

$(1, \alpha)$ – Derivations in Prime Γ – near Rings

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Abstract The purpose of this paper is to investigate $(1, \alpha)$ – derivations satisfying certain differential identities on prime gamma near rings. Some well-known results characterizing commutativity of prime gamma near rings by derivations have been generalized.

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I. Introduction

The notion of a Γ – ring a concept more general than a ring was defined by Nobusawa [7]. As a generalization of near rings, Γ – rings were introduced by satyanarayana[8]. The derivation of a Γ – near ring has been introduced by Bell and Mason[2]. They obtained some basic properties of derivations in Γ – near ring. The recent literature contains numerous results on commutativity in prime and semiprime rings admitting suitably constrained derivations and generalized derivations, and several authors have proved comparable results on near-rings. Some of our results, which deal with conditions on $(1, \alpha)$ – derivations, extend earlier commutativity results involving similar conditions on derivations and semi-derivations.

II. PRELIMINARIES

Throughout this paper, M stands for a zero symmetric right Γ – near ring.

Definition 2.1 [3] A Γ – near ring is a triple $(M, +, \Gamma)$, where

1. $(M, +)$ is a (not necessarily abelian) group
2. Γ is a non- empty set of binary operations on M such that $(M, +, \gamma)$ is a near ring for each $\gamma \in \Gamma$.
3. $(x\beta y)\gamma z = x\beta(y\gamma z)$ for all $x, y, z \in M$ and $\beta, \gamma \in \Gamma$..

Definition 2.2 [10] A Γ – near ring M is said to be **semiprime** Γ – near ring if $x\Gamma M\Gamma x = \{0\}$ for $x \in M$ implies $x = 0$ and **prime** Γ – near ring if $x\Gamma M\Gamma y = \{0\}$ for $x, y \in M$ implies $x = 0$ or $y = 0$.

Definition 2.3 [3] Let N be a zero symmetric right Γ – near ring. An additive mapping $d: N \rightarrow N$ is said to be a **derivation** on N if $d(x\alpha y) = xad(y) + d(x)\alpha y$ for all $x, y \in N, \alpha \in \Gamma$.

Definition 2.4 [6] An additive endomorphism $d: M \rightarrow M$ of a Γ – near ring M is called a $\Gamma - (\alpha, \beta)$ – derivation on M if there exist two functions $\alpha, \beta: M \rightarrow M$ such that the following product holds:
 $d(x\gamma y) = d(x)\gamma\alpha(y) + \beta(x)\gamma d(y)$ for all $x, y \in M, \gamma \in \Gamma$.

In the previous definition if we take $\beta = 1$ then we get $\Gamma - (\alpha, 1)$ – **derivation**. Similarly if we take $\alpha = 1$ and $\beta = \alpha$ we get $\Gamma - (1, \alpha)$ – **derivation**.

An additive mapping $d: M \rightarrow M$ is called a **two sided $\Gamma - \alpha$ – derivation** if d is a $\Gamma - (\alpha, 1)$ – **derivation** as well as $\Gamma - (1, \alpha)$ – **derivation**. Throughout this paper we denote $\Gamma - (1, \alpha)$ derivation as $(1, \alpha)$ derivation.

Definition 2.5 [5] The symbol C will denote the **multiplicative centre** of M ,
i.e $C = \{ x \in M: x\gamma m = m\gamma x \text{ for all } x \in M, \gamma \in \Gamma \}$

Definition 2.6 [4] A Γ – near ring N is called a **2-torsion-free** if $(N, +)$ has no elements of order 2.

Definition 2.7 [1] For any $x, y \in M$ and $\alpha \in \Gamma$, the notations $[x, y]_\alpha$ and $(x \circ y)_\alpha$ will denote $x\alpha y - y\alpha x$ and $x\alpha y + y\alpha x$, respectively.

For all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, the following basic commutator identities hold:

- (i) $[x\beta y, z]_{\alpha} = x\beta[y, z]_{\alpha} + [x, z]_{\alpha}\beta y + x\beta z\alpha y - x\alpha z\beta y$,
- (ii) $[x, y\beta z]_{\alpha} = y\beta[x, z]_{\alpha} + [x, y]_{\alpha}\beta z + y\alpha x\beta z - y\alpha x\beta z$,
- (iii) $(x\beta y \circ z)_{\alpha} = (x \circ z)_{\alpha}\beta y + \beta[y, z]_{\alpha} + x\alpha z\beta y - x\beta z\alpha y$, and
- (iv) $(x \circ y\beta z)_{\alpha} = y\beta(x \circ z)_{\alpha} + [x, y]_{\alpha}\beta z + y\beta x\alpha z - y\alpha x\beta z$. for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.8[9] A Γ – near ring N is said to be **sub-commutative** if $\alpha\gamma N = N\gamma\alpha$ for all $\alpha \in N$ and for all $\gamma \in \Gamma$.

III.(1, α) – derivations in prime Γ – near rings

In this section, we investigate some results of Γ – near rings satisfying certain differential identities on prime Γ – near rings.

Lemma 3.1 Let d be a $(1, \alpha)$ – derivation. Then M satisfies the following partial distributive law $\alpha(x)\mu(d(y)\gamma t + \alpha(y)\gamma d(t)) = \alpha(x)\mu d(y)\gamma t + \alpha(x)\mu\alpha(y)\gamma d(t)$ for all $\gamma, \mu \in \Gamma, x, y, t \in M$.

Proof

By the definition of d we have

$$d(x\mu y\gamma t) = d(x)\mu y\gamma t + \alpha(x)\mu d(y\gamma t) = d(x)\mu y\gamma t + \alpha(x)\mu(d(y)\gamma t + \alpha(y)\gamma d(t)) \quad (1)$$

for all $x, y, t \in M$ and $\mu, \gamma \in \Gamma$.

$$\begin{aligned} \text{On the other hand, } d(x\mu y\gamma t) &= d((x\mu y)\gamma t) \\ &= d(x)\mu y\gamma t + \alpha(x)\mu d(y)\gamma t + \alpha(x)\mu\alpha(y)\gamma d(t) \end{aligned} \quad (2)$$

for all $x, y, t \in M$ and $\mu, \gamma \in \Gamma$. Comparing (1) and (2) we have

$$\alpha(x)\mu(d(y)\gamma t + \alpha(y)\gamma d(t)) = \alpha(x)\mu d(y)\gamma t + \alpha(x)\mu\alpha(y)\gamma d(t)$$

Lemma 3.2

Let M be a Γ – near ring. If M admits an additive mapping d , then the following statements are equivalent:

- (i) d is a $(1, \alpha)$ – derivation
- (ii) $d(x\gamma y) = \alpha(x)\gamma d(y) + d(x)\gamma y$ for all $x, y \in M$ and $\gamma \in \Gamma$

Proof

(i) $\Rightarrow$ (ii)

Since d is a $(1, \alpha)$ – derivation for all $x, y \in M$ we get,

$$d((x+x)\gamma y) = d(x)\gamma y + d(x)\gamma y + \alpha(x)\gamma d(y) + \alpha(x)\gamma d(y)$$

for all $x, y \in M$ and $\gamma \in \Gamma$

$$d((x+x)\gamma y) = d(x)\gamma y + \alpha(x)\gamma d(y) + d(x)\gamma y + \alpha(x)\gamma d(y)$$

for all $x, y \in M$ and $\gamma \in \Gamma$

Comparing two expressions of $d((x+x)\gamma y)$ we get

$$d(x\gamma y) = \alpha(x)\gamma d(y) + d(x)\gamma y$$

Analogously, we can prove the other implication.

Theorem 3.3

Let M be a prime Γ – near ring satisfying $a\alpha b\beta c = a\beta b\alpha c$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ and d be a non-zero $(1, \alpha)$ – derivation associated with a homomorphism α . Then the following assertions are equivalent

- (i) $d(M) \subseteq C$
- (ii) $d([x, y]_{\gamma}) = 0$ for all $x, y \in M$ and $\gamma \in \Gamma$
- (iii) M is commutative .

Proof

(i) $\Rightarrow$ (iii)

Assume that $d(M) \subseteq C$. Then

$$d(x\gamma y)\mu\alpha(t) = \alpha(t)\mu d(x\gamma y) \text{ for all } t, x, y \in M \text{ and } \gamma, \mu \in \Gamma$$

Using Lemma 3.1, we get

$$d(y)\gamma\alpha(x\mu t) + d(x)\gamma y\mu\alpha(t) = \alpha(t)\mu\alpha(x)\gamma d(y) + \alpha(t)\mu d(x)\gamma y \quad (3)$$

Replacing y by $d(x)$ in (3) we get

$$\alpha(x\mu t)\gamma d^2(x) = \alpha(t)\mu \alpha(x)\gamma d^2(x) \\ \Rightarrow (\alpha(x\mu t) - \alpha(t)\mu \alpha(x))\gamma d^2(x) = 0 \text{ for all } t, x, y \in M \text{ and } \gamma, \mu \in \Gamma \quad (4)$$

If $\alpha(x\mu t) = \alpha(t)\mu \alpha(x)$, a contradiction to the hypothesis.

Therefore consider the case $d^2(x) = 0$

Suppose $d^2(x) = 0$ for all $x \in M$. Then by the definition of d we have

$$d(d(xy\gamma y)) = d^2(x)\gamma y + \alpha(d(x))\gamma d(y) + d(\alpha(x))\gamma d(y) + \alpha(\alpha(x))\gamma d^2(y) \\ \Rightarrow \alpha(d(x))\gamma d(y) + d(\alpha(x))\gamma d(y) = 0 \\ \Rightarrow \alpha(d(x))\gamma d(y) = d(-\alpha(x))\gamma d(y) \quad (5)$$

Replace $-\alpha(x)$ by x we get

$$\alpha(d(x))\gamma d(y) = d(x)\gamma d(y) \\ \Rightarrow (\alpha(d(x)) - d(x))\gamma d(y) = 0 \\ \Rightarrow (\alpha(d(x)) - d(x)) = 0 \text{ or } d(y) = 0 \quad (6)$$

Since $d \neq 0$, $\alpha(d(x)) = d(x)$

In this case substituting $x\gamma y$ for x , and using the Lemma 3.2 we have

$$\alpha(d(x\gamma y)) = d(x\gamma y) \\ \Rightarrow \alpha(d(x)\gamma y) + \alpha(\alpha(x))\gamma d(y) = \alpha(x)\gamma d(y) + d(x)\gamma y$$

Replace $\alpha(d(x))$ by $d(x)$

$$d(x)\gamma y + \alpha(\alpha(x))\gamma d(y) = \alpha(x)\gamma d(y) + d(x)\gamma y \\ \Rightarrow d(x)\gamma y + \alpha^2(x)\gamma d(y) = \alpha(x)\gamma d(y) + d(x)\gamma y \\ \Rightarrow (\alpha^2(x) - \alpha(x))\gamma d(y) = 0 \text{ for all } \gamma \in \Gamma \quad (7)$$

Since $d \neq 0$, $\alpha^2(x) = \alpha(x) \Rightarrow \alpha = Id_M$

Since $(M) \subseteq C$, $x\mu d(y\gamma t) = d(y\gamma t)\mu x$

By the above expression and by the Lemma 3.1 we have

$$x\mu d(y)\gamma t + x\mu \alpha(y)\gamma d(t) = d(y)\gamma t\mu x + \alpha(y)\gamma d(t)\mu x \quad (8)$$

Substituting x for t and $\alpha(y)$ for y in (8) we get

$$x\mu y\gamma d(x) = y\mu x\gamma d(x) \\ \Rightarrow [x, y]_\mu \gamma d(x) = 0 \quad (9)$$

Since M is prime,

$$[x, y]_\mu = 0 \text{ (or) } d(x) = 0$$

Since $d \neq 0$, $[x, y]_\mu = 0 \Rightarrow M$ is commutative

(ii) $\Rightarrow$ (iii)

Assume that

$$d([x, y]_\gamma) = 0 \text{ for all } x, y \in M \text{ and } \gamma \in \Gamma \quad (10)$$

Substituting $x\mu y$ for x in (10) and using $[x\mu y, y]_\gamma = [x, y]_\gamma \mu$

$$\alpha([x, y]_\gamma)\mu d(y) = 0 \Rightarrow \alpha(x)\gamma \alpha(y)\mu d(y) = \alpha(y)\gamma \alpha(x)\mu d(y) \\ \Rightarrow x_1\gamma y_1\mu d(y) = y_1\gamma x_1\mu d(y) \text{ for all } x_1, y_1 \in M \text{ and } \gamma, \mu \in \Gamma \quad (11)$$

Replace x_1 by $x\eta y_1$ in (11), we get

$$(x\eta y_1)\gamma y_1\mu d(y) = y_1\gamma x\eta y_1\mu d(y) \\ \Rightarrow [x, y_1]_\gamma \gamma \mu d(y) = 0 \text{ for all } x, y, y_1 \in M$$

$$\Rightarrow [x, y_1]_\gamma = 0 \text{ or } d(y) = 0 \text{ for all } x, y \in M$$

If $d(y) = 0$ for all $y \in M$ then $d = 0$, a contradiction.

Hence $[x, y_1]_\gamma = 0 \Rightarrow x\gamma y_1 = y_1\gamma x$ for all $x, y_1 \in M$

Therefore M is commutative.

(iii) $\Rightarrow$ (i)

Since M is a commutative, $M = C$ Since $d(M) \subseteq M = C$, $d(M) \subseteq C$

(iii) $\Rightarrow$ (ii)

Since M is commutative for all $x, y \in M$ and $\gamma \in \Gamma$

$$x\gamma y = y\gamma x \Rightarrow d(x\gamma y) = d(y\gamma x) \Rightarrow d([x, y]_\gamma) = 0$$

Theorem 3.4

Let M be a Prime Γ – near ring. If M admits a $(1, \alpha)$ – derivation d associated with a homomorphism α such that $d([x, y]_\mu) = [x, y]_\mu$ for all $x, y \in M, \mu \in \Gamma$ then M is commutative.

Proof

Suppose that $d([x, y]_\mu) = [x, y]_\mu$ for all $x, y \in M$ (12)

Substituting $y\gamma x$ for y in (12), one can easily verify that $d([x, y]_\mu)\gamma x + \alpha([x, y]_\mu)\gamma d(x) = [x, y]_\mu\gamma x$ for all $x, y \in M$ and $\gamma \in \Gamma$. (13)

Replace x by $[x, t]_\gamma$ in (13) we obtain

$$\begin{aligned} \alpha([x, t]_\gamma)\mu y\gamma[x, t]_\gamma &= y\mu[x, t]_\gamma\gamma[x, t]_\gamma \text{ for all } x, y, t \in M, \gamma, \mu \in \Gamma \\ \Rightarrow \alpha([x, t]_\gamma)\mu y_1\gamma[x, t]_\gamma &= y_1\mu\alpha([x, t]_\gamma)\gamma[x, t]_\gamma \end{aligned} \quad (14)$$

Substituting $z\eta y$ for y_1 in (14) we get

$$[\alpha([x, t]_\gamma), z]_\mu \Gamma M \Gamma [x, t]_\gamma = 0 \quad (15)$$

By the primeness of M , (15) implies

$$\alpha([x, t]_\gamma) \in C \text{ or } [x, t]_\gamma = 0 \text{ for all } x, t \in M, \gamma \in \Gamma$$

If $\alpha([x, t]_\gamma) \in C$ for all $x, t \in M, \gamma \in \Gamma$ then replacing t by $t\mu x$ and using the primeness of M , we find that $[x, t]_\gamma = 0$. Therefore M is a commutative ring.

Assume that there exist $x, t \in M$ such that $\alpha([x, t]_\gamma) \notin C$. In particular $\alpha([x, t]_\gamma) \neq 0$ and $[x, t]_\gamma = 0$ so that $d([x, t]_\gamma\mu z\eta y) = [x, t]_\gamma\mu z\eta y$ for all $z, y \in M$ and $\gamma, \mu \in \Gamma$ (16)

On the other hand

$$\begin{aligned} d([x, t]_\gamma\mu z\eta y) &= d([x, t]_\gamma\mu z)\eta y \\ &= ([x, t]_\gamma\mu z)\eta y + \alpha([x, t]_\gamma\mu z)\eta d(y) \end{aligned} \quad (17)$$

Comparing (16) and (17) we get

$$[x, t]_\gamma\mu z\eta y = ([x, t]_\gamma\mu z)\eta y + \alpha([x, t]_\gamma\mu z)\eta d(y) \quad (18)$$

Substituting $[u, v]_\gamma$ for y in (18), we get

$$\begin{aligned} \alpha([x, t]_\gamma)\eta\alpha(z)\mu d([u, v]_\gamma) &= 0 \\ \Rightarrow \alpha([x, t]_\gamma)\Gamma M \Gamma [u, v]_\gamma &= 0 \end{aligned} \quad (19)$$

By the primeness of M , $[u, v]_\gamma = 0$ for all $u, v \in M$

$\Rightarrow M$ is commutative.

Theorem 3.5

Let M be a 2- torsion free prime Γ – near ring. There is no non zero $(1, \alpha)$ – derivation d associated with a homomorphism α such that $d(x \circ y)_\mu = 0$ for all $x, y \in M$ and $\mu \in \Gamma$.

Proof

Assume that M admits a non zero $(1, \alpha)$ – derivation d such that

$$d(x \circ y)_\mu = 0 \text{ for all } x, y \in M. \quad (20)$$

Replace x by $x\gamma y$ in (20) we get

$$\begin{aligned} \alpha(x \circ y)_\mu\gamma d(y) &= 0 \\ \Rightarrow x_1\mu y_1\gamma d(y) + y_1\mu x_1\gamma d(y) &= 0 \end{aligned} \quad (21)$$

Substituting $z\eta x_1$ for x_1 and $-y_1$ for y_1 in (21) we obtain

$$\begin{aligned} z\eta x_1 \mu (-y_1) \gamma d(y) &= y_1 \mu z \eta x_1 \gamma d(y) \\ \Rightarrow [y_1, z]_{\eta} \Gamma M \Gamma d(y) &= 0 \text{ for all } y_1, z \in C \\ \Rightarrow y_1 \in C \text{ or } d(y) &= 0 \end{aligned}$$

Let $y_1 \in C$ since M is a 2- torsion free, and using (20)

$$d(x) \gamma y_1 + \alpha(x) \gamma d(y_1) = 0 \text{ for all } x, y_1 \in M \text{ and } \gamma \in \Gamma \quad (22)$$

Substituting $(x \circ z)_{\mu}$ for x in (22) we get

$$(x_1 \circ z_1)_{\mu} \gamma d(y_1) = 0 \quad (23)$$

Replace z_1 by $t\eta z$ and x_1 by $-x$ in (23), one can easily see that

$$\begin{aligned} [-x, t]_{\mu} \Gamma M \Gamma d(y) &= 0 \\ \Rightarrow [-x, t]_{\mu} &= 0 \text{ or } d(y) = 0 \text{ for all } x, y, t \in M \end{aligned} \quad (24)$$

We get either $d(y) = 0$ or M is a commutative ring .

But in the latter case, our hypothesis reduces to

$$d(x\gamma y) = 0 \text{ for all } x, y \in M \text{ and } \gamma \in \Gamma$$

Replacing x by $x\mu z$ in above equation, we get

$$\begin{aligned} x_1 \mu z_1 \gamma d(y) &= 0 \text{ for all } x_1, z_1 \in M \text{ and } \gamma, \mu \in \Gamma \\ \Rightarrow d &= 0 \end{aligned}$$

Hence in both cases, we have $d = 0$ a contradiction.

Theorem 3.6

Let M be a subcommutative 2-torsion free prime Γ – near ring admitting a two sided α – derivation d associated with homomorphism α . If $d(x \circ y)_{\mu} = (x \circ y)_{\mu}$ for all $x, y \in M$ and $\gamma \in \Gamma$ then $d = -\alpha + Id$.

Proof

We assume that

$$d(x \circ y)_{\mu} = (x \circ y)_{\mu} \quad (25)$$

Replace y by $x\eta y$ in (25), we get

$$\begin{aligned} d(x \circ x\eta y)_{\mu} &= (x \circ x\eta y)_{\mu} \\ d\left((x \circ y)_{\mu} \eta x\right) &= d(x \circ y)_{\mu} \eta x + \alpha(x \circ y)_{\mu} \eta d(x) \\ &= (x \circ y)_{\mu} \eta x \text{ for all } x, y \in M \text{ and } \gamma, \eta \in \Gamma \end{aligned} \quad (26)$$

Substituting $(x \circ t)_{\gamma}$ for x in (26), we obtain

$$\alpha((x \circ t)_\gamma \circ y)_\mu \eta(x \circ t)_\gamma = 0$$

$$\Rightarrow \alpha((x \circ t)_\gamma) \mu y_1 \eta(x \circ t)_\gamma + y_1 \mu \alpha((x \circ t)_\gamma) \eta(x \circ t)_\gamma = 0 \quad (27)$$

Putting $y_1 \gamma z$ instead of y_1 in (27), we find that

$$(-x \circ t)_\gamma \gamma z \eta y_1 \mu \alpha((x \circ t)_\gamma) = z \gamma (-x \circ t)_\gamma \eta y_1 \mu \alpha((x \circ t)_\gamma)$$

$$\Rightarrow [z, (x \circ t)_\gamma]_\gamma \Gamma M \Gamma \alpha((x \circ t)_\gamma) = 0 \quad (28)$$

Let $u = (x \circ t)_\gamma \in C$

By the hypothesis,

$$d(u \circ k)_\mu = (u \circ k)_\mu \text{ for all } u, k \in M$$

$$\Rightarrow \alpha(u) \mu d(k) + d(u) \mu k + \alpha(u) \mu d(k) = k \mu u \text{ for all } \mu \in \Gamma \quad (29)$$

By using Lemma 3.2 we get,

$$d(u \mu k) + \alpha(u) \mu d(k) = k \mu u \Rightarrow 2\alpha(u) \mu d(k) = 0$$

Since M is 2-torsion free prime Γ -near ring

$$\alpha(u) \mu d(k) = 0 \text{ for all } u, k \in M, \mu \in \Gamma \quad (30)$$

$$\Rightarrow \alpha(u) = 0$$

Suppose that $\alpha((x \circ t)_\gamma) = 0$ then set $u = (x \circ t)_\gamma$ we get

$$d(u \gamma k \mu v) = d(u \gamma (k \mu v))$$

$$= u \gamma k \mu v \text{ for all } k \in M \text{ and } \gamma \in \Gamma$$

On other hand

$$d(u \gamma k \mu v) = u \gamma k \mu \alpha(v) + u \gamma k \mu d(v)$$

Comparing the last two expressions we get

$$(\alpha(v) + d(v) - v) \mu u \gamma k = 0 \text{ for all } k, v \in M, \gamma, \mu \in \Gamma$$

$$\Rightarrow (\alpha(v) + d(v) - v) \Gamma M \Gamma u = 0$$

By the primeness of M , $u = 0$ or $\alpha(v) + d(v) - v = 0$ for all $v \in M$

If $u = 0$ then $(x \circ t)_\gamma = 0$ for all $y, t \in M$

Therefore $(x \circ t)_\gamma = 0$ or $\alpha(v) + d(v) - v = 0$ for all $v \in M$

Now assume that $(x \circ t)_\gamma = 0$ for all $y, t \in M$

Since M is a 2-torsion free, $x \gamma t = 0$ for all $x, t \in M$

And hence $(x \circ t)_\gamma = 0 \Rightarrow x \gamma t + t \gamma x = 0$

$$\Rightarrow (xyt + tyx)\mu t = 0$$

$$\Rightarrow t\gamma y\mu t = 0$$

$$\Rightarrow t\Gamma M\Gamma t = 0 \text{ for all } t \in M \text{ which forces that } M = \{0\} \text{ a contradiction. Consequently}$$

we have

$$d(v) = -\alpha(v) + v \Rightarrow d = -\alpha + Id.$$

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