International Journal of Mathematics Trends and Technology (IJMTT) - Volume 57 Issue 4 - May 2018

3-Equitable Prime Cordial Labeling of Middle Graph of Different Graphs

J. P.Jena [1], G. V. Ghodasara [2]

[1] *Research Scholar, RK. University, Rajkot 360020, Gujarat - INDIA*

[2] *Assistant Professor in Mathematics, H. & H. B. Kotak Institute of Science, Rajkot - 360001, Gujarat - INDIA*

Abstract

In this paper we discuss 3-equitable prime cordial labeling of middle graph of cycle, cycle with one chord, path and tadpole. **Key words:** Middle graph, cycle, path, 3-equitable prime cordial graph.

2010 Mathematics Subject Classification : 05C78

I Introduction

We consider simple, finite, connected and undirected graph $G = (V, E)$. For various graph theoretic notations and terminology we follow Gross and Yellen[2] and for the concepts and terminology of number theory we follow Burton[1]. If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling. A useful survey to know about the numerous graph labeling methods is given by J. A. Gallian[3].

Definition 1. *[5] A 3-equitable prime cordial labeling of a graph G with vertex set $V(G)$ and edge set $E(G)$ is a bijection $f : V(G) \rightarrow \{1, 2, 3, \dots |V(G)|\}$ such that the induced edge function $f^* : E(G) \rightarrow \{0, 1, 2\}$ defined by*

$$f^*(uv) = \begin{cases} 1 & \text{if } gcd(f(u), f(v)) = 1 \text{ and} \\ & gcd(f(u) + f(v), f(u) - f(v)) = 1; \\ 2 & \text{if } gcd(f(u), f(v)) = 1 \text{ and} \\ & gcd(f(u) + f(v), f(u) - f(v)) = 2; \\ 0 & \text{otherwise} \end{cases}$$

satisfies the condition $|e_f(i) - e_f(j)| \leq 1$, $0 \leq i, j \leq 2$, where $e_f(0)$, $e_f(1)$, $e_f(2)$ denote the number of edges with label 0, 1 and 2 respectively under f^*.

A graph which admits 3-equitable prime cordial labeling is called a 3-equitable prime cordial graph. S. Murugesan et al.[5] introduced 3-equitable prime cordial labeling and proved that paths P_n, cycles C_n for $n \geq 4$, stars $K_{1,n}$ when $n \equiv 1(mod\ 3)$ and complete graphs K_n for $n \leq 2$ admit 3-equitable prime cordial labeling.

II MAIN RESULTS

Definition 2. *The middle graph, $M(G)$, of a graph G is the graph whose vertex set is $V(G) \bigcup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.*

Theorem 1. *$M(C_n)$ is 3-equitable prime cordial.*

Proof. Let $v_1, v_2, \ldots, v_n, v'_1, v'_2, \ldots, v'_n$ be the vertices of $M(C_n)$, where $v_1, v_2, \ldots, v_n$ are the vertices corresponding to cycle C_n and $v'_1, v'_2, \ldots, v'_n$ are the vertices added corresponding to the edges $e_1, e_2, \ldots, e_n$ of C_n in order to obtain $M(C_n)$.
To define $f : V(M(C_n)) \rightarrow \{1, 2, \ldots, 2n\}$, we consider the following cases.
Case 1: $M(C_3)$ is not 3-equitable prime cordial graph.
(Refer Case-1: $n \equiv 0, 1, 4(mod6)$, Theorem-3.2[4]).
Case 2: $n \equiv 0(mod\ 4)$

$$\begin{aligned} f(v_1) &= 3 \\ f(v'_1) &= 5 \\ f(v'_n) &= 1 \end{aligned}$$

For $2 \leq i \leq n$:

$$f(v_i) = \begin{cases} 2i-1, & i \equiv 0(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2(i-1), & i \equiv 2(mod\ 4). \\ 2i, & i \equiv 3(mod\ 4). \end{cases}$$

For $2 \leq i \leq n-1$:

$$f(v'_i) = \begin{cases} 2i+1, & i \equiv 0,1(mod\ 4). \\ 2i, & i \equiv 2(mod\ 4) \\ 2(i+1), & i \equiv 3(mod\ 4). \end{cases}$$

Case 3: $n \equiv 1, 3(mod\ 4)$

$$\begin{aligned} f(v_2) &= 2 \\ f(v_3) &= 4 \\ f(v_n) &= 2n \\ f(v'_2) &= 6 \\ f(v'_n) &= 1 \end{aligned}$$

For $i = 1, 4 \leq i \leq n-1$:

$$f(v_i) = \begin{cases} 2i-1, & i \equiv 0(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2(i-1), & i \equiv 2(mod\ 4). \\ 2i, & i \equiv 3(mod\ 4). \end{cases}$$

For $1 \leq i \leq (n-1), i \neq 2$:

$$f(v'_i) = \begin{cases} 2i+1, & i \equiv 0,1(mod\ 4). \\ 2i, & i \equiv 2(mod\ 4). \\ 2(i+1), & i \equiv 3(mod\ 4). \end{cases}$$

Case 4: $n \equiv 2(mod\ 4)$

$$\begin{aligned} f(v_1) &= 3 \\ f(v_n) &= 2n \\ f(v'_1) &= 5 \\ f(v'_2) &= 2 \end{aligned}$$

For $2 \leq i \leq n-1$:

$$f(v_i) = \begin{cases} 2i, & i \equiv 0(mod\ 4). \\ 2i-1, & i \equiv 1(mod\ 4). \\ 2i+3, & i \equiv 2(mod\ 4). \\ 2(i-1), & i \equiv 3(mod\ 4). \end{cases}$$

For $3 \leq i \leq n$:

$$f(v_i') = \begin{cases} 2(i+1), & i \equiv 0(mod\ 4). \\ 2i+1, & i \equiv 1,2(mod\ 4). \\ 2i, & i \equiv 3(mod\ 4). \end{cases}$$

Thus in each case, the condition for 3-equitable prime cordial labeling is satisfied.
i.e. $|e_f(i) - e_f(j)| \leq 1, 1 \leq i, j \leq n$. Hence the graph $M(C_n)$ is 3-equitable prime cordial graph. □

Example 1. *3-equitable prime cordial labeling of $M(C_4)$ is shown in* Figure 1. *It is the case related to $n \equiv 0(mod\ 4)$.*

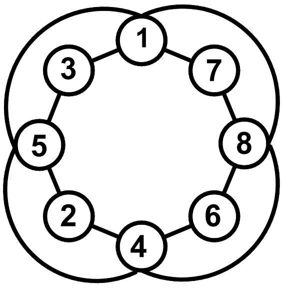

Fig. 1 3-equitable prime cordial labeling of the graph obtained by $M(C_4)$

Theorem 2. *Middle graph of cycle C_n with one chord is 3-equitable prime cordial.*

Proof. Let G be the cycle C_n with one chord. Let $v_1, v_2, \ldots, v_n$ be the vertices of G and let $e = v_2v_n$ be the chord in G. Let $v_1, v_2, \ldots, v_n, v_1', v_2', \ldots, v_{n+1}'$ be the vertices of $M(G)$, where $v_1', v_2', \ldots, v_{n+1}'$ are the vertices added corresponding to the edges $e_1, e_2, \ldots, e_n, e$ respectively in order to obtain $M(G)$.
We define $f : V(M(G)) \to \{1, 2, \ldots, 2n+1\}$ as follows.
$f(v_{n+1}') = 3$.
Case 1: $n \equiv 0(mod\ 4)$

$$f(v_1) = 4$$
$$f(v_3) = 1.$$

For $i = 2,\ 4 \leq i \leq n$:

$$f(v_i) = \begin{cases} 2i-1, & i \equiv 0(mod\ 4). \\ 2i+1, & i \equiv 1(mod\ 4). \\ 2(i+2), & i \equiv 2(mod\ 4). \\ 2(i-1), & i \equiv 3(mod\ 4). \end{cases}$$

For $1 \leq i \leq n$:

$$f(v_i') = \begin{cases} 2i+1, & i \equiv 0(mod\ 4). \\ 2i, & i \equiv 1(mod\ 4). \\ 2(i+1), & i \equiv 2(mod\ 4). \\ 2i-1, & i \equiv 3(mod\ 4). \end{cases}$$

Case 2: $n \equiv 1(mod\ 4)$

$$\begin{aligned} f(v_1) &= 5 \\ f(v_2) &= 6 \\ f(v_n) &= 2 \\ f(v'_1) &= 7 \\ f(v'_{n-1}) &= 4 \\ f(v'_n) &= 1 \end{aligned}$$

For $3 \leq i \leq n-1$:

$$f(v_i) = \begin{cases} 2i, & i \equiv 0(mod\ 4). \\ 2i+4, & i \equiv 1(mod\ 4). \\ 2i+3, & i \equiv 2(mod\ 4). \\ 2i+5, & i \equiv 3(mod\ 4). \end{cases}$$

For $2 \leq i \leq n-2$:

$$f(v'_i) = \begin{cases} 2(i+2), & i \equiv 0(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2i+5, & i \equiv 2(mod\ 4). \\ 2i+4, & i \equiv 3(mod\ 4). \end{cases}$$

Case 3: $n \equiv 2(mod\ 4)$

$$\begin{aligned} f(v_1) &= 1 \\ f(v_2) &= 7 \\ f(v_3) &= 8 \\ f(v_n) &= 4 \\ f(v'_1) &= 5 \\ f(v'_2) &= 9 \\ f(v'_n) &= 6 \\ f(v'_{n-1}) &= 2. \end{aligned}$$

For $4 \leq i \leq n-1$:

$$f(v_i) = \begin{cases} 2i+5, & i \equiv 0(mod\ 4). \\ 2i, & i \equiv 1(mod\ 4). \\ 2i+4, & i \equiv 2(mod\ 4). \\ 2i+3, & i \equiv 3(mod\ 4). \end{cases}$$

For $3 \leq i \leq n-2$:

$$f(v'_i) = \begin{cases} 2i+4, & i \equiv 0(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2i+5, & i \equiv 2(mod\ 4). \\ 2(i+2), & i \equiv 3(mod\ 4). \end{cases}$$

Case 4: $n \equiv 3(mod\ 4)$

$$\begin{aligned} f(v_3) &= 8 \\ f(v_n) &= 4 \\ f(v_{n-2}) &= 2(i+1) \\ f(v'_2) &= 6 \\ f(v'_n) &= 1 \\ f(v'_{n-1}) &= 2 \end{aligned}$$

For $1 \le i \le n-2,\ i \neq 3:$

$$f(v_i) = \begin{cases} 2(i+2), & i \equiv 0(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2i+5, & i \equiv 2(mod\ 4). \\ 2i, & i \equiv 3(mod\ 4). \end{cases}$$

For $i = 1,\ 3 \le i \le n-2:$

$$f(v'_i) = \begin{cases} 2i+3, & i \equiv 0(mod\ 4). \\ 2i+5, & i \equiv 1(mod\ 4). \\ 2(i+2), & i \equiv 2,3(mod\ 4). \end{cases}$$

Thus in each case we have $|e_f(0) - e_f(1)| \le 1$.
Hence middle graph of C_n with one chord is 3-equitable prime cordial graph. □

Example 2. *3-equitable prime cordial labeling of middle graph of C_5 with one chord is shown in* Figure 2. *It is the case related to $n \equiv 1(mod\ 4)$.*

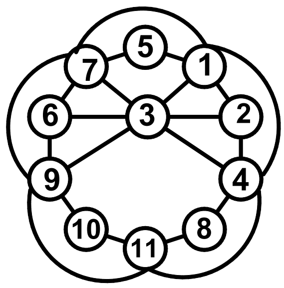

Fig. 2 3-equitable prime cordial labeling of the graph obtained by middle graph of C_5 with one chord

Theorem 3. *$M(P_n)$ is 3-equitable prime cordial.*

Proof. Let $v_1, v_2, \ldots, v_n, v'_1, v'_2, \ldots, v'_n$ be the vertices of $M(P_n)$, where $v_1, v_2, \ldots, v_n$ are the vertices corresponding to path P_n and $v'_1, v'_2, \ldots, v'_n$ are the vertices added corresponding to the edges $e_1, e_2, \ldots, e_n$ of P_n in order to obtain $M(P_n)$.
To define $f : V(M(P_n)) \rightarrow \{1, 2, \ldots, 2n-1\}$, we consider the following cases.
Case 1: $n \equiv 1, 3(mod\ 4)$ For $1 \le i \le n:$

$$f(v_i) = \begin{cases} 2i, & i \equiv 0(mod\ 4). \\ 2i+4, & i \equiv 1(mod\ 4). \\ 2i+3, & i \equiv 2(mod\ 4). \\ 2i+5, & i \equiv 3(mod\ 4) \end{cases}$$

$$f(v_i') = \begin{cases} 2(i+2), & i \equiv 0(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2i+5, & i \equiv 2(mod\ 4). \\ 2i+4, & i \equiv 3(mod\ 4) \end{cases}$$

Case 2: $n \equiv 2(mod\ 4)$
$f(v_n) = 2(n-1)$.
The remaining vertices are labeled same as in Case-1.
Case 3: $n \equiv 0(mod\ 4)$
$f(v_n) = 2n-1, f(v_n') = 2(n-1)$.
The remaining vertices are labeled same as in Case-1.
Thus in each case we have $|e_f(0) - e_f(1)| \leq 1$.
Hence the graph $M(P_9)$ is 3-equitable prime cordial graph. □

Example 3. *3-equitable prime cordial labeling of $M(P_9)$ is shown in* Figure 3. *It is the case related to $n \equiv 1, 3(mod\ 4)$.*

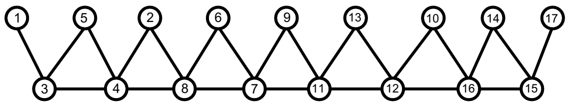

Fig. 3 3-equitable prime cordial labeling of the graph obtained by $M(P_9)$

Definition 3. *Tadpole $T(n, m)$ is the graph in which path P_m is attached to any one vertex of cycle C_n by a bridge.*

Theorem 4. *$M(T(n, m))$ is 3-equitable prime cordial graph.*

Proof. Let $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_m$ be the vertices of tadpole $T(n, m)$, where $v_1, v_2, \ldots\ v_n$ be the vertices of cycle C_n and $u_1, u_2, \ldots, u_m$ be the vertices of path P_m of length $m-1$ attached to the vertex v_n of cycle C_n by a bridge. Note that u_1 is adjacent to v_n.

Let $V(M(T(n, m))) = v_1, v_1', v_2, v_2', \ldots, v_n, v_n', u_1', u_1, u_2', u_2, \ldots, u_m', u_m$, where $v_1', v_2', \ldots,\ v_n'$ be the vertices added corresponding to the edges $e_1, e_2, \ldots, e_n$ of cycle C_n and $u_1', u_2', \ldots,\ u_m'$ be the vertices added corresponding to the edges $e_{n+1}, e_{n+2}, \ldots, e_{n+m}$ in order to obtain middle graph of $T(n, m)$ (e_{n+1} is a bridge and $e_{n+2}, e_{n+3}, \ldots, e_{n+m}$ are edges of P_m).
To define labeling function $f : V(G) \to \{1, 2, \ldots, 2(n+m)\}$ we consider the following cases.
Case 1: $n = 3$.

$$\begin{aligned} f(v_1) &= 4 \\ f(v_2) &= 1 \\ f(v_3) &= 5 \\ f(v_1') &= 6 \\ f(v_2') &= 3 \\ f(v_3') &= 2. \end{aligned}$$

Subcase 1: $m \equiv 0, 3(mod\ 4)$
For $1 \leq j \leq m$:

$$f(u_j) = \begin{cases} 2(n+j)-1, & j \equiv 0(mod\ 4). \\ 2(n+j)+3, & j \equiv 1(mod\ 4). \\ 2(n+j-1), & j \equiv 2(mod\ 4). \\ 2(n+j), & j \equiv 3(mod\ 4). \end{cases}$$

$$f(u'_j) = \begin{cases} 2(n+j), & j \equiv 0(mod\ 4). \\ 2(n+j)-1, & j \equiv 1,2(mod\ 4). \\ 2(n+j-1), & j \equiv 3(mod\ 4). \end{cases}$$

Subcase 2: $m \equiv 1(mod\ 4)$

$f(u_m) = 2(n+m)$.

The remaining vertices are labeled same as in Subcase-1.

Subcase 3: $m \equiv 2(mod\ 4)$

$$\begin{aligned} f(u_m) &= 2(n+m) \\ f(u'_m) &= 2(n+m-1) \\ f(u_{m-1}) &= 2(n+m)-1. \end{aligned}$$

The remaining vertices are labeled same as in Subcase-1.

Case 2: $n \equiv 0(mod\ 4)$

$$\begin{aligned} f(v_1) &= 4 \\ f(v_2) &= 1 \\ f(v'_1) &= 6 \\ f(v'_2) &= 3 \\ f(v'_n) &= 2. \end{aligned}$$

Subcase 1: $m = 1$

$$\begin{aligned} f(v_n) &= 2i+1 \\ f(u_1) &= 2(n+1) \\ f(u'_1) &= 2(n) \end{aligned}$$

For $3 \leq i \leq n-1$:

$$f(v_i) = \begin{cases} 2i+3, & i \equiv 0(mod\ 4). \\ 2(i-1), & i \equiv 1(mod\ 4). \\ 2i, & i \equiv 2(mod\ 4). \\ 2i-1, & i \equiv 3(mod\ 4). \end{cases}$$

$$f(v'_i) = \begin{cases} 2i+1, & i \equiv 0,3(mod\ 4). \\ 2i, & i \equiv 1(mod\ 4). \\ 2(i+1), & i \equiv 2(mod\ 4). \end{cases}$$

Subcase 2: $m \equiv 2,3(mod\ 4)$ For $3 \leq i \leq n$:

$$f(v_i) = \begin{cases} 2i+3, & i \equiv 0(mod\ 4). \\ 2(i-1), & i \equiv 1(mod\ 4). \\ 2i, & i \equiv 2(mod\ 4). \\ 2i-1, & i \equiv 3(mod\ 4). \end{cases}$$

For $3 \leq i \leq n-1$:

$$f(v'_i) = \begin{cases} 2i+1, & i \equiv 0,3(mod\ 4). \\ 2i, & i \equiv 1(mod\ 4). \\ 2(i+1), & i \equiv 2(mod\ 4). \end{cases}$$

For $1 \leq j \leq m$:

$$f(u_j) = \begin{cases} 2(n+j)+3, & j \equiv 0(mod\ 4). \\ 2(n+j-1), & j \equiv 1(mod\ 4). \\ 2(n+j), & j \equiv 2(mod\ 4). \\ 2(n+j)-1, & j \equiv 3(mod\ 4). \end{cases}$$

$$f(u'_j) = \begin{cases} 2(n+j)-1, & j \equiv 0,1(mod\ 4). \\ 2(n+j-1), & j \equiv 2(mod\ 4). \\ 2(n+j), & j \equiv 3(mod\ 4). \end{cases}$$

Subcase 3: $m \equiv 1(mod\ 4)$

$$\begin{aligned} f(u_m) &= 2(n+m) \\ f(u'_m) &= 2(n+m)-1 \\ f(u_{m-1}) &= 2(n+m)+1 \end{aligned}$$

The remaining vertices are labeled same as in Subcase-2.
Subcase 4: $m \equiv 0(mod\ 4)$

$$\begin{aligned} f(u_m) &= 2(n+m) \\ f(u'_m) &= 2(n+m)-1 \end{aligned}$$

The remaining vertices are labeled same as in Subcase-2.
Case 3: $n \equiv 3(mod\ 4), n \neq 3$

$$\begin{aligned} f(v_1) &= 6 \\ f(v_2) &= 1 \\ f(v'_1) &= 4 \\ f(v'_2) &= 3 \\ f(v'_n) &= 2 \end{aligned}$$

For $3 \leq i \leq n$:

$$f(v_i) = \begin{cases} 2i+3, & i \equiv 0(mod\ 4). \\ 2(i-1), & i \equiv 1(mod\ 4). \\ 2i, & i \equiv 2(mod\ 4). \\ 2i-1, & i \equiv 3(mod\ 4). \end{cases}$$

For $3 \leq i \leq n-1$:

$$f(v'_i) = \begin{cases} 2i+1, & i \equiv 0,3(mod\ 4). \\ 2i, & i \equiv 1(mod\ 4). \\ 2(i+1), & i \equiv 2(mod\ 4). \end{cases}$$

For $1 \leq j \leq m$:
Subcase 1: $m \equiv 0,3(mod\ 4)$

$$f(u_j) = \begin{cases} 2(n+j)-1, & j \equiv 0(mod\ 4). \\ 2(n+j)+3, & j \equiv 1(mod\ 4). \\ 2(n+j-1), & j \equiv 2(mod\ 4). \\ 2(n+j), & j \equiv 3(mod\ 4). \end{cases}$$

$$f(u'_j) = \begin{cases} 2(n+j), & j \equiv 0(mod\ 4). \\ 2(n+j)-1, & j \equiv 1,2(mod\ 4). \\ 2(n+j-1), & j \equiv 3(mod\ 4). \end{cases}$$

Subcase 2: $m \equiv 1(mod\ 4)$

$f(u_m) = 2(n+m)$.

The remaining vertices are labeled same as in Subcase-1.

Subcase 3: $m \equiv 2(mod\ 4)$

$$\begin{aligned} f(u_m) &= 2(n+m) \\ f(u_{m-1}) &= 2(n+m)+1 \\ f(u'_m) &= 2(n+m-1) \end{aligned}$$

The remaining vertices are labeled same as in Subcase-1.

Case 4: $n \equiv 2(mod\ 4)$

$$\begin{aligned} f(v_n) &= 1 \\ f(v'_n) &= 3 \\ f(v'_{n-1}) &= 4 \end{aligned}$$
$$f(v_i) = 2i, \quad 1 \leq i \leq 2$$

For $3 \leq i \leq n-1$:

$$f(v_i) = \begin{cases} 2i, & i \equiv 0(mod\ 4). \\ 2(i+2), & i \equiv 1(mod\ 4). \\ 2i+3, & i \equiv 2,3(mod\ 4). \end{cases}$$

For $1 \leq i \leq 2$:

$$f(v'_i) = \begin{cases} 2i+5. \end{cases}$$

For $3 \leq i \leq n-2$:

$$f(v'_i) = \begin{cases} 2(i+2), & i \equiv 0,3(mod\ 4). \\ 2i+3, & i \equiv 1(mod\ 4). \\ 2(i+3)+1, & i \equiv 2(mod\ 4). \end{cases}$$

$f(u_1) = 2$.

For $2 \leq j \leq m$:

Subcase 1: $m \equiv 0,1(mod\ 4)$

$$f(u_j) = \begin{cases} 2(n+j), & j \equiv 0(mod\ 4). \\ 2(n+j)-1, & j \equiv 1(mod\ 4). \\ 2(n+j)+3, & j \equiv 2(mod\ 4). \\ 2(n+j-1), & j \equiv 3(mod\ 4). \end{cases}$$

For $1 \leq j \leq m$:

$$f(u'_j) = \begin{cases} 2(n+j-1), & j \equiv 0(mod\ 4). \\ 2(n+j), & j \equiv 1(mod\ 4). \\ 2(n+j)-1, & j \equiv 2,3(mod\ 4). \end{cases}$$

Subcase 2: $m \equiv 2(mod\ 4)$

$$f(u_m) = 2(n+m)$$

The remaining vertices are labeled same as in Subcase-1.

Subcase 3: $m \equiv 3(mod\ 4)$

$$\begin{aligned} f(u_m) &= 2(n+m) \\ f(u'_m) &= 2(n+m-1) \\ f(u_m) &= 2(n+m)+1 \end{aligned}$$

The remaining vertices are labeled same as in Subcase-1.

Case 5: $n \equiv 1(mod\ 4)$

For $1 \leq i \leq n$:

$$f(v_i) = \begin{cases} 2i+1, & i \equiv 0(mod\ 4). \\ 2i, & i \equiv 1(mod\ 4). \\ 2(i+2), & i \equiv 2(mod\ 4). \\ 2i-1, & i \equiv 3(mod\ 4). \end{cases}$$

$$f(v_i') = \begin{cases} 2i+3, & i \equiv 0(mod\ 4). \\ 2(i+1), & i \equiv 1(mod\ 4). \\ 2(i+1), & i \equiv 2(mod\ 4). \\ 2i+1, & i \equiv 3(mod\ 4). \end{cases}$$

$$\begin{aligned} f(u_m) &= 1 \\ f(u_m') &= 4 \\ f(u_1') &= 6 \\ f(u_2') &= 3. \end{aligned}$$

Subcase 1: $m \equiv 1, 3(mod\ 4)$

For $1 \leq j \leq m-1$:

$$f(u_j) = \begin{cases} 2(n+j)+1, & j \equiv 0(mod\ 4). \\ 2(n+j), & j \equiv 1(mod\ 4). \\ 2(n+j+1), & j \equiv 2(mod\ 4). \\ 2(n+j)-3, & j \equiv 3(mod\ 4). \end{cases}$$

For $3 \leq j \leq m-1$:

$$f(u_j') = \begin{cases} 2(n+j)-1, & j \equiv 0, 3(mod\ 4). \\ 2(n+j-1), & j \equiv 1(mod\ 4). \\ 2(n+j), & j \equiv 2(mod\ 4). \end{cases}$$

Subcase 2: $m \equiv 2(mod\ 4)$

$$f(u_m) = 2(n+m)-1$$

The remaining vertices are labeled same as in Subcase-1.

Subcase 3:$m \equiv 0(mod\ 4)$

$$f(u_m) = 2(n+m)$$

The remaining vertices are labeled same as in Subcase-1.

Thus in each cases we have $|e_f(0) - e_f(1)| \leq 1$.

Hence the graph $M(T(7, 3))$ under consideration is 3-equitable prime cordial graph.

□

Example 4. *3-equitable prime cordial labeling of* $M(T(7, 3))$ *is shown in* Figure 4. *It is the case related to* $n \equiv 3(mod\ 4), n \neq 3$ *and* $m \equiv 0, 3(mod\ 4)$.

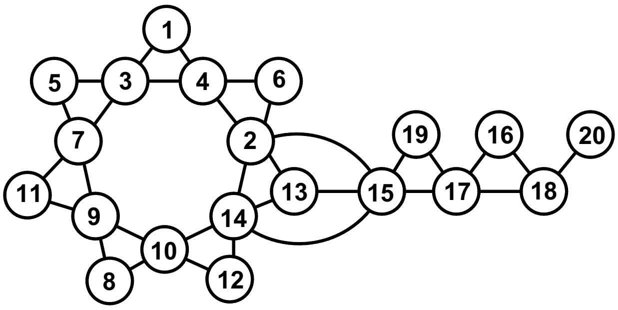

Fig. 4 3-equitable prime cordial labeling of the graph obtained by $M(T(7,3))$

Definition 4. *The crown, $(C_n \bigodot K_1)$, is the graph obtained by joining a pendant vertex to each vertex of the cycle C_n by an edge.*

Theorem 5. *$M(C_n \bigodot K_1)$ is 3-equitable prime cordial.*

Proof. Let $v_1, v_2, \dots, v_n$ be the rim vertices and $v_{n+1}, v_{n+2}, \dots, v_{2n}$ be the pendant vertices of crown $M(C_n \bigodot K_1)$, where $v'_1, v'_2, \dots, v'_n$ are the vertices divides corresponding to the rim edges $e_1, e_2, \dots, e_n$ and $v'_{n+1}, v'_{n+2}, \dots, v'_{2n}$ are the vertices divides corresponding to the pendant edges $e_{n+1}, e_{n+2}, \dots, e_{2n}$ in order to obtain $M(C_n \bigodot K_1)$.
To define $f : V(M(C_n \bigodot K_1)) \to \{1, 2, \dots, 4n\}$, we consider the following cases.
Case 1: $n \equiv 0(mod\ 3)$

$$f(v_1) = 4n$$

For $1 \le i \le n$:

$$f(v_i) = \begin{cases} 4(i-1), & i \equiv 0(mod\ 3). \\ 4i, & i \equiv 1(mod\ 3). \\ 4i-1, & i \equiv 2(mod\ 3). \end{cases}$$

$$f(v'_i) = \begin{cases} 4i-3, & i \equiv 0(mod\ 3). \\ 4(i-1), & i \equiv 1(mod\ 3). \\ 4i-7, & i \equiv 2(mod\ 3). \end{cases}$$

For $n+1 \le i \le 2n$:

$$f(v_i) = \begin{cases} 4i-2, & i \equiv 0(mod\ 3). \\ 4i+2, & i \equiv 1(mod\ 3). \\ 4i-3, & i \equiv 2(mod\ 3). \end{cases}$$

$$f(v'_i) = \begin{cases} 4i-1, & i \equiv 0(mod\ 3). \\ 4i-2, & i \equiv 1(mod\ 3). \\ 4i-5, & i \equiv 2(mod\ 3). \end{cases}$$

Case 2: $n \equiv 1(mod\ 3)$

$$\begin{aligned} f(v_{2n-1}) &= 4i-2. \\ f(v'_n) &= 4i-2. \\ f(v_{2n}) &= 4i-3. \\ f(v'_{n-1}) &= 4i-3. \\ f(v'_{2n-1}) &= 4(2n-1). \\ f(v'_{2n}) &= 8n+3. \end{aligned}$$

The remaining vertices are labeled same as in Case-1.

Case 3: $n \equiv 2(mod\ 3)$

$$\begin{aligned} f(v_n) &= 4n-3. \\ f(v_{2n}) &= 8n-5. \\ f(v_{2n-2}) &= 8n-6. \\ f(v'_{2n}) &= 4n-1. \\ f(v_{n-2}) &= 8n-10. \\ f(v'_{n-2}) &= 8n-10. \\ f(v'_{2n-2}) &= 4n-1. \\ f(v_{n-3}) &= 4n-7. \end{aligned}$$

The remaining vertices are labeled same as in Case-1.
Thus in each case we have $|e_f(0) - e_f(1)| \leq 1$.
Hence the graph under consideration is 3-equitable prime cordial graph. □

Example 5. *3-equitable prime cordial labeling of* $M(C_3 \bigodot K_1)$ *is shown in* Figure 5. *It is the case related to* $n \equiv 0(mod\ 3)$.

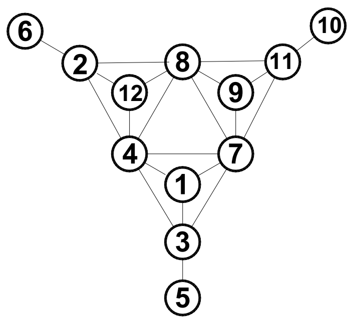

Fig. 5 3-equitable prime cordial labeling of the graph obtained by $M(C_3 \bigodot K_1)$

III Conclusion

In this paper we investigated five new 3-equitable prime cordial graphs. All the results in this paper are novel. For the better understanding of the proofs of the theorems, the labeling pattern defined in each theorem is demonstrated by illustration.

References

[1] D. M. Burton, *Elementary Number Theory* (Sixth edition), Tata McGraw-Hill, (2006).

[2] J. Gross and J. Yellen, *Graph Theory and its Applications*, CRC Press, 1999.

[3] J. A. Gallian, *A dynamic survey of graph labeling*, The Electronics Journal of Combinatorics, December 22 (2017),pp. 1-415.

[4] J. P. Jena and G. V. Ghodasara, "3-equitable prime cordial labeling of different graphs", *International Journal of Mathematics and Soft Computing*, 7(2), 2017.

[5] S. Murugesan, D. Jayaraman and J. Shiama,"3-equitable prime cordial labeling of graphs", *International Journal of Applied Information Systems*, 5(9), 2013.

[6] Dr. Sweta Srivastav and Dr. Sangeeta Gupta, "3-equitable prime cordial labeling of some graphs", *International Journal of Engineering Research*, 4(3), 2015.