(J(G)) is minimum degree of J(G).

Proof; Let v be a vertex of minimum degree in J(G) since diam(J(G))=2,there exists a vertex u is not adjacent to v. Hence it follows that N9V) is a split dominating set of J(G) Thus (6) holds

The proof of the following proposition follow easily.

Proposition 8;

(i) For any cycle J(C_p) with p ≥ 4 vertices $\surd_s$(J(C_p) = ┌ p/3 ┐ …..(7) where ┌ x ┐ is the least positive integer not less than x.

(ii) For any wheel J(W_p) with p≥ 5 vertices $\surd_s$(J(w_p)) = 3 …..(8)

(iii) For any complete bipartite jump graph $K_{m,n}$ with 2 ≤ m ≤ n

$\surd_s$(J($K_{m,n}$))=m…..(9)

We obtain a sufficient condition for a cut vertex to be in every $\surd_s$-set of J(G).

Theorem9 . If J(G) have one cut vertex v and at least two blocks B_1 and B_2 then v is in every $\surd_s$-set of J(G).

Proof; Let D be a $\surd_s$-set of J(G).Suppose v ∈ V-D.Then each B_1 and B_2 contributes at least one vertex to D say u and w respectively. This implies that

D' = D – {u, w} ∪ {v} is a split dominating set of J(G) a contradiction. Hence v is in every

$\surd_s$-set of J(G).

Theorem 10; Let v be a cut-vertex of J(G).If there is a block H of J(G) such that v is the only cut-vertex of H, and v is adjacent to all vertices of H, then there is a $\surd_s$-set of J(G) containing v,

Proof; If there exists at least two blocks in J(G) satisfying the given condition, then by Theorem 9. V is in every $\surd_s$-set of J(G) and hence the result.Suppose there exists only one block H in J(G).satisfying the given condition. Let D be a $\surd_s$-set of J(G). suppose v ∈ V-D Then for some vertex u ∈ H. [u] ⊂ D. This proves that D'= D – {u} ∪ {v} is a $\surd_s$-set of J(G).

To prove our next result we make use of the following definitions from [7].

A dominating set D of a jump graph is connected domination set ,if the induced sub graph < D > is connected. The connected domination number $\surd_c$(J(G)) of J(G) is the minimum cardinality of a connected dominating set.

Theorem 11 ; If $\surd_s$(J(G)) ≤ $\surd_c$(J(G)), then for any $\surd_s$-set D of J(G), V- D is a split dominating set of J(G).

Proof; Since D is minimal, by Theorem 4 V-D is a dominating set of J(G) and further it is split dominating set since < D > is disconnected.

Finally we obtain a
Nordhus-Gaddum type result [8]

Theorem 12 ; Let J(G) be a jump graph such that both J(G) and its complement J($\bar{G}$)are connected then

$$\surd_s(J(G)) + \surd_s(J(\bar{G})) \le p(p-3) \quad .....(10)$$

Proof; By (1) $\surd_s(J(G)) \le \alpha_0(J(G))$ Since J(G) and J($\bar{G}$ 0 are connected

Δ (J(G)) , Δ(J($\bar{G}$)) ≤ p – 1 This implies that β_0(J(G)) β_0(J($\bar{G}$)≥ 2. Hence

$\sqrt{s}(J(G) \le p - 2 \le 2(p-1) - p \le 2q - p$

Similarly $\surd_s(J(\bar{G})) \le 2\bar{q} - p$ Then

$\surd_s(J(G)) + \surd_s(J(\bar{G})) \le 2(q + \bar{q}) - 2p \le$
$p(p-1) - 2p \le p(p-3)$

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