# A study of Solid Fixed Charge Transportation Problem and its Solution by Grey Situation Decision-Making Theory

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**Abstract**- *In this paper, the fixed charge solid transportation problem under uncertainty is investigated. Nowadays finding an efficient solution for transportation problem is one of the main issues for organizations and industries. According to the real world, in this paper, it is assumed that the products are transported with fixed charge solid transportation problem. Besides, the fuzzy values are used conferring to the value of the parameter in the real world. In this paper, we focus on solution of fixed charge solid transportation problem transportation problem by Grey Situation Decision (GSD) making theory with membership function.*

**Keywords —** *fixed charge solid transportation problem, membership function, Grey Situation Decision (GSD) making theory*

## 1. Introduction to Solid transportation problem with fixed charge

It is well-known that the transportation problem (TP) plays an important role in logistics and supply chain management for reducing cost and improving the service quality. In the traditional transportation problem, there are two kinds of constraints taken into consideration, i.e., source constraint and destination constraint. For the TP, various algorithms have been investigated by many researchers such as [1]- [4]. But in real life, we have to deal with other constraints besides source constraint and destination constraint, such as transportation mode constraint. Then, as an extension of the traditional TP, the solid transportation problem (STP) was introduced by [5]. Even since then, the STP has been received much attention and many models have been investigated. For example, Bhatia [6] presented an algorithm for a solid transportation problem wherein the objective function is indefinite quadratic. Reference [7] provided a method based on the principle of zero point method ([8]) to find an optimal solution of the STP. Reference [9] studied a capacitated two-stage time minimization transportation problem. Another important extension of the traditional TP is the fixed charge transportation problem, which was initialized by Reference [10]. Up to now, it has been studied by many researchers such as [12] and so on. In classical models and algorithms, the parameters of the models are supposed to be crisp values. Reference [13] investigated a bicriteria solid transportation problem with stochastic parameters and constructed three models for the problem. References [14, 15] investigated fixed charge transportation problem in the uncertain environment. Reference [16] proposed an expected constrained programming model for STP in the uncertain environment. Reference [17] investigated a transportation problem under conditions of emergency scheduling. Recently, Reference [18] studied a transportation problem with uncertain costs and random supplies. Reference [19] applied type-2 uncertain optimization methods for solving the fixed charge solid transportation problem.

The traditional transportation problem (TP) is a well-known optimization problem in operational research, in which two kinds of constraints are taken into consideration, i.e., source constraint and destination constraint. But in the real system, we always deal with other constraints besides of source constraint and destination constraints, such as product type constraint or transportation mode constraint. For such case, the traditional TP turns into the solid transportation problem (STP). As a generalization of traditional TP, the STP was introduced by [20] in 1962. In recent years, the STP received much attention and many models and algorithms under both crisp environment and uncertain environment have been investigated. For example, Reference [21] presented the fuzzy programming model for a multi-objective STP, Reference [22] studied two kinds of uncertain STP, that is, the supplies, demands and conveyance capacities are interval numbers and fuzzy numbers, respectively. Reference [23] designed an evolutionary algorithm based parametric approach to solving fuzzy STP. In addition, Reference [24] designed a neural network approach for multi-criteria STP, also, they presented an improved genetic algorithm to solve multi-objective STP with fuzzy numbers in [25].

It is easy to see from the literature that the research of STP under fuzzy environment is very popular in recent years. One of the reasons is due to the development of the fuzzy set theory so that the ability to deal with fuzziness is improved. As we know, the fuzzy set theory was introduced by [26] to deal with fuzziness. Up to now, the fuzzy set theory has been applied to broad fields. With respect to fuzzy programming, [29] presented three kinds of ways to construct models in the fuzzy environment, that is, expected value model, chance-constrained programming model, and dependent-chance programming model.

## 2. Mathematical formulation of fixed charge Solid transportation problem

In this section, we shall introduce some knowledge of fixed charge STP. Suppose that there are $m$ sources, $n$ destinations, and $l$ conveyances. The goal of the STP is to make a transport plan so that the total transportation cost is minimized. There are two types of costs will be taken into consideration, which is the direct cost and the fixed charge. The direct cost is the cost with respect to per unit transportation amount. The fixed charge will be paid when the transportation activity between a source and a destination by a conveyance occurs. In order to construct the mathematical model for the fixed charge STP, some notations and assumptions are listed in Table -1.

In order to model the above mentioned fixed charge STP, the following notations are employed:

Table-1: List of notations and assumptions

| | |
|---|---|
| $i \in \{1,2,\dots,m\}$ | The index for sources |
| $j \in \{1,2,\dots,n\}$ | The index for destinations |
| $k \in \{1,2,\dots,l\}$ | The index for conveyances |
| $a_i$ | The amount of products in source $i$ which can be transported to $n$ destinations |
| $b_j$ | The minimal demand of products in destination $j$ |
| $c_k$ | The transportation capacity of conveyance $k$ |
| $d_{ijk}$ | The direct cost of unit transportation amount from source $i$ to destination j by conveyance $k$ |
| $e_{ijk}$ | The fixed charge with respect to transportation activity from source $i$ to destination j by conveyance $k$ |
| $x_{ijk}$ | The quantity transported from source i to destination $j$ by conveyance $k$ |
| $y_{ijk} = \begin{cases} 1; & x_{ijk} > 0 \\ 0; & \text{otherwise} \end{cases}$ | |

Where $i = 1,2,\dots,m$ ; $j = 1,2,\dots,n$ ; $k = 1,2,\dots,l$ respectively. This implies that, if the transportation activity is assigned from source $i$ to destination $j$ by conveyance $k$, then the corresponding fixed charge will occur. Then the total cost, as well as the objective function, is

$$f(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}(d_{ijk}x_{ijk} + e_{ijk}y_{ijk}),$$

where $x$ and $y$ denote the vectors consisting of $x_{ijk}$ and $y_{ijk}$, $i = 1,2,\dots,m$ ; $j = 1,2,\dots,n$ ; $k = 1,2,\dots,l$, respectively. Therefore, the fixed charge STP can be described by the model as follows:

$$\left.\begin{array}{l} \min \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}(d_{ijk}x_{ijk} + e_{ijk}y_{ijk}) \\ \text{subject to the constraints:} \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le a_i, i = 1,2,\dots\dots,m \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge b_j, j = 1,2,\dots\dots,\mathrm{n} \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le c_k, k = 1,2,\dots\dots,l \\ x_{ijk} \ge 0, y_{ijk} = \begin{cases} 1; & x_{ijk} > 0 \\ 0; & \text{otherwise} \end{cases} \\ i = 1,2,\dots,m; j = 1,2,\dots,n; k = 1,2,\dots,l \end{array}\right\} \quad (1)$$

The first constraint requires that the total amount transported from source $i$ is no more than $a_i$ ; the second constraint requires that the total amount transported from $m$ sources should satisfy the demand of destination $j$ and the third constraint is a capacity constraint. The above model assumes $a_i, b_j, c_k, d_{ijk}, e_{ijk}$ are all constants. But in the real world, the transportation plan is always made in advance, thus it is impossible for us to know these values precisely. If there is enough historical data of the information about them, we can regard them as random variables. However, when we are lack of historical data or historical data is invalid because of unexpected events have occurred. We usually have some domain experts to evaluate the belief degree that each event will occur. This expert data is just the subject of the uncertainty theory.

For the fixed charge STP discussed in the uncertain environment, we assume that $a_i, b_j, c_k, d_{ijk}$ and $e_{ijk}$ are all uncertain variables, and rewrite them as $\tilde{a}_i, \tilde{b}_j, \tilde{c}_k, \xi_{ijk}$ and $\eta_{ijk}$ respectively. Also, assume that all the uncertain variables are independent. Then the fixed charge STP becomes uncertain fixed charge STP, denoted by UCSTP. We will see that the uncertainty theory offers a useful tool to deal with UCSTP.

### 3. Grey situation decision-making theory and Fuzzy Programming Technique based approach to find the solution to the fixed charge solid Transportation problem:

Consider some notations to define the variables and the sets in multi-objective transportation problem.

Let $O = \{O_1, O_2, \ldots\ldots, O_m\}$ be the set of m-origins having $a_i \; (i = 1, 2, \ldots, m)$ units of supply respectively. Let $D = \{D_1, D_2, \ldots\ldots, D_n\}$ be the set of n-destinations with $b_j \, (j = 1, 2, \ldots, n)$ units of requirement respectively. Let $E = \{\mathrm{E}_1, \mathrm{E}_2, \ldots\ldots, \mathrm{E}_l\}$ be the set of $l$ -conveyances having $e_k \; (k = 1, 2, \ldots, l)$ units of supply respectively.

There is a penalty $p_{ijk}$ associated with transporting a unit of product from $i^{th}$ source to destination with k conveyance. This penalty may be cost or delivery time or safety of delivery etc. A variable $x_{ijk}$ represents the unknown quantity to be shipped from $i^{th}$ source to $j^{th}$ destination with k conveyance. The problem is to determine the transportation schedule when multiple objectives exist.

Grey situation decision-making theory is used to minimize or maximize the total transportation penalty according to the problem which satisfying supply and demand conditions. Consider the set of m-origins $O = \{O_1, O_2, \ldots\ldots, O_m\}$ as the set of events, the set of n destinations $D = \{D_1, D_2, \ldots\ldots, D_n\}$ as the set of countermeasure, $E = \{\mathrm{E}_1, \mathrm{E}_2, \ldots\ldots, \mathrm{E}_l\}$ as the set of conveyance the penalty $p_{ijk}$ as the situation set denotes by $P = \{ p_{ijk} = (O_i, D_j, \mathrm{E}_k) / O_i \in O, D_j \in D, \mathrm{E}_k \in E\}$

First of all, confirm the decision-making goals (objectives) and seek the corresponding effect measure matrix $U^{(t)}$.

$$U^{(t)} = \left[ u_{ijk}^{(t)} \right] = \begin{bmatrix} u_{111}^{(t)} & u_{112}^{(t)} & \cdots\cdots & u_{1nl}^{(t)} \\ u_{211}^{(t)} & u_{212}^{(t)} & \cdots\cdots & u_{2nl}^{(t)} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ u_{m11}^{(t)} & u_{m12}^{(t)} & \cdots\cdots & u_{mnl}^{(t)} \end{bmatrix}$$

$$u_{ijk}^{(t)} = C_{ijk} = c_{ijk} + \frac{f_{ijk}}{m_{ij}} \quad \text{where } m_{ij} = \min(a_i, b_j)$$

Where for $i = 1, 2, \ldots\ldots, m$, $j = 1, 2, \ldots\ldots, n$ and $k = 1, 2, \ldots, l$

$f_{ijk}$ is fixed cost

Here, the data of decision-making goals for transporting a product is the effective value $u_{ijk}^{(t)}$ of situation $p_{ijk} \in P$ with the objective $t = 1, 2, \ldots, s$ .

Now, find the upper effect measure and lower effect measure by the formula:

- Upper effect measure

$$r_{ijk}^{(t)} = u_{ijk}^{(t)} \Big/ \max_i \max_j \max_k \left\{ u_{ijk}^{(t)} \right\}$$

- Lower effect measure

$$r_{ijk}^{(t)} = \min_i \min_j \min_k \left\{ u_{ijk}^{(t)} \right\} \Big/ u_{ijk}^{(t)}$$

And achieve the consistent matrix of effect measure $R^{(r)}$ by using upper effect measure or lower effect measure as

$$R^{(t)} = \left[ r_{ijk}^{(t)} \right] = \begin{bmatrix} r_{111}^{(t)} & r_{112}^{(t)} & \cdots\cdots & r_{1nl}^{(t)} \\ r_{211}^{(t)} & r_{212}^{(t)} & \cdots\cdots & r_{2nl}^{(t)} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ r_{m11}^{(t)} & r_{m12}^{(t)} & \cdots\cdots & r_{mnl}^{(t)} \end{bmatrix}$$

Subtract each data of comprehensive matrix R of effect measure from 1 to convert combine maximization objective in minimization form and then find solutions of this grey theory based multi-objective problem from the comprehensive matrix $r_{ijk}^{(t)}$ of effect measure and by using given supply, demand, and conveyance as like LPP problem.

In fuzzy programming technique, we first find the lower bound as $L_k$ and the upper bound as $U_k$ for the $k^{th}$ objective function $Z_k, k = 1, 2, \ldots\ldots, K$ where $U_k$ is the highest acceptable level of achievement for objective $k$, $L_k$ the aspired level of achievement for objective $k$ and $d_k = U_k - L_k$ the degradation allowance for objective $k$. When the aspiration levels for each of the objective have been specified, a fuzzy model is formed and then the fuzzy model is converted into a crisp model. Here, in this developed approach we first utilized Grey situation decision-making theory is utilized to Find the lower effect measure $r_{ijk}^{(t)}$ and upper effect measure $r_{ijk}^{(t)}$ and accomplish the consistent matrix of effect measure $R^{(t)} = [r_{ijk}^{(t)}]$ for each objective k. These matrices of each objective are utilized as a cost matrix of each objective in fuzzy programming technique and solution are obtained. So here Grey situation decision-making theory is utilized for normalization of data.

### 3.1 Algorithm for finding the solution of fixed charge solid transportation problem using MGSD theory:

***Input:*** *Effect measure matrix* $U^{(t)} = \left( U^{(1)}, U^{(2)}, \ldots\ldots\ldots, U^{(s)}; n \times m \right)$

***Output:*** *Solution of fixed charge solid transportation problem*

***begin***

***Read:*** *Example*

***while*** *example = fixed charge solid transportation problem do*

***for*** *k=1 to s do*

*enter effect measure matrix* $U^{(t)}$

***end***

-| *Find the lower effect measure* $r_{ijk}^{(t)}$ *and upper effect measure* $r_{ijk}^{(t)}$ *and accomplish the*

*The consistent matrix of effect measure* $R^{(t)} = [r_{ijk}^{(t)}]$.

***for*** *k=1 to s do*

$$r_{ijk}^{(t)} = \min_i \min_j \min_k \left\{ u_{ijk}^{(t)} \right\} \Big/ u_{ijk}^{(t)}$$

$$r_{ijk}^{(t)} = u_{ijk}^{(t)} \Big/ \max_i \max_j \max_k \left\{ u_{ijk}^{(t)} \right\}$$

$$R^{(t)} = \left[ r_{ijk}^{(t)} \right] = \begin{bmatrix} r_{111}^{(t)} & r_{112}^{(t)} & \cdots\cdots & r_{1nl}^{(t)} \\ r_{211}^{(t)} & r_{212}^{(t)} & \cdots\cdots & r_{2nl}^{(t)} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ r_{m11}^{(t)} & r_{m12}^{(t)} & \cdots\cdots & r_{mnl}^{(t)} \end{bmatrix}$$

***end***

-| *Subtract each data of comprehensive matrix R of effect measure from 1 to convert combine maximization objective in minimization form.*

-| *Find solutions for multi-objective transportation problem from the comprehensive matrix* $R = [r_{ijk}]$ *of effect measure using modified distribution method in LINGO package.*

-| *Find pay off matrix by using each objective solution.*

-| *Define linear as well as hyperbolic membership function using payoff matrix.*

-| *Developed single objective optimization problem using fuzzy linear membership Function and hyperbolic function.*

-| *Solve above developed model and find the compromise solution.*

## 4. Numerical illustrations

### 4.1 Numerical illustration-1:

To illustrate the efficiency of the proposed method we consider the following numerical example, consider the following fixed charge solid transportation problem [30]:

Suppose that there are four coal mines to supply the coal for six cities, and two kinds of conveyances are available to be selected i.e., train and cargo ship. Now, the decision maker should make a transportation plan for the next month such that the transportation cost minimized. In the following experiments, the notations $\tilde{a}_i, \tilde{b}_j$ and $\tilde{c}_k$ are employed to denote the supply capacities, demands, and transportation capacities, respectively.

Assume that all uncertain variables are independent zigzag uncertain variables, which are listed as follows:

$\tilde{a}_1 \sim Z\,(15,33,39), \tilde{a}_2 \sim Z\,(18,32,38), \tilde{a}_3 \sim Z\,(17,32,35), \tilde{a}_4 \sim Z\,(20,34,36)$

$\tilde{b}_1 \sim Z\,(10,13,15), \tilde{b}_2 \sim Z\,(12,15,17), \tilde{b}_3 \sim Z\,(8,12,14), \tilde{b}_4 \sim Z\,(11,13,16), \tilde{b}_5 \sim Z\,(13,15,18), \tilde{b}_6 \sim Z\,(9,13,16)$

$\tilde{c}_1 \sim Z\,(50,56,60), \tilde{c}_2 \sim Z\,(58,65,70)$

In addition, we also assume that the direct costs $\xi_{ijk}$ and fixed charges $\eta_{ijk}$ are independent zigzag uncertain variables, which are listed in Tables 2–5, respectively.

**Table- 2: The Direct costs by train** $(\xi_{ij1})$

| $Mines \downarrow / Cities \rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (8,11,12) | (6,7,10) | (10,12,15) | (8,10,13) | (9,11,15) | (7,9,12) |
| 2 | (6,7,10) | (7,10,12) | (8,12,14) | (5,9,12) | (10,12,15) | (8,10,13) |
| 3 | (5,8,12) | (8,10,13) | (6,8,12) | (9,10,13) | (7,10,12) | (8,10,13) |
| 4 | (10,12,13) | (7,9,13) | (7,9,12) | (5,8,10) | (12,13,15) | (10,12,13) |

**Table-3: The fixed charges by train** $(\eta_{ij1})$

| $Mines \downarrow / Cities \rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (4,6,9) | (5,6,8) | (4,6,9) | (3,5,8) | (4,6,9) | (3,5,6) |
| 2 | (4,6,7) | (2,5,6) | (4,5,7) | (3,4,6) | (3,6,7) | (3,5,6) |
| 3 | (3,4,6) | (3,5,6) | (3,4,6) | (5,7,8) | (4,6,7) | (3,4,6) |
| 4 | (4,6,9) | (2,5,6) | (2,3,5) | (3,4,7) | (3,6,8) | (5,6,9) |

**Table: 4: The Direct costs by ship** $(\xi_{ij2})$

| $Mines \downarrow / Cities \rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (10,15,17) | (10,12,15) | (8,10,13) | (10,13,18) | (12,14,17) | (9,12,14) |
| 2 | (8,10,13) | (9,13,16) | (9,13,15) | (10,14,16) | (13,16,18) | (12,15,16) |
| 3 | (10,12,15) | (11,14,15) | (10,14,17) | (7,8,10) | (12,13,15) | (9,10,12) |
| 4 | (8,10,13) | (9,12,14) | (12,15,17) | (10,12,16) | (10,15,18) | (8,10,13) |

**Table-5: The fixed charges by ship** $(\eta_{ij1})$

| $Mines \downarrow / Cities \rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (5,7,8) | (3,6,8) | (5,6,8) | (5,7,8) | (6,8,9) | (5,7,10) |
| 2 | (2,5,6) | (4,7,8) | (4,6,9) | (3,6,8) | (5,8,10) | (4,8,10) |
| 3 | (3,6,8) | (4,7,8) | (2,6,9) | (2,3,5) | (6,8,11) | (4,7,9) |
| 4 | (2,5,7) | (5,7,8) | (5,7,8) | (4,7,9) | (4,8,10) | (2,5,7) |

**Solution:**

Step-1: Using the formula $C_{ijk} = c_{ijk} + \dfrac{f_{ijk}}{m_{ij}}$ for lower, middle and upper values.

For lower values

$$U^{(1)} = \begin{bmatrix} 8.4 & 10.5 & 6.4167 & 10.25 & 10.5 & 8.625 & 8.2727 & 10.4545 & 9.3077 & 12.4615 & 7.3333 & 9.5556 \\ 6.4 & 8.2 & 7.1667 & 9.3333 & 8.5 & 9.5 & 5.2727 & 10.2727 & 10.2308 & 13.3846 & 8.3333 & 12.4444 \\ 5.3 & 10.3 & 8.25 & 11.3333 & 6.375 & 10.25 & 9.4545 & 7.1818 & 7.3077 & 12.4615 & 8.3333 & 9.4444 \\ 10.4 & 8.2 & 7.1667 & 9.4167 & 7.25 & 12.625 & 5.2727 & 10.3636 & 12.2308 & 10.3077 & 10.5556 & 8.2222 \end{bmatrix}$$

For middle values

$$U^{(2)} = \begin{bmatrix} 11.4615 & 15.5385 & 7.4000 & 12.4000 & 12.5000 & 10.5000 & 10.3846 & 13.5385 & 11.4000 & 14.5333 & 9.3846 & 12.5385 \\ 7.4615 & 10.3846 & 10.3333 & 13.4667 & 12.4167 & 13.5 & 9.3077 & 14.4615 & 12.4 & 16.5333 & 10.3846 & 15.6154 \\ 8.3077 & 12.4615 & 10.3333 & 14.4667 & 8.3333 & 14.5 & 10.5385 & 8.2308 & 10.4 & 13.5333 & 10.3077 & 10.5385 \\ 12.4615 & 10.3846 & 9.3333 & 12.4667 & 9.25 & 15.5833 & 8.3077 & 12.5385 & 13.4 & 15.5333 & 12.4615 & 10.3846 \end{bmatrix}$$

For upper values

$$U^{(3)} = \begin{bmatrix} 12.6 & 17.5333 & 10.4706 & 15.4706 & 15.6429 & 15.3714 & 13.5 & 18.5 & 15.5 & 17.5 & 12.3750 & 14.625 \\ 10.4667 & 13.4 & 12.3529 & 16.4706 & 14.5 & 15.6429 & 12.375 & 16.5 & 15.3889 & 18.5556 & 13.375 & 16.625 \\ 12.4 & 15.3333 & 13.3529 & 15.406 & 12.4286 & 17.6429 & 13.5 & 10.3125 & 12.3889 & 15.6111 & 13.375 & 12.5625 \\ 13.6 & 13.4667 & 13.3529 & 14.4706 & 12.3571 & 17.5714 & 10.4375 & 16.5625 & 15.4444 & 18.5556 & 13.5625 & 13.4375 \end{bmatrix}$$

Step-2: Select the minimum number from $c_{ijk}$ ; $c_{ijk}$ = element of $i$ supply, $j$ demand, $k$ conveyance , row-wise and take the ratio of minimum number and $c_{ijk}$ for all objective and select the minimum number from $c_{ijk}$ column wise and take the ratio of minimum number and $c_{ijk}$ for all objective.

For transporting a product, time and cost are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{111}^{(1)} = \dfrac{\min_1 \min_1 \min_1 \{u_{111}\}}{u_{111}} = \dfrac{5.3}{8.4}$

Similarly, obtain lower effect measure for each data. Therefore the consistent matrices of effect measure are given below and subtract each data from 1 to convert combine maximization objective in minimization form, therefore, we have;

For lower values

$$R^{(1)} = \begin{bmatrix} 0.3690 & 0.3889 & 0 & 0.3740 & 0.3929 & 0.2560 & 0.3626 & 0.3862 & 0.3106 & 0.4851 & 0.1250 & 0.3285 \\ 0.1761 & 0.3570 & 0.2643 & 0.4351 & 0.3797 & 0.4450 & 0 & 0.4867 & 0.4846 & 0.6061 & 0.3673 & 0.5763 \\ 0 & 0.4854 & 0.3576 & 0.5324 & 0.1686 & 0.4829 & 0.4423 & 0.2620 & 0.2747 & 0.5747 & 0.3640 & 0.4388 \\ 0.4930 & 0.3570 & 0.2643 & 0.4401 & 0.2727 & 0.5824 & 0 & 0.4912 & 0.5689 & 0.4885 & 0.5005 & 0.3587 \end{bmatrix}$$

Similarly, for middle values

$$R^{(2)} = \begin{bmatrix} 0.3544 & 0.5238 & 0 & 0.4032 & 0.4080 & 0.2952 & 0.2874 & 0.4534 & 0.3509 & 0.4908 & 0.2115 & 0.4098 \\ 0 & 0.2815 & 0.2839 & 0.4459 & 0.3091 & 0.4473 & 0.1984 & 0.4840 & 0.3983 & 0.5487 & 0.2815 & 0.5222 \\ 0.1019 & 0.3395 & 0.2839 & 0.4311 & 0.0123 & 0.4324 & 0.2190 & 0 & 0.2086 & 0.3918 & 0.2015 & 0.2190 \\ 0.4930 & 0.3570 & 0.2643 & 0.4401 & 0.2727 & 0.5824 & 0 & 0.4912 & 0.5689 & 0.4885 & 0.5005 & 0.3587 \end{bmatrix}$$

Similarly, for upper values

$$R^{(3)} = \begin{bmatrix} 0.1693 & 0.4028 & 0 & 0.3232 & 0.3306 & 0.2285 & 0.2269 & 0.4426 & 0.3245 & 0.4017 & 0.1539 & 0.2841 \\ 0 & 0.2189 & 0.1527 & 0.3645 & 0.2782 & 0.3309 & 0.1566 & 0.3750 & 0.3199 & 0.4359 & 0.2174 & 0.3704 \\ 0.1683 & 0.3361 & 0.2277 & 0.3334 & 0.1703 & 0.4155 & 0.2361 & 0 & 0.1676 & 0.3394 & 0.2290 & 0.1791 \\ 0.2325 & 0.2249 & 0.2183 & 0.2787 & 0.1553 & 0.4060 & 0 & 0.3774 & 0.3242 & 0.4375 & 0.2304 & 0.2233 \end{bmatrix}$$

Step-3: Find solutions for fixed charge solid transportation problem from the consistent matrix of effect measure using the simplex method. And we have

$$\text{Pay-off matrix} = \begin{bmatrix} Z_1(X^1) & Z_2(X^1) & Z_3(X^1) \\ Z_1(X^2) & Z_2(X^2) & Z_3(X^2) \\ Z_1(X^3) & Z_2(X^3) & Z_3(X^3) \end{bmatrix} = \begin{bmatrix} 19.4708 & 23.2397 & 21.4155 \\ 26.183 & 13.5647 & 14.9609 \\ 24.3851 & 13.8484 & 12.2381 \end{bmatrix}$$

Applying fuzzy linear membership function, we get the solution of this model is

$$\lambda = 0.6549527 \quad \& \quad Z_1 = 668.8022544, Z_2 = 876.6911763, Z_3 = 1151.075837$$

Applying fuzzy hyperbolic membership function, we get the solution of this model is

(Where, $a_1 = a_2 = a_3 = 1$)

$\lambda = 0.865230729$ & $Z_1 = 668.8022544, Z_2 = 876.6911763, Z_3 = 1151.075837$

### 4.1.1 Comparison of fixed charge solid transportation problem of Grey situation decision-making theory approach with other approaches

Table-6 indicates the grey situation decision-making theory approach can be provided an alternative approach to find the solution of numerical illustration-1.

**Table-6: Comparison of results obtained by using different approaches for numerical illustration-1**

| **by Grey situation decision-making theory approach** | **Other approach [30]** |
|---|---|
| $Z_1 = 668.8022544, Z_2 = 876.6911763, Z_3 = 1151.075837$ | $Z = 730.31$ |

### 4.2 Numerical illustration-2:

Consider the following fixed charge solid transportation problem. As we know, coal is a kind of crucial energy source in the development of economy and society. Accordingly, how to transport the coal from mines to the different areas economically is also an important issue in the coal transportation. For the convenience of description, we summarize the problem as follows. Suppose that there are four coal mines to supply the coal for four cities. During the process of transportation, two kinds of conveyances are available to be selected, i.e., train and cargo ship. Now, the task for the decision-maker is to make the transportation plan for the next month. At the beginning of this task, the decision maker needs to obtain the basic data, such as supply capacity, demand, transportation capacity, transportation cost of unit product, and so on. In fact, since the transportation plan is made in advance, we generally cannot get these data exactly. For this condition, the usual way is to obtain the fuzzy data by means of experience evaluation or expert advice. In this example, the notations $\tilde{a}_i, \tilde{b}_j$ and $\tilde{e}_k$ are employed to denote the supply capacity, demand, and transportation capacity, respectively. And the corresponding fuzzy data are listed as follows [31]:

$\tilde{a}_1 = (15,28,29,31), \tilde{a}_2 = (6,20,23,25), \tilde{a}_3 = (22,34,36,38), \tilde{a}_4 = (18,30,32,34)$

$\tilde{b}_1 = (12,13,14,15), \tilde{b}_2 = (20,23,26,27), \tilde{b}_3 = (19,21,23,24), \tilde{b}_4 = (25,27,29,31),$

$\tilde{e}_1 = (39,45,50,55), \tilde{e}_2 = (60,65,70,75)$

For the same reason, we cannot obtain the transportation cost of unit amount accurately in advance, and we also treat it as a fuzzy variable by experience or expert advice. For this example, the transportation cost of a unit amount is listed in Tables- 6 and 7.

In the real situation, before the transportation activity occurs, we always spend some money in checking and repairing the conveyances. In addition, the depreciation of conveyance should also be considered. These two kinds of costs may be treated as the fixed charge, which may be also treated as fuzzy variables. For the convenience of computing, we suppose that the fixed charge $\eta_{ijk} = 0.5\,\xi_{ijk}$ .

For this solid transportation problem, if the decision maker prefers treating the uncertain constraints as chance constraints, we may construct chance-constrained programming model for the problem.

**Table-7:** The direct costs by train $(\xi_{ij1})$

| Mines $\downarrow$ / Cities $\rightarrow$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | (3,5,8,10) | (7,8,9,10) | (15,17,19,20) | (13,15,17,19) |
| 2 | (5,7,8,9) | (6,9,10,12) | (3,4,6,7) | (16,20,23,25) |
| 3 | (5,8,9,10) | (13,15,17,19) | (3,6,8,9) | (7,8,11,13) |
| 4 | (18,19,21,24) | (9,12,13,15) | (7,10,11,14) | (9,10,13,15) |

**Table-8:** The direct costs by ship $(\xi_{ij2})$

| Mines $\downarrow$ / Cities $\rightarrow$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | (7,9,12,13)) | (5,9,10,12) | (9,12,13,15) | (20,25,26,28) |
| 2 | (10,12,14,15) | (12,14,16,18) | (11,17,18,20) | (3,5,8,10) |
| 3 | (8,10,12,14) | (20,23,25,27) | (23,25,27,29) | (6,8,10,12) |
| 4 | (6,8,9,10) | (26,28,30,32) | (30,32,33,35) | (30,32,38,40) |

**Solution:**

(1) Using the formula $C_{ijk} = c_{ijk} + \frac{f_{ijk}}{m_{ij}}$ for lower, middle and upper values.

For lower values

$$U^{(1)} = \begin{bmatrix} 3.125 & 7.2917 & 7.1750 & 5.125 & 15.3947 & 9.2368 & 13.26 & 20.4 \\ 5.2083 & 10.4167 & 6.1875 & 12.3750 & 3.0978 & 11.3438 & 16.5 & 3.0938 \\ 5.2083 & 8.3333 & 13.3250 & 20.5 & 3.0789 & 23.6053 & 7.14 & 6.12 \\ 18.75 & 6.25 & 9.225 & 26.65 & 7.1842 & 30.7895 & 9.18 & 30.6 \end{bmatrix}$$

For middle 1 values

$$U^{(2)} = \begin{bmatrix} 5.1923 & 9.3462 & 8.1739 & 9.1957 & 17.4048 & 12.2857 & 15.2778 & 25.463 \\ 7.2692 & 12.4615 & 9.225 & 14.35 & 4.1 & 17.425 & 20.5 & 5.125 \\ 8.3077 & 10.3846 & 15.3261 & 23.5 & 6.1429 & 25.5952 & 8.1481 & 8.1481 \\ 19.7308 & 8.3077 & 12.2609 & 28.6087 & 10.2381 & 32.7619 & 10.1852 & 32.5926 \end{bmatrix}$$

For middle 2 values

$$U^{(3)} = \begin{bmatrix} 8.3077 & 12.4615 & 9.1957 & 10.2174 & 19.4524 & 13.3095 & 17.3148 & 26.4815 \\ 8.3077 & 14.5385 & 10.2174 & 16.3478 & 6.1429 & 18.4286 & 23.5 & 8.1739 \\ 9.3462 & 12.4615 & 17.3696 & 25.5435 & 8.1905 & 27.6429 & 11.2307 & 10.1852 \\ 21.8077 & 9.3462 & 13.2826 & 30.6522 & 11.2619 & 33.7857 & 13.2407 & 38.7037 \end{bmatrix}$$

For upper values

$$U^{(4)} = \begin{bmatrix} 10.3333 & 13.4333 & 10.1852 & 12.2222 & 20.4167 & 15.3125 & 19.3065 & 28.4516 \\ 9.3 & 15.5 & 12.24 & 18.36 & 7.1458 & 20.4167 & 25.5 & 10.2 \\ 10.3333 & 14.4667 & 19.3519 & 27.5 & 9.1875 & 29.6042 & 13.2097 & 12.1935 \\ 24.8 & 10.3333 & 15.2778 & 32.5926 & 14.2917 & 35.7292 & 15.2419 & 40.6452 \end{bmatrix}$$

(2) Select the minimum number from $c_{ijk}$ row wise and take the ratio of minimum number and $c_{ijk}$ for all objective and Select the minimum number from $c_{ijk}$ column wise and take the ratio of minimum number and $c_{ijk}$ for all objectives.

For transporting a product, time and cost are less than its the batter, so use lower effect measure. So the lower effect measure for first data $r_{111}^{(1)} = \frac{\min_1 \min_1 \min_1 \{u_{111}\}}{u_{111}} = \frac{3.125}{3.125}$

Similarly, obtain lower effect measure for each data. Therefore the consistent matrices of effect measure are given below and subtract each data from 1 to convert combine maximization objective in minimization form, therefore, we have,

For lower values

$$R^{(1)} = \begin{bmatrix} 0 & 0.5714 & 0.5645 & 0.3902 & 0.8 & 0.6617 & 0.7643 & 0.8483 \\ 0.4060 & 0.7030 & 0.5000 & 0.7500 & 0.0061 & 0.7273 & 0.8125 & 0.0000 \\ 0.4088 & 0.6305 & 0.7689 & 0.8498 & 0 & 0.8696 & 0.5688 & 0.4969 \\ 0.8333 & 0 & 0.3293 & 0.8077 & 0.5714 & 0.7970 & 0.3192 & 0.8989 \end{bmatrix}$$

For middle_1 values

$$R^{(2)} = \begin{bmatrix} 0 & 0.4444 & 0.3648 & 0.4354 & 0.7644 & 0.5774 & 0.6601 & 0.7987 \\ 0.4360 & 0.6710 & 0.5556 & 0.7143 & 0 & 0.7647 & 0.8 & 0.2 \\ 0.3750 & 0.4085 & 0.5992 & 0.7386 & 0.3326 & 0.76 & 0.2461 & 0.3710 \\ 0.7368 & 0 & 0.3333 & 0.7096 & 0.5995 & 0.7464 & 0.2 & 0.8428 \end{bmatrix}$$

For middle_2 values

$$R^{(3)} = \begin{bmatrix} 0 & 0.3333 & 0.0966 & 0.1869 & 0.6842 & 0.3758 & 0.5202 & 0.6913 \\ 0.2606 & 0.5775 & 0.3988 & 0.6242 & 0 & 0.6667 & 0.7386 & 0.2485 \\ 0.1237 & 0.3427 & 0.5285 & 0.6794 & 0.25 & 0.7037 & 0.2689 & 0.1975 \\ 0.619 & 0 & 0.3077 & 0.6951 & 0.4545 & 0.7234 & 0.2941 & 0.7888 \end{bmatrix}$$

Similarly, for upper values

$$R^{(4)} = \begin{bmatrix} 0.1000 & 0.2418 & 0 & 0.1667 & 0.65 & 0.3348 & 0.4724 & 0.6420 \\ 0.2316 & 0.5390 & 0.4162 & 0.6108 & 0 & 0.65 & 0.7198 & 0.2994 \\ 0.1109 & 0.3649 & 0.5252 & 0.6659 & 0.2222 & 0.6897 & 0.3045 & 0.2465 \\ 0.6250 & 0 & 0.3333 & 0.6830 & 0.5 & 0.7108 & 0.3220 & 0.7490 \end{bmatrix}$$

(3) Find solutions for fixed charge solid transportation problem from the consistent matrix of effect measure using the simplex method and we have,

$$\text{Pay-off matrix} = \begin{bmatrix} Z_1(X^1) & Z_2(X^1) & Z_3(X^1) & Z_4(X^1) \\ Z_1(X^2) & Z_2(X^2) & Z_3(X^2) & Z_4(X^2) \\ Z_1(X^3) & Z_2(X^3) & Z_3(X^3) & Z_4(X^3) \\ Z_1(X^4) & Z_2(X^4) & Z_3(X^4) & Z_4(X^4) \end{bmatrix} = \begin{bmatrix} 13.1371 & 23.6375 & 20.8804 & 23.4353 \\ 21.57705 & 20.3323 & 15.70688 & 19.0209 \\ 26.6195 & 23.1246 & 13.0434 & 14.792 \\ 30.348 & 24.9936 & 14.2584 & 14.6675 \end{bmatrix}$$

Applying fuzzy linear membership function, we get the solution of this model is

$$\lambda = 0.6175727 \quad \& \quad Z_1 = 490.59103, Z_2 = 704.1253, Z_3 = 867.1813, Z_4 = 1021.3589$$

Applying fuzzy hyperbolic membership function, we get the solution of this model is (Where, $a_1 = a_2 = a_3 = a_3 = 1$)

$$\lambda = 0.803904 \quad \& \quad Z_1 = 490.59103, Z_2 = 704.1253, Z_3 = 867.1813, Z_4 = 1021.3589$$

### 4.2.1 Comparison of fixed charge solid transportation problem of Grey situation decision-making theory approach with other approaches

Table-9 indicates the grey situation decision-making theory approach can provide an alternative approach to find the solution of numerical illustration-2.

**Table-9: Comparison of results obtained by using different approaches for numerical illustration-2**

| by Grey situation decision-making theory approach | Other approaches [31] |
|---|---|
| $Z_1 = 490.59103, Z_2 = 704.1253, Z_3 = 867.1813, Z_4 = 1021.3589$ | $Z = 1080$ |

**Conclusion:**

In this paper, linear membership function & hyperbolic membership function are used to solve fixed charge solid transportation problem using grey situation decision-making theory. Using GSD theory fixed charge solid transportation problem converted into simple transportation problem. And it can be solved by the different standard method to get an optimal solution. It provides an alternative optimal solution of fixed charge solid transportation problem.

## REFERENCES


1) Srinivasan, V., & Thompson, G. L. (1972). An operator theory of parametric programming for the transportation problem-I. Naval Research Logistics Quarterly, 19(2), 205–225.
2) Lotfi, M. M., & Tavakkoli-Moghaddam, R. (2013). A genetic algorithm using priority based encoding with new operators for fixed charge transportation problems. Applied Soft Computing, 13(5), 2711–2726.
3) Altassan, K. M., EI-Sherbiny, M. M., & Abid, A. D. (2014). Artificial immune algorithm for solving the fixed charge transportation problem. Applied Mathematics Information Sciences, 8(2), 751–759.
4) Sabbagh, M. S., Ghafari, H., & Mousavi, S. R. (2015). A new hybrid algorithm for the balanced transportation problem. Computers Industrial Engineering, 82,115–126.
5) Haley, K. B. (1962). The solid transportation problem. Operational Research, 10(4), 448–463.
6) Bhatia, H. L. (1981). Indefinite quadratic solid transportation problem. Journal of Information Optimization Sciences, 2(3), 297–303.
7) Pandian, P., & Anuradha, D. (2010). A new approach for solving solid transportation problems. Applied Mathematical Sciences, 4, 3603–3610.
8) Pandian, P., & Natarajan, G. (2010). A new method for finding an optimal solution for transportation problems. International Journal of Mathematical Sciences and Engineering Applications, 4(2), 59–65.
9) Sharma, V., Dahiya, K., & Verma, V. (2010). Capacitated two-stage time minimization transportation problem. Asia-Pacific Journal of Operational Research, 27(4), 457–476.

10) Hirsch, W. M., & Dantzig, G. B. (1968). The fixed charge problem. Naval Research Logistics Quarterly, 15(3), 413–424.
11) Adlakha, V., Kowalski, K., Wang, S., Lev, B., & Shen, W. (2014). On approximation of the fixed charge transportation problem. Omega, 43(2), 64–70.
12) Buson, E., Roberti, R., & Toth, P. (2014). A reduced-cost iterated local search heuristic for the fixed-charge transportation problem. Operations Research, 62(5), 1095–1106.
13) Yang, L., & Feng, Y. (2007). A bicriteria solid transportation problem with fixed charge under stochastic environment. Applied Mathematical Modelling, 31(12),2668–2683.
14) Sheng, Y., & Yao, K. (2012a). Fixed charge transportation problem in uncertain environment. Industrial Engineering and Management Systems, 11(2), 183–187.
15) Sheng, Y., & Yao, K. (2012b). A transportation model with uncertain costs and demands. Information: An International Interdisciplinary Journal, 15(8),3179–3186.
16) Cui, Q., & Sheng, Y. (2013). Uncertain programming model for solid transportation problem. Information: An International Interdisciplinary Journal, 16(2A), 1207–1214.
17) Mou, D., Zhou, W., & Chang, X. (2013). A transportation problem with uncertain truck times and unit costs. Industrial Engineering Management Systems, 12(1), 30–35.
18) Guo, H., Wang, X., & Zhou, S. (2015). A transportation problem with uncertain costs and random supplies. International Journal of e-Navigation and Maritime Economy, 2, 1–11.
19) Yang, L., Liu, P., Li, S., Gao, Y., & Ralescu, D. A. (2015). Reduction methods of type-2 uncertain variables and their applications to solid transportation problem.Information Science, 291, 204–237.
20) K.B. Haley, The solid transportation problem, Oper. Res. 11 (1962) 446–448.
21) A.K. Bit, M.P. Biswal, S.S. Alam, Fuzzy programming approach to multi-objective solid transportation problem, Fuzzy Sets Syst. 57 (1993) 183–194.
22) F. Jime´nez, J.L. Verdegay, Uncertain solid transportation problems, Fuzzy Sets Syst. 100 (1998) 45–57.
23) F. Jime´nez, J.L. Verdegay, Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach, Eur. J. Oper. Res.117 (1999) 485–510.
24) Y. Li, K. Ida, M. Gen, R. Kobuchi, Neural network approach for multicriteria solid transportation problem, Comput. Ind. Eng. 33 (1997) 465–468.
25) Y. Li, K. Ida, M. Gen, Improved genetic algorithm for solving multi-objective solid transportation problem with fuzzy numbers, Comput. Ind.Eng. 33 (1997) 589–592.
26) L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353.
27) U. Deshpande, A. Gupta, A. Basu, Task assignment with imprecise information for real-time operation in a supply chain, Appl. Soft Comput.5 (2004) 101–117.
28) M. Papadrakakis, N.D. Lagaros, Soft computing methodologies for structural optimization, Appl. Soft Comput. 3 (2003) 283–300.
29) B. Liu, Theory and Practice of Uncertain Programing, Physica-Verlag,New York, 2002.
30) Bo Zhang, Jin Peng, Shengguo Li, Lin Chen, Fixed charge solid transportation problem in uncertain environment and its algorithm ,Computers & Industrial Engineering 102 (2016) 186–197
31) Lixing yang,Linzhong Liu, fuzzy Fixed charge solid transportation problem and algorithm, applied soft computing 7(2007)879-889.