

Review of Applications of Graph Theory in Engineering

¹S. N. Banasode, ²Y.M.Umathar.

¹Department of Mathematics, R. L. Science Institute Belagavi, Karnataka, India.

²Department of Mathematics, K.L.E.Technological University, Hubballi, Karnataka, India.

Abstract: The use of Mathematics is significant in every field of Engineering, like Computer Science engineering, Networking, Electrical engineering, and many more That improves the effectiveness and applicability of existing methods and algorithms. [3] Graphs are excellent mathematical tools that are used to model the various types of relation between many physical circumstances. Most of the real world problem can be represented by graphs. This paper explains the concepts of graph theory and its applications in different field of engineering, like

- Network Engineering
- Computer science engineering
- Electrical Engineering etc

These applications demonstrate the objectives and importance of graph in Engineering. It explains the how the graph can be used to model many engineering problems.

Key words: Graph, Connectivity, Path, Shortest path, Electronic circuit, Networking, truth Table, Link, Impedence

1. Introductions:

1.1. Graph theory is branch of mathematics that deals with the study of graph, that are considered to be the mathematical structure helpful to have mathematical model with pair wise relation between objectives.

Graph is made up of two things. Set of vertices and set of edges Graphs give us many techniques and flexibility while defining and solving real world problems. Graphs have many features such as,

- Establishes relation between objects.
- Helps in modeling
- Helps in decision making.

As an example in networking Engineering, Network is system of points with distances between them. A network can represent a road, pipeline, cables, etc. The problem with network involves finding the shortest path between one point to another point in the network. [1]

2. Applications of graph in the various engineering field

2.1. Network Engineering: In [1] and [5] author have explained an application of graph in networking. In addition to that, Graph theoretic concepts have many applications in network engineering, such as connectivity, data gathering, Energy efficiency, traffic analysis, Finding shortest path and many more, where the term Graph and Network are equivalent. In graph theory nodes and edges are used, in networking links and lines are used. The term graph is used in mathematics and Network is used in Engineering. Particularly in computer engineering.

Graph based representation for the network system makes the problem much easier and will provide much accurate results.

2.1.1. Illustration: Graph representation of circuit network

Circuit network showing the Resistors in series and parallel

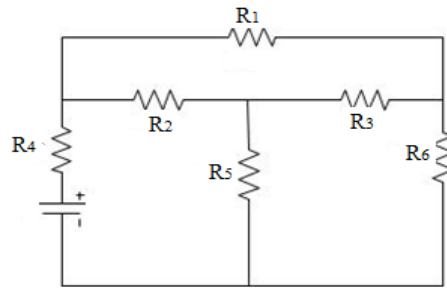


Figure 1

The logical truth table for the circuit can be shown as

	R_1	R_2	R_3	R_4	R_5	R_6
R_1	0	1	1	1	0	1
R_2	1	0	1	1	1	0
R_3	1	1	0	0	1	1
R_4	1	1	0	0	1	1
R_5	0	1	1	1	0	1
R_6	1	0	1	1	1	0

Graph model that is used to represent circuit network

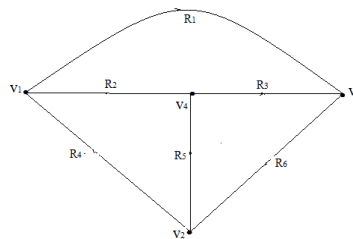


Figure 2

Again graph can be represented in one of the matrix form that is adjacency matrix taking the vertex set as $V = \{R_1, R_2, R_3, R_4, R_5, R_6\}$

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

From the adjacency matrix the energy of a graph, minimum dominating energy of a are defined. And the energy of a graph is defined as absolute sum of the Eigen values of adjacency matrix of a graph. The characteristic equation of adjacency matrix A is

$$(A - \lambda I) = \begin{bmatrix} \lambda & 1 & 1 & 1 & 0 & 1 \\ 1 & \lambda & 1 & 1 & 1 & 0 \\ 1 & 1 & \lambda & 0 & 1 & 1 \\ 1 & 1 & 0 & \lambda & 1 & 1 \\ 0 & 1 & 1 & 1 & \lambda & 1 \\ 1 & 0 & 1 & 1 & 1 & \lambda \end{bmatrix} = 0$$

That is, the characteristic equation is $\lambda^6 - 12\lambda^4 - 16\lambda^3 = 0$ and eigen values are

$\lambda = 0, 0, 0, -2, -2, \text{ and } 4$. Therefore energy of a graph is

$$E(G) = |0|\{\text{three times}\} + |-2|\{\text{two times}\} + |4|(1) = 8$$

2.2. Electrical Engineering: Electrical Circuits are closed loop formed by Source, Wires, Load and Switches. When switch is turned on electrical circuit is complete. Then current flows from negative terminal of source of power. Here we apply the concept of Graph Theory to solve Electrical Circuit Problems. In [4] author have discussed regarding an application of graph in electric circuits. Using the definition of a link and cycle matrix for the graph, we consider one more application of graph in electric field.

2.2.1. Link: A branch of a graph which does not belongs to particular tree under consideration.

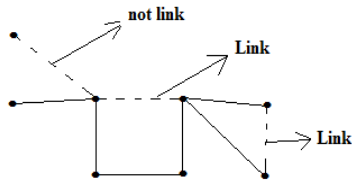


Figure 3

2.2.2. Cycle matrix (Loop matrix) : It is matrix with elements as b_{ij}

$$b_{ij} = \begin{cases} 1 & \text{Branch } b_j' \text{ direction in the loop is same as direction of loop} \\ -1 & \text{Branch } b_j' \text{ direction in the loop is opposite as direction of loop} \\ 0 & \text{When } b_j \text{ is not in the loop} \end{cases}$$

Loop matrix is a $i \times b$ matrix where i is the number of loops and b is the number of branches. A set of branches contained in a loop such that each loop contains one link and the remaining are the tree branches.

2.2.3. Illustration:



Figure 4

Selecting (2,4,5) as a tree and the co-tree is (1,3). For the above fig oriented cycle matrix (loop matrix) is

$$M = B = \begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

2.2.4. Impedence: Electric impedance is the measure of opposition that a circuit presents to a current when voltage is applied.

Consider an electric circuit shown bellow, the graph for the circuit is shown.

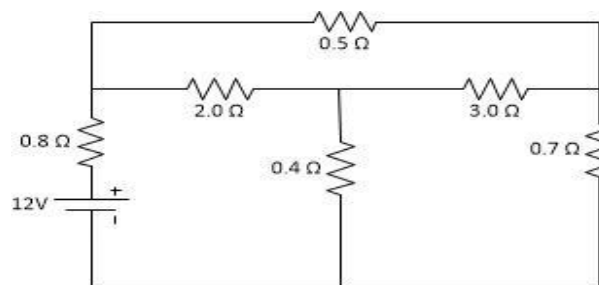
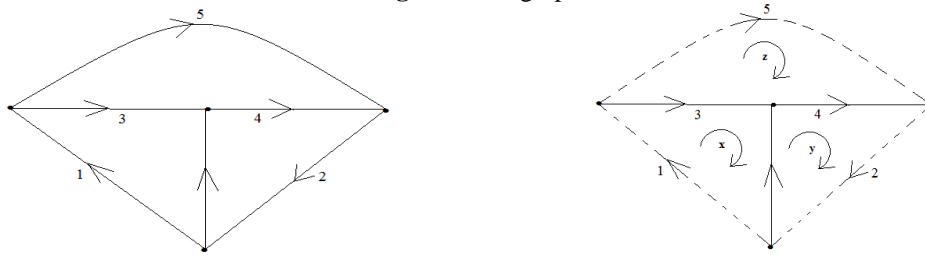


Figure 5

Now we discuss the graphical method of finding branch currents

For the electric circuit shown in the **figure 5**, the graph for the circuit is shown.



Graph representing circuit

Figure 6

Tree and Co-tre

The oriented cycle matrix is,
$$B = \begin{matrix} x \\ y \\ z \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix}$$

If branch impedance matrix is denoted by Z_b , then $(i, j)^{th}$ the elements of the matrix Z_b are defined as

$$Z_b = \begin{cases} R_i & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases}$$

$$\Rightarrow \text{Branch impedance matrix, } Z_b = \begin{bmatrix} 0.8 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.7 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 2.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 3.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.4 \end{bmatrix}$$

Then branch source voltage vector is obtained as the product of oriented cycle matrix and branch impedance matrix, That is,

$$BZ_b = \begin{bmatrix} 0.8 & 0 & 2 & 0 & 0 & -4 \\ 0 & 0.7 & 0 & 3 & 0 & 4 \\ 0 & 0 & -2 & -2 & 0.5 & 0 \end{bmatrix} = E_{gb} \text{ (Say)}$$

And the mess (loop) impedance matrix is defined as $E_{gb}B'$

Therefore we get,
$$E_{gb}B' = BZ_bB' = \begin{bmatrix} 6.8 & -4 & -2 \\ -4 & 7.7 & -3 \\ -2 & -3 & 5.5 \end{bmatrix} = Z_l \text{ (say)}$$

Mess source voltage vector is given by BE_b which is obtained as the product of the matrix B

And E_b . That is
$$BE_b = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

If I_l represents the currents in the loops, then
$$I_l = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Loop equations are $Z_l I_l = BE_b$

$$\Rightarrow \begin{bmatrix} 6.8 & -4 & -2 \\ -4 & 7.7 & -3 \\ -2 & -3 & 5.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x = 6.672A, \quad y = 5.602A, \quad z = 5.482A.$$

Hence proposed graph theoretical method can be applied to solve electrical circuit problems to branch currents in the circuit.

2.3. Computer Science Engineering: Graph theory can be used in research areas of computer science. In [2] [3] uses of graph in computer engineering are explained. Along with those few more application are explained. Some of them are data structure, Image processing, Web designing, data mining, clustering, etc. Some of them are discussed here.

2.3.1. Data structure: It is a systematic method of organizing and storing the data. It is designed to suit a specific purpose so that it can be accessed and worked with appropriate ways. The selection of the model for the data depends on the information of the real world and the structure should be simple enough that can effectively process data whenever it is required.

2.3.2. Image processing: It is process of analysis and manipulation of digitized image, especially in order to improve the quality of an image. It is a method by which the information from the image is extracted. Using graph theory concept image processing method can be improved. The graph theory provides the calculation of alignment of the picture. It finds the mathematical constraints using **minimal spanning tree**.

2.3.3. Web designing: Web designing is also method of creating the web site that encompasses several different aspects including web page, layout, content production and graphic design. While the term web design and web development are often used interchangeably. Web design is a subset of web development. Here web pages are represented by the vertices and the links between the web pages are represented by the edges of the graph. In the web community the vertices are representing the classes of objects and each vertex represents one nature of the object and each vertex representing one type of object is connected to every vertex representing the other kind of the object. In graph theory same concept is explained by **complete bipartite graphs**.

The concepts of weighted graph in graph theory, where weights have been assigned to the edges of the graph are used to represent the structure, wherein the paired connections have numerical values. If the edges represent the roads between the places then problem is to find shortest path connecting all places which is solved by minimum spanning tree. There are many methods of finding minimum spanning tree, the new and simple approach that we propose is an **Edge Elimination Method** is illustrated here.

Considering the weighted graph shown in the following figure

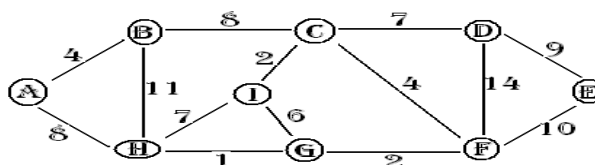


Figure 7 (weighted Graph)

The algorithm for the edge elimination method is

Step 1: The edge with highest weight from each smallest cycle is eliminated with care that graph is not disconnected.

Step 2: step 1 repeated for the cycles formed after the step 1.

Step 3: Continue the process of elimination of edges of highest weight from each smallest cycle obtained in the step 2 using step 1, until no cycle remains in the graph. So that there are exactly (n-1) edges, n is the number of vertices. The resulting graph is the minimum spanning tree as shown.

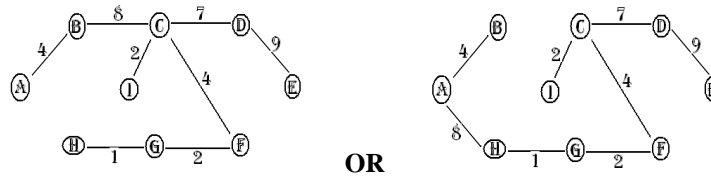


Figure 8 (Minimum spanning tree of 37)

3. Conclusion: This paper is prepared to help the students of engineering to gain the depth knowledge of graph theory and its relevance with other subject of engineering. In this paper we have focused on the application of graph theory in few branches of engineering. We conclude that, this paper motivate not only the students of engineering but also engineering faculty.

4. References:

- 1) B. Sadavare, R V Kulkarni, *A Review of Application of Graph Theory for Network*, International Journal of Computer science and Information technologies, 3(6), 2012.
- 2) Ferozuddin Riaz, Khidir M Ali, *Application of Graph Theory in Computer Science*, 2011, International conference on Computational Intelligence System and Network.
- 3) Rishi Pal Singh, Vandana, *Application of Graph Theory in Computer Science and Engineering*, 104(1), October 2014.
- 4) Samai'la Abdullah, *An Application of Graph theory to the Electric Circuit Using Matrix Method*, ISRO Journal of Mathematics, 10(2), Mar-Apr 2014.
- 5) Suman Deswal and Anita Singhrova, *Application of Graph Theory in Communication Networks*, 1(2), October 2012.