A Tale of Revolution: Discovery and Development of TSP

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Abstract—*In recent days combinatorial optimization problems are attracting too many researchers to spend their time after it for its high socio economic impacts and at the same time complexity in finding an exact solution. Traveling salesman problem is one of the most important and widely studied optimization problems. It has attracted a large number of researchers over the last few decades because it is simple to formulate and always it has a solution. Researchers are using TSP as a testing ground for new algorithms to solve optimization problems. Sothe importance of TSP in the field of combinatorial optimization problems is obvious. This paper reveals discovery and development of Traveling Salesman Problem.*

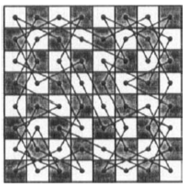
Keywords—*Graph Theory, Traveling salesman problem, combinatorial optimization*

I. Introduction

The historical background of traveling salesman problem goes back to the 19th century, precisely 1832. The problem was first mentioned by a handbook for traveling salesman including exampletours through Germany and Switzerland, but containing no mathematical treatment. But the origins of the travelling salesman problem are not clear. First mathematical formulation of traveling salesman problem was given by the Irish mathematician W.R. Hamilton and by the British mathematician Thomas Kirkman. Actually, the concept of entire graph theory are humble, even frivolous.Because it was not motivated by many fundamental problems of calculation, motion and measurement. Development of entire graph theory were inspired by those problems which were often little more than puzzles. Because they were designed to test the ingenuity rather than to stimulate and imagination.[1]Since the goal of TSP is to find an optimal Hamiltonian path, so the origin of TSP is related with the origin of Hamiltonian type problem. For that reason we can classify the invention and development of TSP into two major phase named (a) pre-traveling salesman work and (b) development of traveling salesman problem. In this paper we will separately discuss about this two phase.

II. Pre-traveling salesman work

The earliest known example of a Hamiltonian-type problem is that of the knight's tour problem on a chessboard: Can a knight visit all sixty-four squares of a chessboard just once and return to its starting position? [2] This kind of Chessboard problems was very interesting and many mathematicians was attracted to spend their time after it for a good solution. That is why it has a long history, and solutions are known to exist as far back as the 14th century. A solution to the knight's tour problem is given in Figure 1.



50	11	24	63	14	37	26	35
23	62	51	12	25	34	15	38
10	49	64	21	40	13	36	27
61	22	9	52	33	28	39	16
48	7	60	1	20	41	54	29
59	4	45	8	53	32	17	42
6	47	2	57	44	19	30	55
3	58	5	46	31	56	43	18

Figure 1: A solution of Knight Tour problem

Literature [3,4,5] shows that, De Montmort and De Moivre first gave a solution method at the end of seventeenth century. In 1759 wrote a paper for the Academy of the Royal Society of Berlin. In that paper he gave systematic approach to the problem describing various method to solve. He also discussed the similar type problem for chessboards of different shapes and sizes. Probably, he is the first person who proved that if there

are odd number of squares on a chessboard then no solution is possible. In 1771, Vandermonde made some significant advance. In nineteenth century, Rouse Ball and Jaenisch made some quality full work.
Then we may mention about the work of kirkman. He was too much interested onpolyhedra (or polyedm). Kirkman was fascinated to find theexistence of a closed cycle passing through every vertex of the polyhedra. In his paper[6] he gave a sufficient condition for the existence of such a cycle, but this proved to be incorrect. Kirkman also showed that no such cycle is possible if the polyhedron contains even-sided faces but odd number of vertices.
Cycles of polyhedral attracted many mathematicians like Kirkman. Willium Rowan Hamilton is one of them. From his interest on polyhedral Hamilton invented icosiancalculus[7].Hamilton became too much influenced by these ideas that soon he invented the icosian game[8]. Hamilton's icosian game played an important rule for the formulation of TSP because the main purpose of the game wasto find a path along the edges of a dodecahedron such that every vertex is visited exactly once, and the ending point is the same as the starting point. And this is the core idea of traveling salesman problem. Literatures give us evidence of a number of results of greater or less significance. In 1891, Brunel wrote a short note considering the graph with 64 vertices and 168 edges. Edges were determined by possible knight's-moves. He then applied the solution of the knight's tour problem in a related electrical network of same number of edges.
In 1934 a surprising prove was given byRQdei[9], where he told about the certainty of havinga directed path passing through every vertex in every tournament.This result gave a clear concept of Hamiltonian cycle but surprisingly was not proved until 1957.A detailed discussion on Eulerian and Hamiltonian graphs were given by DQnesKonig in 1936 [10]. And the word 'Hamiltonian' was established forever.

III. Development of traveling salesman problem

Now let's look through the TSP. We will classify all efforts step by step from nineteenth century; the first manual where the problem was first formulated mentioning all efforts till now.

A manual of 1832

The traveling salesman problem has a natural interpretation and Müller-Merbach[11] detected that the problem was formulated in a 1832 manual for the successful traveling salesman. The manual describe the problem totally analytically describing no mathematical formulation. A translation from the booklet is as follows:
"Business brings the traveling salesman now here, then there, and no travel routes can be properly indicated that are suitable for all cases occurring; but sometimes, by an appropriate choice and arrangement of the tour, so much time can be gained, that we don't think we may avoid giving some rules also on this. Everybody may use that much of it, as he takes it for useful for his goal; so much of it however we think we may assure, that it will not be well feasible to arrange the tours through Germany with more economy in view of the distances and, which the traveler mainly has to consider, of the trip back and forth. The main point always consists of visiting as many places as possible, without having to touch the same place twice."
The manual suggests five tours through Germany (one of them partly through Switzerland).

Contribution of Karl Menger and Hassler Whitney[12,13,14,15,16]

It is believed that the term traveling salesman problem was introduced in mathematical circles in 1931-1932 by Hassler Whitney. Mathematician K. Menger first wrote about traveling salesman problem. He also defined the problem. A major contribution of K. Menger is to give the idea of exact and approximate algorithms solving TSP. He is the first mathematician who proved the obvious ofhaving a solution in brute-force algorithm and the non-optimality of nearest neighbor.

Menger posed the traveling salesman problem, as follows:

We denote by messenger problemthe task to find, for finitely many points whose pairwise distances are known, the shortest route connecting the points. Of course, this problem is solvable by finitely many trials. Rules which would push the number of trials below the number of permutations of the given points are not known. The rule that one first should go from the starting point to the closest point, then to the point closest to this, etc., in general does not yield the shortest route.

Menger spent six months as visiting lecturer at Harvard University.He presented his results on lengths of arcs and shortest paths through finite sets of points in one of his seminar talks at Harvard.In his time we find another name Hassler Whitney who was a National Research Council Fellow at Princeton University. He also mentioned the problem of finding the shortest route along the 48 States of America. According to Flood, Hassler Whitney mentioned this problem Whitney in a seminar talk at Princeton University in 1934.

Contribution of Flood and Koopmans[17]

Like K. menger, Flood has a large contribution for TSP. He showed a number of connections of the TSP with Hamiltonian games and Hamiltonian paths in graphs in 1937. In literature we find an interview of Flood that was taken by A.W. Tucker where he relate the TSP with school bus routing problem and mentioned about Koopmas. He told that Koopmans first became interested in the "48 States Problem" of Hassler Whitney when he had been with him in the Princeton Surveys.

Development in 1940's[18,19,20,21,22]

In 1940, a new stage for TSP started. First mathematical results of TSP seems to be appeared in this stage. Mathematicians were trying to find shortest path through a given set of points in a metric space. An investigations of Milgram on the shortest Jordan curve that covers a given, not necessarily finite, set of points in a metric space influenced this idea. The problem of shortest curve was also investigated by Fejes through n points in the unit square. In consequence of this, Verblunsky showed that its length is less than 2 + sqrt(2:8n). Later work in this direction includes Few and Beardwood, Halton, and Hammersley.

Development in 1950's[23]

Dantzig expressed the TSP as an integer linear program and developed the cutting plane method for its solution. This cutting plane method later used in the development of a general algorithm. Using these new methods they solved a problem of 49 cities finding the shortest tour.

Development in 1960's[24]

According to Hoffman and Wolfe , Gomory proposed several algorithms, in 1958, 1960 and 1963, to solve the TSP which have continued to affect the theory on integer programming. The most popular of them is the branch and bound method.

Development in 1970's and 1980's[25]

Karp is the first person who showed that the Hamiltonian cycle problem was NP-complete : a*n exact solution requires a number of computational steps that grows exponentially with the size of the problem*, which implies the NP-hardness of TSP. This supplied a scientific explanation for the apparent computational difficulty of finding optimal tours. In the late 1970's and 1980's, great progress was made when Grötschel and others researchers managed to solve instances with up to 2392 cities, using cutting planes and branch and bound.

Development in 1990's and 2000's

Literature shows that[25], in the 1990's, Applegate, Bixby, Chvátal and Cook developed the program Concorde. Concorde has been used in many recent record solutions. Applegate, Bixby, Chvátal and Cook gave the solution of a TSP through 15112 cities in Germany in 2001. For this calculation they used a network of 110 processors and it took 22.6 years of computer time. Next biggest TSP problem of 24978 cities in Sweden was solved in 2004. A tour of length approximately 72500 kilometers was found.

Two years later in 2006 an instance with 85900 points was solved using Concorde TSP Solver, taking over 136 CPU years, scaled to a 2.4GHz AMP Opteron 250 computer node. For many other instances with millions of cities, solutions can be found that are probably within 1% of the optimal tour. Year by year milestones in the solution of TSP are presented below:

Year	Research team	Problem Size
1954	G. Dantzig, R. Fulkerson and S. Johnson	49 cities
1971	M. Held and R.M. Karp	64 cities
1975	P.M. Camerini, L. Fratta and F. Maffioli	67 cities
1977	M. Grötschel	120 cities
1980	H. Crowder and M.W. Padberg	318 cities

1987	M. Padberg and G. Rinaldi	532 cities
1987	M. Grötschel and O. Holland	666 cities
1987	M. Padberg and G. Rinaldi	2392 cities
1994	D. Applegate, R. Bixby, V. Chvátaland W. Cook	7397 cities
1998	D. Applegate, R. Bixby, V. Chvátal and W. Cook	13509 cities
2001	D. Applegate, R. Bixby, V. Chvátal and W. Cook	15112 cities
2004	D. Applegate, R. Bixby, V. Chvátal and W. Cook	24978 cities
2006	D. Applegate, R. Bixby, V. Chvátal and W. Cook	85900 cities

Table 1: Milestones in the solution of TSP instances[16]

IV. Conclusion

From the above articles, it is clear to us that traveling salesman problem has a very sophisticated historical background. A lot of mathematicians worked and working for it. For its huge applications to real life TSP is important. But till now it is confusion either there exists a general solution or not. If we can find a globally applicable solution method for solving TSP, then our world will be 200 years ahead from now. So, it is easy to guess that work for TSP will continue with full swing.

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