Irreducible Polynomial on Finite Field

Poonam^{#1},Shilpa Aggrwal^{#2} ^{#1} Department of Mathematics, Govt. College,Hisar. ^{#2} Department of Mathematics, Govt. College,Hisar.

Abstract

The aim of this paper is to study about irreducible polynomials over finite field and to find irreducible polynomials of degree 2 and 3 over the field Z_2 and Z_3 .

Key Words: Irreducible, Polynomial, Reducible, Field.

Introduction

Irreducible polynomial plays very important role in field theory. An irreducible polynomial is a non-constant polynomial that can't factorised into the product of two non constant polynomials. A polynomial that is irreducible over any field containing coefficients is absolutely irreducible.

Notations

$$\begin{split} &Z_2 = \{0, 1\} \text{ is field under addition modulo 2 and} \\ & \text{multiplication modulo 2.} \\ & Z_2 \left[x\right] \text{ is a polynomial ring.} \\ & Z_{3=} \left\{0, 1, 2\right\} \text{ is a field under addition modulo 3 and multiplication modulo 3.} \end{split}$$

 Z_3 [x] is polynomial ring.

Irreducible Polynomial

Let F be a field. Let $f(x)\epsilon F[x]$ be a non zero, non-constant polynomial. Then f(x) is said to be irreducible if it can not be factorised into the product of two non-constant polynomial with coefficients in F.

Theorem - Reducibility test for degree 2 and 3. Let f be a field. If $f(x)\epsilon F[x]$ and deg f(x) = 2 or 3, then f(x) is reducible over F if and only if f(x) has a zero in field F.

2	Now we calculate number of irreducible polynomials of degree 2 over \mathbf{Z}_2
$ax^2 + b$	$bx + C$ over Z_2
	Clearly $a \neq 0$, so $a = 1$, equation becomes
	$x^2 + bx + C$
	Now we have two choice for c and two for b.
Case – I	
Case – II	If $C = 1$ then polynomial becomes
	$x^2 + bx + 1$ Now we have two choice for b
	If $b = 0$, then polynomial $x^2 + 1$ is reducible over Z_2 .
	If $b = 1$, then polynomial $x^2 + x + 1$ is irreducible over Z_2 .
	Now we calculate number of irreducible polynomials of degree 2 over \mathbb{Z}_3
	Now $ax^2 + bx + C$ over Z_3
	Clearly $a \neq 1$, so a have two choices.
Case – I	Clearly a $\neq 1$, so a have two choices. If a = 1, then polynomial becomes $x^2 + bx + C$, here C have three choices.
Sub Case-(i) If	$C = 0$, then polynomial x^2+bx is reducible over
Z3	
Sub Case-(ii) If	$C = 1$, then polynomial becomes $x^2 + bx + 1$
	Now b has three choices.
	If $b = 0$, then polynomial $x^2 + 1$ is irreducible over
Z_3 .	

	If $b = 1$, then polynomial $x^2 + x + 1$ is reducible
	$\begin{array}{l} \text{Yer } Z_3.\\ \text{If } b=2, \text{ then polynomial} \\ \text{Yer } Z_3. \end{array} \qquad \qquad x^2+2x+1 \text{ is reducible} \\ \end{array}$
Sub Case-(iii)	If C = 2, then polynomial becomes $x^2 + bx + 2$ Now b has three choices. If b = 0, then polynomial x^2+bx+2 is reducible Over Z ₃ .
	If $b = 1$, then polynomial x^2+x+2 is irreducible over Z_3 .
	If $b = 2$, then polynomial x^2+2x+1 is irreducible over Z_3 .
Case – II	If $a = 2$, then the polynomial becomes $2x^2 + bx + C$. Now C have again three choices.
Sub Case-(i)	If $C = 0$, then polynomial $2x^2 + bx$, is reducible over Z_3 .
Sub Case-(ii)	C = 1, then polynomial becomes $2x^2 + bx + 1$, here b has three choices. If b = 0, then polynomial $2x^2 + 1$ is reducible over Z ₃ . If b = 1, then polynomial $2x^2+x+1$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^2 + 2x + 1$ is irreducible over Z ₃ .
Sub Case-(iii)	C = 2, then polynomial becomes $2x^2 + bx + 2 = 0$, here b has three choices. If b = 0, then polynomial $2x^2 + 2$ is irreducible over Z_3 . If b = 1, then polynomial $2x^2 + x + 2$ is reducible over Z_3 . If b = 2, then polynomial $2x^2 + 2x + 2$ is reducible over Z_3 . Now we calculate irreducible polynomials of degree 3 over Z_2 . Now $ax^3 + bx^2 + cx + d$ over Z_2 Clearly $a \neq 0$, so $a = 1$, then the polynomial becomes $x^3 + bx^2 + cn + d$. Here d have two choices.
Case – I	If $d = 0$, then polynomial $x^3 + bx^2 + cx$, is reducible over Z_2 .
Case – II	If d = 1, then polynomial becomes $x^3 + bx^2 + cx + 1$ here C has two choices.
Sub Case (i) If If $b = 0$, then p	C = 0, then polynomial becomes $x^3 + bx^2 + 1$, here b has two choices. olynomial $x^3 + 1$ is reducible over Z_2 . If b = 1, then polynomial $x^3 + x + 1$ is irreducible over Z_2 .
Sub Case (ii)	If C =1, then polynomial $x^3 + bx^2 + x + 1$, here b has two choices. If b = 0, then polynomial $x^3 + x + 1$ is reducible over Z ₂ .
	If $b = 1$, then polynomial $x^3 + x^2 + x + 1$ is irreducible over Z_2 .
	Now we calculate irreducible polynomials of degree 3 over Z ₃ . Now $ax^3 + bx^2 + cx + d = 0$ over Z ₃ Clearly $a \neq 0$ clearly a has two choices.
Case – I	If $a = 1$, then polynomial becomes $x^3 + bx^2 + cx + d$, here d has three choices.
Sub Case (i)	If $d = 0$, then polynomial becomes $x^3 + bx^2 + cx$ reducible over Z_3
Sub Case (ii)	If $d = 1$, then polynomial becomes $x^3 + bx^2+cx+1$ here C has three choices.
(A)	If C = 0, then polynomial becomes $x^3 + bx^2 + 1$, here b has three choices. If b = 0, then polynomial $x^3 + 1$ is reducible over Z ₃ .

(D)		$x^{3} + x^{2} + 1$ is reducible over Z_{3} . $x^{3} + 2x^{2} + 1$ is irreducible over Z_{3}
(B)	If $C = 1$ then polynomial b	becomes $x^3 + bx^2 + x + 1$, here b has three choices.
		x^3+x+1 is reducible over Z_3 .
	If $b = 1$, then polynomial	$x^3 + x^2 + x + 1$ is reducible over Z ₃ .
	If $b = 2$, then polynomial	$x^3 + 2x^2 + x + 1$ is irreducible over Z_3 .
(C) If $C = C$	2, then polynomial becomes	$x^3 + bx^2 + 2x + 1$, here b has three choices.
$\Pi C = 2$	If $b = 0$ then polynomial vectories	$x^{3} + 0x^{2} + 2x + 1$, here b has unce choices. $x_{3}^{3} + x_{4}^{2} + 2x + 1$ is irreducible over Z ₃ .
	If $b = 0$, then polynomial f . If $b = 1$, then polynomial f	$x^3 + x^2 + 2x + 1$ is irreducible over Z ₃ .
	If $b = 2$, then polynomial	$x^3 + 2x^2 + 2x + 1$ is reducible over Z_3 .
	f d = $\frac{2}{2}$, then polynomial	
	$+bx^{2}+cx+2$ here C has the	ree choices.
(A)	If $C = 0$ then polynomial	$x^3 + bx^2 + 2$, here b has three choices.
	If $b = 0$, then polynomial x	$x^{3} + 2$ is reducible over Z_{3} .
	If $b = 1$, then polynomial	$x^{3} + 2$ is reducible over Z_{3} . $x^{3} + x^{2} + 2$ is irreducible over Z_{3} .
	If $b = 2$, then polynomial	$x^3 + 2x^2 + 2$ is reducible over Z_3 .
(B)		3 1 2
	If $C = 1$, then polynomial x .	$x^{3} + bx^{2} + x + 2$, here b has three choices. $x^{3} + x + 2$ is reducible over Z ₃ .
	If $b = 0$, then polynomial f . If $b = 1$, then polynomial f	$x^3 + x^2 + x + 2$ is irreducible over Z_3 .
		$x^3 + 2x^2 + x + 2$ is reducible over Z_3 .
(C)		
	If $C = 2$, then polynomial b	ecomes $x^3 + bx^2 + 2x + 2$ here b has three choices.
	If $b = 0$, then polynomial 2 If $b = 1$ then polynomial	$x^{3} + 2x + 2$ is irreducible over Z ₃ . $x^{3} + x^{2} + 2x + 2$ is reducible over Z ₃ .
	If $b = 1$, then polynomial 2 If $b = 2$ then polynomial	$x^3 + 2x^2 + 2x + 2$ is irreducible over Z_3 . $x^3 + 2x^2 + 2x + 2$ is irreducible over Z_3 .
Case – II	If $a = 2$, then poly becomes	$s^{2} + 2x^{2} + 2x^{2} + 2$ is include ble over 23. $s^{2} + bx^{2} + cx + d = 0$, here d has three choices.
	a = 2, then poly. Decomes	$5 \Delta A + 0A + 0A + 0 = 0$. Here u has unce choices.
Sub Case (i)	If $d = 0$, then polynomial 2	
Sub Case (i) red	If $d = 0$, then polynomial 2 lucible over Z_3	$2x^3 + bx^2 + cx$ is
Sub Case (i)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be	$2x^3 + bx^2 + cx$ is ecomes $2x^3 + bx^2 + cx + 1$
Sub Case (i) red Sub Case (ii)	If $d = 0$, then polynomial 2 lucible over Z_3	$2x^3 + bx^2 + cx$ is ecomes $2x^3 + bx^2 + cx + 1$
Sub Case (i) red	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices.
Sub Case (i) red Sub Case (ii) (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial be 0, then polynomial $2x^3 + 1$ is over Z_3 .	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. is reducible
Sub Case (i) red Sub Case (ii) (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial b D, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z ₃ .
Sub Case (i) red Sub Case (ii) (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial b D, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. is reducible
Sub Case (i) red Sub Case (ii) (A) If b = 0	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial b D, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z ₃ .
Sub Case (i) red Sub Case (ii) (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial be 0, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $C = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. is reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices.
Sub Case (i) red Sub Case (ii) (A) If b = 0	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $C = 1$, then polynomial 2 If $b = 0$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x + 1$ is irreducible over Z_{3} .
Sub Case (i) red Sub Case (ii) (A) If b = 0	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} .
Sub Case (i) red Sub Case (ii) (A) If b = ((B)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x + 1$ is irreducible over Z_{3} .
Sub Case (i) red Sub Case (ii) (A) If b = 0	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2	$2x^{3} + bx^{2} + cx \text{ is}$ ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + bx^{2} + x + 1, \text{ here b has three choices.}$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$
Sub Case (i) red Sub Case (ii) (A) If b = ((B)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial be 0, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 2$, then polynomial 2 If $C = 2$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + 2x^{2} + x + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + x + 1$ is reducible over Z_{3} .
Sub Case (i) red Sub Case (ii) (A) If b = ((B)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial be 0, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + x^{2} + x + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + x + 1$ is reducible over Z_{3} . $2x^{3} + bx^{2} + 2x + 1$ is reducible over Z_{3} . $2x^{3} + bx^{2} + 2x + 1$ is reducible over Z_{3} .
Sub Case (i) red Sub Case (ii) (A) If b = ((B)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial be 0, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + 1$ is irreducible over Z_{3} . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + x^{2} + x + 1$ is irreducible over Z_{3} . $2x^{3} + x^{2} + x + 1$ is reducible over Z_{3} . $2x^{3} + 2x^{2} + x + 1$ is reducible over Z_{3} . $2x^{3} + bx^{2} + 2x + 1$ is reducible over Z_{3} .
Sub Case (i) red Sub Case (ii) (A) If b = ((B) (C) Sub Case (iii) If	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial be 0, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. ereducible $2x^{3} + x^{2} + 1$ is reducible over Z ₃ . $2x^{3} + 2x^{2} + 1$ is irreducible over Z ₃ . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x^{2} + x + 1$ is irreducible over Z ₃ . $2x^{3} + x^{2} + x + 1$ is irreducible over Z ₃ . $2x^{3} + x^{2} + x + 1$ is reducible over Z ₃ . $2x^{3} + 2x^{2} + x + 1$ is reducible over Z ₃ . $2x^{3} + bx^{2} + 2x + 1$ is reducible over Z ₃ . $2x^{3} + bx^{2} + 2x + 1$ is reducible over Z ₃ . $2x^{3} + x^{2} + 2x + 1$ is reducible over Z ₃ . $2x^{3} + x^{2} + 2x + 1$ is reducible over Z ₃ .
Sub Case (i) red Sub Case (ii) (A) If b = ((B) (C) Sub Case (iii) I 2x (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2	$2x^{3} + bx^{2} + cx$ is ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. reducible $2x^{3} + x^{2} + 1$ is reducible over Z ₃ . $2x^{3} + 2x^{2} + 1$ is irreducible over Z ₃ . $2x^{3} + bx^{2} + x + 1$, here b has three choices. $2x^{3} + x^{2} + x + 1$ is irreducible over Z ₃ . $2x^{3} + x^{2} + x + 1$ is irreducible over Z ₃ . $2x^{3} + x^{2} + x + 1$ is reducible over Z ₃ . $2x^{3} + 2x^{2} + x + 1$ is reducible over Z ₃ . $2x^{3} + bx^{2} + 2x + 1$, here b has 3 choices. $2x^{3} + 2x + 1$ is reducible over Z ₃ . $2x^{3} + 2x^{2} + 2x + 1$ is reducible over Z ₃ . $2x^{3} + 2x^{2} + 2x + 1$ is reducible over Z ₃ .
Sub Case (i) red Sub Case (ii) (A) If b = ((B) (C) Sub Case (iii) I 2x (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 I	$2x^{3} + bx^{2} + cx \text{ is}$ ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. reducible $2x^{3} + x^{2} + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + bx^{2} + x + 1, \text{ here b has three choices.}$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + 2x^{2} + x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + bx^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + bx^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$
Sub Case (i) red Sub Case (ii) (A) If b = ((B) (C) Sub Case (iii) I 2x (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three cl If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 0$, then polynomial 2 If $b = 0$, then polynomial 2	$2x^{3} + bx^{2} + cx \text{ is}$ ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. s reducible $2x^{3} + x^{2} + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + bx^{2} + x + 1, \text{ here b has three choices.}$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + 2x^{2} + x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + bx^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + bx^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$
Sub Case (i) red Sub Case (ii) (A) If b = ((B) (C) Sub Case (iii) I 2x (A)	If $d = 0$, then polynomial 2 lucible over Z_3 If $d = 1$, then polynomial be here C has three ch If $C = 0$, then polynomial $2x^3 + 1$ is over Z_3 . If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 2$, then polynomial 2 If $b = 1$, then polynomial 2 If $b = 1$, then polynomial 2	$2x^{3} + bx^{2} + cx \text{ is}$ ecomes $2x^{3}+bx^{2}+cx+1$ hoices. ecomes $2x^{3} + bx^{2} + 1$, here b has three choices. reducible $2x^{3} + x^{2} + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + bx^{2} + x + 1, \text{ here b has three choices.}$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + x^{2} + x + 1 \text{ is irreducible over } Z_{3}.$ $2x^{3} + 2x^{2} + x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + bx^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + bx^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is reducible over } Z_{3}.$ $2x^{3} + 2x^{2} + 2x + 1 \text{ is irreducible over } Z_{3}.$

If C =1, then polynomial becomes $2x^3 + bx^2 + x + 2$, here b has three choices. If b = 0, then polynomial $2x^3 + x + 2$ is irreducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + x + 2$ is irreducible over Z ₃ . (C) If C = 2, then polynomial becomes $2x^3 + bx^2 + 2x + 2$, here b has 3 choices. If b = 0, then polynomial $2x^3 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ .
If b = 1, then polynomial $2x^3 + x^2 + x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3+2x^2+x+2$ is irreducible over Z ₃ . (C) If C = 2, then polynomial becomes $2x^3 + bx^2 + 2x + 2$, here b has 3 choices. If b = 0, then polynomial $2x^3 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ .
(C) If b = 2, then polynomial $2x^3+2x^2+x+2$ is irreducible over Z ₃ . (C) If C = 2, then polynomial becomes $2x^3 + bx^2 + 2x + 2$, here b has 3 choices. If b = 0, then polynomial $2x^3 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . CONCLUSION Irreducible polynomial of degree 2 over Z ₂ is $x^2 + x + 1$
(C) If C = 2, then polynomial becomes $2x^3 + bx^2 + 2x + 2$, here b has 3 choices. If b = 0, then polynomial $2x^3 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . CONCLUSION Irreducible polynomial of degree 2 over Z ₂ is $x^2 + x + 1$
If C = 2, then polynomial becomes $2x^3 + bx^2 + 2x + 2$, here b has 3 choices. If b = 0, then polynomial $2x^3 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . CONCLUSION Irreducible polynomial of degree 2 over Z ₂ is $x^2 + x + 1$
If C = 2, then polynomial becomes $2x^3 + bx^2 + 2x + 2$, here b has 3 choices. If b = 0, then polynomial $2x^3 + 2x + 2$ is reducible over Z ₃ . If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . CONCLUSION Irreducible polynomial of degree 2 over Z ₂ is $x^2 + x + 1$
$\begin{array}{l} \text{If } b=0, \text{ then polynomial } 2x^3+2x+2 \text{ is reducible over } Z_3.\\\\ \text{If } b=1, \text{ then polynomial } 2x^3+x^2+2x+2 \text{ is reducible over } Z_3.\\\\ \text{If } b=2, \text{ then polynomial } 2x^3+2x^2+2x+2 \text{ is irreducible over } Z_3.\\\\ \end{array}$
If b = 1, then polynomial $2x^3 + x^2 + 2x + 2$ is reducible over Z ₃ . If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . CONCLUSION Irreducible polynomial of degree 2 over Z ₂ is $x^2 + x + 1$
If b = 2, then polynomial $2x^3 + 2x^2 + 2x + 2$ is irreducible over Z ₃ . CONCLUSION Irreducible polynomial of degree 2 over Z ₂ is $x^2 + x + 1$
CONCLUSION Irreducible polynomial of degree 2 over Z_2 is $x^2 + x + 1$
Irreducible polynomial of degree 2 over Z_2 is $x^2 + x + 1$
Irreducible polynomial of degree 2 over Z_2 is $x^2 + x + 1$
$x^2 + x + 1$
Irraducible polynomials of degree 2 over 7 are
Irreducible polynomials of degree 2 over Z_3 are
$x^{2} + 1$, $x^{2} + x + 2$, $x^{2} + 2x + 2$, $2x^{2} + x + 1$, $2x^{2} + 2x + 1$
Irreducible polynomials of degree 3 over Z_2 are
$x^3 + x^2 + 1$, $x^3 + x + 1$
Irreducible polynomials of degree 3 over Z_3 are
$x^{3} + 2x^{2} + 1, x^{3} + 2x^{2} + x + 1, x^{3} + 2x + 1, x^{3} + x^{2} + 1, x^{3} + x^{2} + 2, x^{3} + x^{2} + x + 2, x^{3} + 2x + 2, x^{$
$2x^{2} + 2x + 1$, $2x^{3} + 2x^{2} + 1$, $2x^{3} + x + 1$, $2x^{3} + x^{2} + x + 1$, $2x^{3} + 2x^{2} + 2x + 1$, $2x^{3} + x^{2} + 2x^{3} + 2x^{3} + 2x^{2} + 2x^{3} + 2x^{3} + 2x^{3} + 2x^{3} + 2x^{3} + 2x^{2$
$2x^{3} + x + 2$, $x^{3} + 2x^{3} 2x^{2} + x + 2$, $x^{3} + 2x^{3} + x^{2} + 2x + 2$,

Reference

- "Contemporary Abstract Algebra" by Joseph A. Gallian. ISBN 1305887859, 9781305887855 1)
- "A Course in Abstract Algebra" by V.K. Khanna & S.K. Bhambri, Second Edition, Vikas Publication House Pvt 2) Limmited,1999,070698675X,9780706986754.
- "A first Court in Abstract Algebra" (7th Edition) by J.B. Fraleigh. ISBN 13:9780201763904 "Abstract Algebra" (3rd Edition) by David S. Dummit, Richard M.Foote,ISBN 9780471433347 3)
- 4)
- 5) "Abstract Algebra" by I.N. Hersterin. ISBN-10:0471368792