Generalized Pre Closed Sets with Respect to an Ideal

[1]V.Senthilkumaran, [2]R.Krishnakumar, [3]Y.Palaniappan*

[1]*Associate Professor of Mathematics, Arignar Anna Government Arts College, Musiri, Tamilnadu, India.E.mail*
[2]*Assistant Professor of Mathematics,Urumu Dhanalakshmi College Kattur Trichy,Tamilnadu, India*
[3]*Associate Professor of Mathematics (Retd), Arignar Anna Government Arts College, Musiri, Tamilnadu, India.*

Abstract

An ideal on a set X is a non empty collection of subsets of X with heredity property which is also closed under finite unions. The concept of generalized closed sets was introduced by Levine. In this paper, we introduce and investigate the concept of pre generalized closed sets with respect to an ideal.

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1. Introduction

Indeed ideals are very important tools in General Topology. It was the works of Newcomb[9], Rancin[10], Samuels[11] and Hamlet and Jankovic (see[1,2,3,4,6]) which motivated the research in applying the topological ideals to generalize the most basic properties in General Topology. A non empty collection I of subsets on a topological space (X, τ) is called a topological ideal [7] if it satisfies the following two conditions:

1. If AϵI and B $\subset$ A implies BϵI (heredity)

2. If AϵI and BϵI, then A$\cup$B ϵI (finite additivity)

If A is a subset of a topological space(X, τ), cl(A) and int(A) denote the closure of A and interior of A respectively. In (1963), Levine [8] introduces the concept of generalized closed sets. This notion has been studied extensively in recent years by many topologists. A subset A of a topological space (X,τ) is said to be pre open if A$\subset$ int cl(A) and pre closed if cl int(A)$\subset$A. A subset A of a topologicl space(X, τ) is said to be semi open if A$\subset$cl int(A) and semi closed if int cl(A)$\subset$A. A subset A of a topological space (X, τ) is said to be generalized closed (briefly g closed) if cl(A)$\subset$U whenever A$\subset$U and U is open in X. A subset A of a topological space(X, τ) is said to be generalized pre closed (briefly gp closed) if pcl(A) $\subset$U whenever A$\subset$ U and U is open in(X,τ).

In this paper, we introduce and study the concept of gp closed sets with respect to an ideal, which is the extension of the concept of gp closed sets.

2. Generalized pre closed sets with respect to an ideal.

Definition 2.1: Let (X, τ) be a topological space and I be an ideal in X. A subset A of X is said to be generalized closed with respect to an ideal (briefly Ig closed) if and only if clA-UϵI whenever A$\subset$ U and U is open in X.

Definition 2.2: Let (X, τ) be a topological space and I be an ideal in X. A subset A of X is said to be generalized pre closed with respect to an ideal (briefly Igp closed) if and only if pcl(A)-UϵI whenever A$\subset$ U and U is open in X.

Remark 2.3: Every closed, pre closed, g closed set is Igp closed.

The converse of the above remark need not be true can be seen from the following examples.

Example 2.4: Let X={a,b,c}, τ ={φ,{a},{a,b},X}, I={φ,{b},{c},{b,c}}
{a} is Igp closed but not closed, pre closed or g closed.

Theorem 2.5: Every Ig closed set is Igp closed.
Proof: Let A be Ig closed in a topological space X. Let A⊂ U, where U is open. As A is Ig closed Cl(A)-U∊I. Hence pcl(A)-U∊I.

The converse of the above theorem need not be true can be seen from the following example.

Example 2.6: Let X={a,b,c}, τ={φ,{a},{a,b},X}, I={φ,{b}}
{b} is Igp closed but not Ig closed.

Theorem 2.7: If A is semi open and Igp closed, then A is Ig closed.
Proof: Let A⊂ U, where U is open. A is semi open. Hence pcl(A) = cl(A). Ais Igp closed. Hence pcl(A)-U∊I. This implies cl(A)-U∊I.

Intersection of two Igp closed sets need not be Igp closed can be seen from the following example.

Example 2.8: Let X={a,b,c}. τ ={φ,{a},X}. I ={φ,{b}}. {a,b}, {a,c} are Igp closed.
but{a} is not Igp closed.

Union of two Igp closed sets need not be Igp closed can be seen from the following example.

Example 2.9: Let X= {a,b,c}. τ ={φ,{a,b},X}. I ={φ,{b}}.
{a} and {b} are Igp closed but {a,b} is not Igp closed.

Theorem 2.10: Let (X,τ) be a topological space. If PC(X) =O(X), then every subset of X is Igp closed.

Proof: Let A⊂X be such that A⊂U, where U is open in X. U = pcl(U), A⊂U. So pcl(A)⊂pcl(U). pcl(A)-U⊂ pcl(U)-U = φ∊I. Hence A is Igp closed.

Remark 2.11: If PC(X) = O(X), then IgpC(X) = P(X), the power set of X.

Theorem 2.12: Let (X, τ) be a topological space. For p∊X, X-{p} is either open or Igp closed.
Proof: If X-{p} is not open, the only open set containing X-{p} is X. Hence pcl[X-{p}]-X∊I.
So X-{p} is Igp closed.

Definition 2.13: Let (X,τ) be a topological space. Then X is called Igp connected (Ig connected) if there does not exist a set A⊂X such that φ ≠ A ≠ X, which is Igp open and Igp closed (Ig open and Ig closed)

Theorem 2.14: Every Igp connected space is connected.

Proof: Let X be Igp connected, which is not connected. Then there exists a set A⊂X such that
Φ ≠ A ≠ X, which is both open and closed. Every open set is Igp open and every closed set is Igp closed. This implies a contradiction. Hence the assertion.
The converse of the above theorem need not be true can be seen from the following example.

Example 2.15: Let X={a,b,c}. τ = {φ,{a},{a,b},X}. I = {φ,{b},{c},{b,c}}.

(X,τ) is connected but not Igp connected as {a} is Igp open and Igp closed.

Theorem 2.16: Every Igp connected space is Ig connected.
The converse of the above theorem need not be true can be seen from the following example.

Example 2.17: Let X and τ be as in the above example. I = {φ}.
X is Ig connected but not Igp connected as {b} is Igp open and Igp closed.

3. Igp closure.

Definition 3.1: Let (X,τ) be a topological space. Let A⊂X.
Then Igp-cl(A) =⋂{F:F is Igp closed and A⊂F}
Igp-int(A) = ∪{G: G is Igp open and G⊂A}

Lemma 3.2: Let A and B be subsets of (X,τ). Then the following are obvious.
1)Igp-cl(X) = X and Igp-cl(φ) = φ
2) If A⊂B then Igp-cl(A)⊂Igp-cl(B)
3) A⊂Igp-cl(A)
4) Igp-cl(A) = Igp-cl(Igp-cl(A))
5) Igp-cl(A)∪Igp-cl(B) ⊂ Igp-cl(A∪B)

6)Igp-cl(A⋂B) ⊂ Igp-cl(A)⋂Igp-cl(B)

Remark 3.3: Let A be any Igp closed set in (X,τ). Then Igp-cl(A) =A. But the converse need not be true can be seen from the following example.

Example 3.4: Let (X,τ) and I be as in the previous example. Let A ={a}. Igp-cl(A) = {a}=A, which is not Igp closed.

Theorem 3.5: If the family of all Igp closed subsets of (X,τ) is closed under finite unions, then
Igp-cl(A∪B) = A∪B = Igp-cl(A)∪Igp-cl(B).

Proof: A and B are Igp closed. Therefore A∪B is Igp closed.
Igp-cl(A∪B) = A∪B = Igp-cl(A)∪Igp-cl(B).

Theorem 3.6: If PC(X) is closed under finite unions, then IGPC(X) is closed under finite unions.
Proof: Let A,B∈IGPC(X). PC(X) is closed under finite unions. So pcl(A∪B) = pcl(A)∪pcl(B).
Let A∪B⊂U, where U is open in X. Then pcl(A)-U∈I, pcl(B)-U∈I.
pcl(A∪B)-U=(pcl(A)-U)∪(pcl(B)-U)∈I. So A∪B is Igp closed.

Corollary 3.7: If PO(X) is closed under finite intersection, then IGPO(X) is closed under finite intersection.

Theorem 3.8: Let A be an Igp closed set of (X,τ) and A⊂B⊂pcl(A), then Bis Igp closed in X.

Proof: Let B⊂U, where U is open in X. B⊂pcl(A). pcl(B)⊂pcl(pcl(A)) = pcl(A). A is Igp closed and A⊂U. Therefore pcl(A)-U∈I. Hence pcl(B)-U∈I. This implies B is Igp closed.

Theorem 3.9: Let A be an Igp open set of (X,τ) and pint(A)⊂B⊂A. Then B is Igp open.

Proof: pint (A)$\subset$B$\subset$A. $(\text{pint}(A))^c = \text{pcl}(A^c)$. $A^c \subset B^c \subset \text{pcl}(A^c)$. A^c is Igp closed. This implies B^c is Igp closed and B is Igp open.

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