A Decision Making Problem on FAHP with Z-Numbers

R.Karthika [#1], Dr.A.Sahaya Sudha[#2]
[#1]*Asst.prof, Department of Mathematics, Hindusthan Institute of Technology, Cbe-32,India*
[#2]*Asst.prof, Department of Mathematics, Nirmala College for Women, Cbe-18,India*

Abstract — *This paper deals with the study of the model based on FAHP with Z-number and the evaluation of the alternatives with respect to each criterion and is described using z number where the Z-number contains both uncertain variable and its reliability (i.e) the two components constitutes the triangular fuzzy number. In this paper a classical triangular fuzzy number has been transformed into a crisp value and a decision analysis has been proposed using AHP. Finally a practical example with risk assessment factor has been analysed and evaluated using this technique*.

Keywords — *AHP, Decision Making, Z- number, triangular fuzzy number, graded mean.*

I. Introduction

In the real world numerous risk factors exists due to uncertain conditions which has brought various challenges. There were many models and tools to solve these problems such as probability, utility function were proposed .But in (1970) Zadeh[2] proposed the fuzzy decision making model and in 2011 a notion namely Z-number, which constitutes an ordered pair of fuzzy numbers(A,B). The first component A, plays a vital role of a fuzzy restriction and the second component B is a reliability of the first component. Researchers have made an in depth study on fuzzy multi criteria decision making in which uncertainty condition arises due to vagueness in people's natural language, Fuzzy linguistic approach, etc… Fuzzy Analytic hierarchy process gives a wide application in system evaluation. In (2015) Mardani et. al;,[7] reviewed combination methods of fuzzy set and MCDM systematically, as well as the applications and methods of MCDM techniques.

Z-number has a strong ability to describe the knowledge of human. Recently [11], (2012) proposed a method to convert z number to classical fuzzy number for the calculation of the decision making, the method requires the two components of z numbers should be the triangular fuzzy numbers[8]. In (2017), A model based on z number was proposed using FAHP in which the data was analyzed from Nezarat et. al; [9] and the decision for ranking was discussed using z numbers.

In this paper, the same technique as in [10] (2011) was proposed and the classical triangular fuzzy number obtained using z number has been transformed to a crisp value and the decision analysis has been done using normal AHP. To establish this idea a practical example has been introduced using Z-FAHP in which a risk assessment factor has been analyzed and the conclusions has been discussed in detail.

II. PRELIMINARIES

Definition: 2.1 **[20]**

A fuzzy set A is defined on a universe X may be given as A= $\{(x, \mu_{\ddot{A}}(x)) / x \in X\}$

Where $\mu_{\ddot{A}} : X \to [0,1]$ is the membership function A. The membership value $\mu_A(x)$ describes the degree of belongingness of $x \in X$ in A

Definition: 2.2 **[19]**

A fuzzy number $\ddot{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2, and a_3 are real numbers and its membership function is given below.

$$\mu_{\ddot{A}}(x) = \begin{cases} \frac{(x - a_1)}{(a_2 - a_1)} & for \quad a_1 \le x \le a_2 \\ \frac{(a_3 - x)}{(a_3 - a_2)} & for \quad a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

***Definition: 2.3*[10]**

A Z- number is an ordered pair of fuzzy numbers denoted as Z = (A,B), the first component A which is a real valued uncertain variable X plays the role of fuzzy number restriction, R(x), where A is a fuzzy set, The second component B, is referred to as a measure of reliability for the first component.

***Definition: 2.4*[12]**

Though there exist several methods for the ranking of TFN, An efficient approach for comparing fuzzy number is by the use of a ranking function based on their graded means (i.e) for every $\ddot{A}(a, b, c)$, the ranking function $R(\ddot{A}) = \frac{c + 4a + b}{6}$

***Definition: 2.5 (Arithmetic Operations on Triangular Fuzzy Number)*[19]**

As there exist a number of operations on TFN, which are described in detail. The three operations used in this article are as follows. Let $\ddot{A}(a_1, b_1, c_1)$ and $\ddot{B}(a_2, b_2, c_2)$ be two triangular fuzzy number.

Addition : A+B $= (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

Multiplication : A*B $= (a_1 a_2, b_1 b_2, c_1 c_2)$

Inverse: $(a_1, b_1, c_1)^{-1} = \left[\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right]$

III.FUZZY ANALYTIC HIERARCHY PROCESS (FAHP)

Analytic Hierarchy process (AHP) is a multi-criteria decision method that uses hierarchical structures to represent a problem and then develop priorities for alternatives based on the judgment of the user(Saaty, 1980) [15]. In some cases, AHP method is incapable of handling the uncertainty and vagueness involved is the mapping of one's preference to an exact number. The major difficulty with classical AHP is its inability in mapping human judgments. It uses both qualitative and quantitative variables. Although the AHP is to capture the experts knowledge, the traditional AHP still cannot really reflect the human thinking style(Kahraman et. al:, 2003a)[3]. The traditional AHP method uses an exact value to the decision makers opinion in a comparison of alternatives (Wang & Chen 2007)[17]. It is often criticized due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pairwise comparison process (Deng,1999)[18]. To overcome this type of situations, FAHP was developed for solving hierarchical problems. It provides decision makers with interval judgments rather than a fixed value.

3.1 Converting Z number to a fuzzy number:

Assume a Z-number Z $(\ddot{A}, \ddot{B})$ where $\ddot{A} = \{(x, \mu_A(x) / \mu_A(x) \in [0,1], x \in X\}$ and $\ddot{B} = \{(x, \mu_B(x) / \mu_B(x) \in [0,1], x \in X\}$ here $\mu_A = (a_1, b_1, c_1)$ and $\mu_B = (a_2, b_2, c_2)$ are triangular fuzzy number.

(i)Convert the second part (reliability) into a crisp number.

Inorder to convert B into a crisp number, we use the method proposed by Ali Azadeh et. al:, [16]

$\alpha = \frac{\int x \mu_{\ddot{B}}(x) dx}{dx}$ thus $Z = (\ddot{A}, \alpha)$

(ii) Converting weighted z number to fuzzy number.

Add the weight of the $\ddot{B}$ to $\ddot{A}$, weighted z-number can be denoted as

$$\ddot{Z} = \{((x, \mu_A(x) / \mu_A(x) = \sqrt{\alpha} \mu_{\ddot{A}}, x \in X)\}$$

Where $\ddot{Z} = (\sqrt{\alpha}\, a, \sqrt{\alpha}\, b, \sqrt{\alpha}\, c)$ so far, a z-number has been converted to a crisp fuzzy number

3.1.1Modeling of Z- FAHP:

Though Fuzzy Analytic Hierarchy Process is a good decision making tool, it still has ambiguity in language description and cannot respond well to the evaluators, natural language and so on. Z number in turn appears to be a good solution to this defect. Combining them together and converting it to a crisp value gives most approximate and nearby solution to the decision maker. Steps are as follows

Step-1: Construction of the detailed hierarchy of the problem:

The hierarchy is constructed based on all criteria, sub criteria and alternatives to the practical research problem

Step-2: Constructing a pairwise comparison matrix:

Once the hierarchy was established and a series of equations were asked to direct pairwise comparisons, each expert performed a pairwise comparison. Assuming expert gives his or her opinion as follows:

Consider $\ddot{A}(a_1, b_1, c_1)$ and $\ddot{B}(a_2, b_2, c_2)$ as an Z number, where $\ddot{A}$ and $\ddot{B}$ are triangular fuzzy numbers such as $[(a_1, b_1, c_1), (a_2, b_2, c_2)]$

Converting $\ddot{B}$ to a crisp number, secondly we add weight of $\ddot{B}$ to the $\ddot{A}$ according to the equation

$\ddot{z} = (\ddot{A}, \alpha) = [(a_1, b_1, c_1), \alpha]$, $\ddot{z}(\ddot{A}, \alpha)$ to a fuzzy numbers $\ddot{Z} = (\ddot{A}, \ddot{B})$ as $\ddot{Z} = (\sqrt{\alpha}\, a, \sqrt{\alpha}\, b, \sqrt{\alpha}\, c)$ again construct a pairwise comparison matrix according to the experts opinion as triangular fuzzy number and applying into FAHP.

Step-3: Convert the above pairwise comparison matrix of a triangular fuzzy number to a normal pairwise crisp value using the ranking formula such as $R(\ddot{Z}) = \dfrac{c + 4a + b}{6}$

Step-4: Finally the decision making has been evaluated using the normal AHP and the priorities have been determined.

IV. PRACTICAL EXAMPLE OF FAHP:

Many factors leads to uncertain and unpredictable condition for contracting dengue fever which is one of the most dangerous disease, spread all over the world. It is a global problem, it has been estimated that 390 million dengue infections occur worldwide each year, with about 96 million resulting in illness. India also saw a drastic increase in the incidence of dengue in the last few years. The world health organization(WHO) has declared a global health emergency regarding the spread of zika virus, which spread among the various states in India. If the decision makers use improper methods, this will result in unimaginable losses, hence the government has taken several steps to curtail this risk factor. In this paper, several geographic areas in India, which was affected by dengue with risk factors has been considered and FAHP has been used to access the level of risk and has been ranked.

4.1 Structuring the hierarchy:

Our goal is to decide the best safety measure to control the disease

Some of the measures taken by the government to improve this global problem are assigned as several criteria.

4.1.1TABLE
DECISION TABLE

Criteria	Risk factors
C_1	Domestic breeding checkers to take up mosquito control steps
C_2	Household survey of containers and awareness to store water inside homes to protect houses getting infested with larvae
C_3	People were asked not to let the water get stagnated
C_4	Fogging
C_5	Medical Camps being held; mosquito breeding curtailed

4.1.2TABLE

LINGUISTIC VARIABLES

Fuzzy number	**Linguistic variables (level of measures taken in all states)**
(1,1,1)	Equal importance
((1,1,3)	10-30%
(1,3,5)	30-50%
(3,5,7)	50-70%
(5,7,9)	70-90%
(7,9,11)	Above 90%

4.1.3TABLE

FUZZY PAIRED COMPARISON MATRIX OF DECISION VARIABLES:

Triangular fuzzy scale	Fuzzy number	Triangular Fuzzy scale	Fuzzy number
(7,9,11)	9	(1,1/9,1/7)	1/9
(6,8,9)	8	(1/9,1/8,1/6)	1/8
(5,7,9)	7	(1/9,1/7,1/5)	1/7
(4,6,8)	6	(1/8,1/6,1/4)	1/6
(3,5,7)	5	(1/7,1/5,1/3)	1/5
(2,4,6)	4	(1/6,1/4,1/2)	1/4
(1,3,5)	3	(1/5,1/3,1)	1/3
(1,2,4)	2	(1/4,1/2,1))	1/2
(1,1,3)	1	(1/3,1,1)	1
(1,1,1)	1	(1,1,1)	1

Using the concept of z number to improve the reliability of the evaluation , opinion about the decision variable has been given as in [1],[5]

4.1.4TABLE

Z-NUMBER WEIGHT OF DECISION VARIABLE

Z number Weight of decision variable	**A**			**B**		
	a	b	c	a	b	c
C_1	1	1	1	0.8	0.9	1
C_2	1	3	5	0.76	0.88	1
C_3	1	2	4	0.8	0.9	1
C_4	1	3	7	0.76	0.88	1
C_5	1	1	3	0.8	0.9	1

4.1.5TABLE

CONVERTING Z NUMBER TO CRISP NUMBER

The value of α	C_1	C_2	C_3	C_4	C_5
A	0.9	0.88	0.9	0.88	0.9

4.2 Converting weighted z number to fuzzy number:

4.2.1TABLE

CONVERTING Z NUMBER TO CRISP NUMBER

	a	**b**	**c**
C_1	0.9486	0.9486	0.9486
C_2	0.938	2.81	4.69
C_3	0.948	1.897	3.795
C_4	0.938	2.81	6.57
C_5	0.948	0.948	2.85

4.2.2TABLE

FUZZY PAIRWISE COMPARISON MATRIX

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(1/5,1/3,1)	(1/4,1/2,1)	(1/7,1/3,1)	(1/3,1,1)
C_2	(1,3,5)	(1,1,1)	(2,4,6)	(1,1,2)	(5,7,9)
C_3	(1,2,4)	(1/6,1/4,2)	(1,1,1)	(1/7,1/5,1/3)	(1/9,1/7,1/5)
C_4	(1,3,7)	(1/2,1,1)	(3,5,7)	(1,1,1)	(5,8,9)
C_5	(1,1,3)	(1/9,1/7,1/5)	(5,7,9)	(1/9,1/8,1/5)	(1,1,1)

***4.3Converting the weighted z number to triangular fuzzy number and to a crisp value*:**

4.3.1TABLE

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(0.2,0.31,0.95)	(0.24,0.47,0.95)	(0.135,0.31,0.95)	(0.313,0.95,0.95)
C_2	(0.938,2.81,4.69)	(1,1,1)	(1.88,3.75,5.63)	(0.94,0.94,1.88)	(4.69,6.57,8.4)
C_3	(0.948,1.897,3.795)	(0.16,0.24,0.47)	(1,1,1)	(0.14,0.20,0.31)	(0.10,0.14,0.2)
C_4	((0.938,2.81,6.57)	(0.469,0.94,0.94)	(2.81,4.69,6.57)	(1,1,1)	(4.69,7.51,8.44)
C_5	((0.948,0.948,2.85)	(0.104,0.13,0.19)	(4.74,6.64,8.54)	(0.104,0.12,0.2)	(1,1,1)

4.3.2TABLE

WEIGHTED CRISP VALUE USING Z NUMBER

	C_1	C_2	C_3	C_4	C_5	Priority
C_1	1.000	0.325	0.358	0.267	0.419	0.090
C_2	1.563	1.000	2.505	1.097	5.308	0.344
C_3	1.423	0.212	1.000	0.168	0.117	0.090
C_4	1.877	0.548	3.437	1.000	5.315	0.319
C_5	1.265	0.120	5.373	0.120	1.000	0.157
SUM	7.128	2.204	12.673	2.652	12.159	

4.3.3TABLE

NORMAL WEIGHTS OF DECISION VARIABLE

Decision Variable	C_1	C_2	C_3	C_4	C_5
Normal weight of decision variable	0.091	0.344	0.090	0.319	0.157

From the above table it has been observed that $C_2>C_4>C_5>C_1>C_3$, i.e To prevent the dengue, government has given more importance to Criteria C_2in which household survey is made in every area to protect the houses being infested with larvae by storing the water inside the houses, Fogging (C_2) has been done in all the areas, medical camps (C_5) to create the awareness among the people about the disease has been spread all over the areas. Thus all the measures have been taken in the above order by the government to protect the areas.

Similarly we calculate the normal weights for the various states in India such as Kerala, Karnataka, Tamil Nadu, West Bengal and New Delhi of all criteria with step 2 to step 4

4.4 Final Priorities:

The decision making and the final priorities and rating of each criterion are as follows

4.4TABLE

DECISION MAKING WITH ORIGINAL AHP

Five Criteria	C_1	C_2	C_3	C_4	C_5	Final Priority
	0.091	**0.344**	**0.09**	**0.319**	**0.157**	
A_1	0.131	0.150	0.105	0.079	0.230	**0.13449**
A_2	0.321	0.059	0.135	0.127	0.149	**0.12563**
A_3	0.090	0.293	0.191	0.207	0.155	**0.21668**
A_4	0.222	0.188	0.163	0.318	0.203	**0.23278**
A_5	0.236	0.309	0.405	0.269	0.263	**0.29142**

From the above table it has been observed that $A_5 > A_4 > A_3 > A_1 > A_2$ i.e.. Kerala (A5) has been most affected region with more death rates and cases, followed by West Bengal (A4) and the third is New Delhi(A3), Comparing Tamil Nadu(A1) with Karnataka (A2), Tamil Nadu is under high risk. Thus in spite of all safety measures taken by the government, it has been found that Kerala has been considered to be under highest risk, hence the central Government has been requested to take necessary steps to curtail this disease.

V. Conclusion

In this paper FAHP with Z number , enhances the reliability of traditional FAHP , has been taken into consideration which may be under certain risk factors such as if more number of criteria's used or more number of fuzzy numbers such as triangular , trapezoidal , hexagonal etc…. , Thus we have introduced an additional concept of converting triangular fuzzy number to a crisp value using basic ranking method ,hence the decision making has been done using Analytic Hierarchy process and the same has been applied to a practical social problem and the results has been produced and verified with the experts.

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