New Operations over Interval Value Intuitionistic Fuzzy Sets of Cube Root Type

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Abstract
In this paper, we defined arithmetic mean operation and geometric mean operation over Interval Value of Intuitionistic Fuzzy Sets of Cube Root Type(IVIFSCRT) were proposed and few theorem are proved. In addition, some of the basic properties of the new operations were discussed

Keyword
Interval Value of Intuitionistic Fuzzy Sets of Cube Root Type(IVIFSCRT),Intuitionistic Fuzzy Sets (IFS),Interval-Valued Intuitionistic Fuzzy Set(IVIFS),arthimetic mean operation, geometric mean operation

1 Introduction

In 1965, Fuzzy sets theory was proposed by L. A. Zadeh[1]. In 1986, the concept of intuitionistic fuzzy sets (IFSs), as a generalization of fuzzy set were introduced by K. Atanassov[2]. After the introduction of IFS, many researchers have shown interest in the IFS theory and applied in numerous fields, such as pattern recognition, machine learning, image processing, decision making and etc.In 1989, the notion of Interval-Valued Intuitionistic Fuzzy Sets which is a generalization of both Intuitionistic Fuzzy Sets and Interval-Valued Fuzzy Sets were proposed by K.T Atanassov and G.Gargov [3]. After the introduction of IVIFS, many researchers have shown interest in the IVIFS theory and applied it to the various field.

In this paper, our aim is to propose two new operations @ and $ over IVIFSCRT and we will discuss their properties and propose a multi-criteria group decision making method based on the new operations.

2 Preliminaries

For completeness, some operators and definition on IVIFSCRT are reviewed in this section.Let X be non empty set.

Definition 2.1(L.A.Zadeh,1965) Let X be an nonempty set.A fuzzy set A is defined as

$$A = \{\langle x, \mu_A(x)\rangle : x \in X\}$$

*Mr.Mohammed Nabeel I is a part time research scholar in Bharathiar University,Coimbatore,India where the functions $\mu_A(x) : X \to [0,1]$ is the membership function of the fuzzy set A.Fuzzy set is a collection of objects with graded membership i.e having degree of membership.

Definition 2.2(K.T.Atanassov,1986) An IFS A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the membership and non-membership function of A respectively, and

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \text{ for each } x \in X$$

Furthermore,we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e $\pi_A(x) : X \to [0,1]$ and $0 \leq \pi_A(x) \leq 1$ for each $x \in X$.$\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

Remark 2.3

1. Every fuzzy set is an intuitionistic fuzzy set, but the reverse is not true.
2. $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$.

Definition 2.4 (K.T.Atanassov,1986) Let X be a non empty set. An Intuitionistic Fuzzy Set of Second Type(IFSST) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the membership and non-membership function of A respectively, and

$$0 \leq [\mu_A(x)]^2 + [\nu_A(x)]^2 \leq 1 \text{ for each } x \in X$$

Remark: 2.5 It is obvious that for all real numbers $a, b \in [0,1]$if $0 \leq a + b \leq 1$ then $0 \leq a^2 + b^2 \leq 1$

Definition 2.6 (Srinivasan R and Palaniappan N, 2006) Let X be a non empty set. An Intuitionistic Fuzzy Set of Root Type(IFSRT) A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the membership and non-membership function of A respectively, and

$$0 \leq \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\nu_A(x)}}{2} \leq 1 \text{ for each } x \in X$$

Definition 2.7(U Rizwan and Nabeel, 2015) An Intuitionistic fuzzy set of cube root type(IFSCRT) A in X, is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denotes the degree of membership and non-membership function of A, respectively and where

$$0 \leq \frac{\sqrt[3]{\mu_A(x)} + \sqrt[3]{\nu_A(x)}}{\sqrt{3}} \leq 1 \text{ for each } x \in X$$

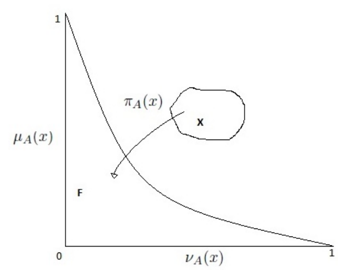

Figure 1: Geometrical representation of IFSCRT

Definition 2.8(Atanassov and Gargov,1989) Let X be a non empty set. Interval Value Intuitionistic fuzzy sets(IVIFS) A in X, is defined as an object of the form

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$$

where the functions $M_A : X \to [I]$ and $N_A : X \to [I]$ denotes the degree of membership and non-membership function of A respectively
where

$$M_A(x) = [M_{AL}(x), M_{AU}(x)]$$
$$N_A(x) = [N_{AL}(x), N_{AU}(x)]$$
$$0 \leq M_{AU}(x) + N_{AL}(x) \leq 1 \text{ for each } x \in X$$

Definition 2.9 Let [I] be the set of all closed subintervals of the interval [0,1] and

$$M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$$

and

$$N_A(x) = [N_{AL}(x), N_{AU}(x)] \in [I]$$

then $N_A(x) \leq M_A(x)$ if and only if $N_{AL}(x) \leq M_{AL}(x)$ and $N_{AU}(x) \leq M_{AU}(x)$

Definition 2.10(U Rizwan and Nabeel, 2016) Let X be a non empty set, Interval value of Intuitionistic fuzzy set of cube root type (IVIFSCRT) A in X, is defined as an object of the form

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$$

where the functions $M_A : X \to [I]$ and $N_A : X \to [I]$ denotes the degree of membership and non-membership function of A respectively
and

$$M_A(x) = [M_{AL}(x), M_{AU}(x)]$$
$$N_A(x) = [N_{AL}(x), N_{AU}(x)]$$

where

$$0 \leq \frac{\sqrt[3]{M_{AU}(x)} + \sqrt[3]{N_{AU}(x)}}{\sqrt{3}} \leq 1 \text{ for each } x \in X$$

A IVIFSCRT value is denoted by $A = ([M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)])$ for convenience

Definition 2.11 The degree of non-determinacy (uncertainty) of an element $x \in X$ to the IVIFSCRT A is defined by

$$\pi_A(x) = [\pi_{AL}(x), \pi_{AU}(x)]$$
$$= [(1 - M_{AU}(x)^{1/3} - N_{AU}(x)^{1/3})^3, (1 - M_{AL}(x)^{1/3} - N_{AL}(x)^{1/3})^3]$$

Definition 2.12 For every IVIFSCRT $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$,we define the modal logic operators "neccessity" and "possibility".

The Neccesity Measure on A

$$\Box A = \left\{\langle x, [M_{AL}(x), M_{AU}(x)], [(1 - M_{AU}(x)^{1/3})^3, (1 - M_{AL}(x)^{1/3})^3]\rangle : x \in X\right\}$$

The Possibility measure on A

$$\Diamond A = \left\{\langle x, [(1 - N_{AU}(x)^{1/3})^3, (1 - N_{AL}(x)^{1/3})^3], [N_{AL}(x), N_{AU}(x)]\rangle : x \in X\right\}$$

3 Main Results

Here we will introduce new operations over the IVIFSCRT which extend two operations in the literature related to IVIFS.Let X is a non empty finite set

Definition 3.1 For every IVIFSCRTs as

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$$
$$B = \{\langle x, M_B(x), N_B(x)\rangle : x \in X\}$$

we define the arthimetic mean operation and geometric mean operation as follows

$(i) A@B = \{\langle x, M_{A@B}(x), N_{A@B}(x)\rangle : x \in X\}$

$$M_{A@B}(x) = [(\frac{1}{2}(M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3}))^3, (\frac{1}{2}(M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}))^3]$$
$$N_{A@B}(x) = [(\frac{1}{2}(N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3}))^3, (\frac{1}{2}(N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3}))^3]$$

$(ii) A\$B = \{\langle x, M_{A\$B}(x), N_{A\$B}(x)\rangle : x \in X\}$

$$M_{A\$B}(x) = [\sqrt{M_{AL}(x).M_{BL}(x)}, \sqrt{M_{AU}(x).M_{BU}(x)}]$$
$$N_{A\$B}(x) = [\sqrt{N_{AL}(x).N_{BL}(x)}, \sqrt{N_{AU}(x).N_{BU}(x)}]$$

Example 3.2 It $A = \{\langle x, [0.2, 0.3], [0.3, 0.4]\rangle : x \in X\}$, $B = \{\langle x, [0.3, 0.4], [0.2, 0.4]\rangle : x \in X\}$then the arithmetic mean operation and geometric mean operation as follows

Soluiton

$$(i) A@B = \{\langle x, [(\frac{1}{2}(0.2^{1/3} + 0.3^{1/3}))^3, (\frac{1}{2}(0.3^{1/3} + 0.4^{1/3}))^3],$$
$$[(\frac{1}{2}(0.3^{1/3} + 0.2^{1/3}))^3, (\frac{1}{2}(0.4^{1/3} + 0.4^{1/3}))^3]\rangle : x \in X\}$$

$$= \{\langle x, [0.2466, 0.3476], [0.2466, 0.4]\rangle : x \in X\}$$

$$(ii) A\$B = \left\{\left\langle x, [\sqrt{0.2X0.3}, \sqrt{0.3X0.4}], [\sqrt{0.3X0.2}, \sqrt{0.4X0.4}]\right\rangle : x \in X\right\}$$

$$= \{\langle x, [0.2449, 0.3446], [0.2449, 0.4]\rangle : x \in X\}$$

Theorem 3.3 For every two IVIFSCRTs A and B, we have

$(i) A@B$is a IVIFSCRT

$(ii) A\$B$ is a IVIFSCRT

Proof

$$(i) M_{A@B}(x)^{1/3} + N_{A@B}(x)^{1/3} = \tfrac{1}{2}[M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}] + \tfrac{1}{2}[N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3}]$$

$$= \frac{1}{2}[M_{AU}(x)^{1/3} + N_{AU}(x)^{1/3}] + \frac{1}{2}[M_{BU}(x)^{1/3} + N_{BU}(x)^{1/3}]$$

$$\leq \frac{1}{2} + \frac{1}{2} = 1$$

therefore it can be concluded that $A@B \in$ IVIFSCRT

(ii) By using defintion 2.11 and $\sqrt{a^{1/3}.b^{1/3}} \leq \frac{1}{2}(a^{1/3} + b^{1/3})$ we have

$$M_{(A\$B)U}(x)^{1/3} + N_{(A\$B)U}(x)^{1/3} = [\sqrt{[M_{AU}(x).M_{BU}(x)]^{1/3}} + \sqrt{[N_{AU}(x).N_{BU}(x)]^{1/3}}]$$

$$\leq \frac{1}{2}[M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}] + \frac{1}{2}[N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3}]$$

$$\leq \tfrac{1}{2} + \tfrac{1}{2} \leq 1$$

Finally,it can be concluded that $A\$B \in$ IVIFSCRT

Theorem 3.4 For every two IVIFSCRTs A,B(Commutative laws)

$(i) A@B = B@A$

$(ii) A\$B = B\$A$

Proof: (i) By using Definition 3.1,we have

$$(i) M_{A@B}(x) = [(\tfrac{1}{2}[M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3}])^3, (\tfrac{1}{2}[M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}])^3]$$

$$= [(\frac{1}{2}[M_{BL}(x)^{1/3} + M_{AL}(x)^{1/3}])^3, (\frac{1}{2}[M_{BU}(x)^{1/3} + M_{AU}(x)^{1/3}])^3]$$

$$= M_{B@A}(x)$$

$$N_{A@B}(x) = [(\tfrac{1}{2}[N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3}])^3, (\tfrac{1}{2}[N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3}])^3]$$

$$= [(\frac{1}{2}[N_{BL}(x)^{1/3} + N_{AL}(x)^{1/3}])^3, (\frac{1}{2}[N_{BU}(x)^{1/3} + N_{AU}(x)^{1/3}])^3]$$

$= N_{B@A}(x)$

the proof is completed. Proof(ii) is similar to that of (i)

Theorem 3.5 For every two IVIFSCRTs A and B (Idempotent Law)

$(i) A@A = A$

$(ii) A\$A = A$

Proof : the proof is true from Defintion 3.1

Theorem 3.6 For every two IVIFSCRTs A and B (Complementary Law)

$(i) \overline{\overline{A}@\overline{B}} = A@B$

$(ii) \overline{\overline{A}\$\overline{B}} = A\$B$

Proof:

(i)Since $\overline{A} = \{\langle x, N_A(x), M_A(x)\rangle : x \in X\}$, $\overline{B} = \{\langle x, N_B(x), M_B(x)\rangle : x \in X\}$

we have $\overline{A}@\overline{B} = \left\{\left\langle x, M_{\overline{A}@\overline{B}}(x), N_{\overline{A}@\overline{B}}(x)\right\rangle : x \in X\right\}$

$\overline{\overline{A}@\overline{B}} = \left\{\left\langle x, N_{\overline{A}@\overline{B}}(x), M_{\overline{A}@\overline{B}}(x)\right\rangle : x \in X\right\}$
since
$M_{\overline{A}@\overline{B}}(x) = [(\frac{1}{2}[N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3}])^3, (\frac{1}{2}[N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3}])^3]$

$= N_{A@B}(x)$

$N_{\overline{A}@\overline{B}}(x) = [(\frac{1}{2}[M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3}])^3, (\frac{1}{2}[M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}])^3]$

$= M_{A@B}(x)$

$\overline{\overline{A}@\overline{B}} = \{\langle x, M_{A@B}(x), N_{A@B}(x)\rangle : x \in X\}$

$= A@B$

this proof is completed

Proof of (ii) is similiar to that of (i)

Theorem 3.7 For every three IVIFSCRTs A,B and C (Associative Laws)

$(i)(A@B)@C = (A@C)@(B@C)$

$(ii)(A\$B)\$C = (A\$C)\$(B\$C)$

Proof

$(i)M_{(A@B)@C}(x) = [[\frac{1}{2}(M_{(A@B)L}(x)^{1/3} + M_{CL}(x)^{1/3})]^3, [\frac{1}{2}(M_{(A@B)U}(x)^{1/3} + M_{CU}(x)^{1/3})]^3]$

$= [[\frac{1}{2}(\frac{1}{2}[M_{AL}(x)^{1/3}+M_{BL}(x)^{1/3}]+M_{CL}(x)^{1/3})]^3, [\frac{1}{2}(\frac{1}{2}[M_{AU}(x)^{1/3}+M_{BU}(x)^{1/3}]+M_{CU}(x)^{1/3})]^3]$

$$= [[\frac{1}{2}(\frac{1}{2}[M_{AL}(x)^{1/3} + M_{CL}(x)^{1/3}] + \frac{1}{2}[M_{BL}(x)^{1/3} + M_{CL}(x)^{1/3}])]^3, \\ [\frac{1}{2}(\frac{1}{2}[M_{AU}(x)^{1/3} + M_{CU}(x)^{1/3}] + \frac{1}{2}[M_{BU}(x)^{1/3} + M_{CU}(x)^{1/3}])]^3]$$

$= M_{(A@C)@(B@C}(x)$

$N_{(A@B)@C}(x) = [[\frac{1}{2}(N_{(A@B)L}(x)^{1/3} + N_{CL}(x)^{1/3})]^3, [\frac{1}{2}(N_{(A@B)U}(x)^{1/3} + N_{CU}(x)^{1/3})]^3]$

$= [[\frac{1}{2}(\frac{1}{2}[N_{AL}(x)^{1/3}+N_{BL}(x)^{1/3}]+N_{CL}(x)^{1/3})]^3, [\frac{1}{2}(\frac{1}{2}[N_{AU}(x)^{1/3}+N_{BU}(x)^{1/3}]+N_{CU}(x)^{1/3})]^3]$

$$= [[\frac{1}{2}(\frac{1}{2}[N_{AL}(x)^{1/3} + N_{CL}(x)^{1/3}] + \frac{1}{2}[N_{BL}(x)^{1/3} + N_{CL}(x)^{1/3}])]^3, \\ [\frac{1}{2}(\frac{1}{2}[N_{AU}(x)^{1/3} + N_{CU}(x)^{1/3}] + \frac{1}{2}[N_{BU}(x)^{1/3} + N_{CU}(x)^{1/3}])]^3]$$

$= N_{(A@C)@(B@C}(x)$

This completes the proof

$(ii)M_{(A\$B)\$C}(x) = [\sqrt{M_{(A\$B)L}(x)M_{CL}(x)}, \sqrt{M_{(A\$B)U}(x)M_{CU}(x)}]$

$= [\sqrt{\sqrt{M_{AL}(x)M_{BL}(x)}M_{CL}(x)}, \sqrt{\sqrt{M_{AU}(x)M_{BU}(x)}M_{CU}(x)}]$

$= [\sqrt{\sqrt{M_{AL}(x)M_{CL}(x)}.\sqrt{M_{BL}(x)M_{CL}(x)}}, \sqrt{\sqrt{M_{AU}(x)M_{CU}(x)}.\sqrt{M_{BU}(x)M_{CU}(x)}}]$

$= M_{(A\$B)\$(B\$C)}(x)$

$N_{(A\$B)\$C}(x) = [\sqrt{N_{(A\$B)L}(x)N_{CL}(x)}, \sqrt{N_{(A\$B)U}(x)N_{CU}(x)}]$

$= [\sqrt{\sqrt{N_{AL}(x)N_{BL}(x)}N_{CL}(x)}, \sqrt{\sqrt{N_{AU}(x)N_{BU}(x)}N_{CU}(x)}]$

$= [\sqrt{\sqrt{N_{AL}(x)N_{CL}(x)}.\sqrt{N_{BL}(x)N_{CL}(x)}}, \sqrt{\sqrt{N_{AU}(x)N_{CU}(x)}.\sqrt{N_{BU}(x)N_{CU}(x)}}]$

$= N_{(A\$B)\$(B\$C)}(x)$

The proof is complete

Theorem 3.8 For every three IVIFSCRTs A,B and C (Distributive Laws)

$(i)(A \cup B)@C = (A@C) \cup (B@C)$

$(ii)(A \cap B)@C = (A@C) \cap (B@C)$

$(iii)(A \cup B)\$C = (A\$C) \cup (B\$C)$

$(iv)(A \cap B)\$C = (A\$C) \cap (B\$C)$

Proof

(i)Since $A \cup B = \{\langle x, max(M_A(x), M_B(x)), min(N_A(x), N_B(x))\rangle : x \in X\}$

we have

$$M_{(A\cup B)@C}(x) = [[\frac{1}{2}(max(M_{AL}(x), M_{BL}(x))^{1/3} + M_{CL}(x)^{1/3})]^3, [\frac{1}{2}(max(M_{AU}(x), M_{BU}(x))^{1/3} + M_{CU}(x)^{1/3})]^3]$$

$$= [[\tfrac{1}{2}(max(M_{AL}(x)^{1/3}, M_{BL}(x)^{1/3}) + M_{CL}(x)^{1/3})]^3, [\tfrac{1}{2}(max(M_{AU}(x)^{1/3}, M_{BU}(x)^{1/3}) + M_{CU}(x)^{1/3})]^3]$$

$$= [[max(\frac{1}{2}(M_{AL}(x)^{1/3} + M_{CL}(x)^{1/3}), \frac{1}{2}(M_{BL}(x)^{1/3} + M_{CL}(x)^{1/3}))]^3, [max(\frac{1}{2}(M_{AU}(x)^{1/3} + M_{CU}(x)^{1/3}), \frac{1}{2}(M_{BU}(x)^{1/3} + M_{CU}(x)^{1/3}))]^3]$$

$$= M_{(A@C)\cup(B@C)}(x)$$

$N_{(A\cup B)@C}(x) = [[\frac{1}{2}(min(N_{AL}(x), N_{BL}(x))^{1/3} + N_{CL}(x)^{1/3})], [\frac{1}{2}(min(N_{AU}(x), N_{BU}(x))^{1/3} + N_{CU}(x)^{1/3})]]$

$$= [[\tfrac{1}{2}(min(N_{AL}(x)^{1/3}, N_{BL}(x)^{1/3}) + N_{CL}(x)^{1/3})]^3, [\tfrac{1}{2}(min(N_{AU}(x)^{1/3}, N_{BU}(x)^{1/3}) + N_{CU}(x)^{1/3})]^3]$$

$$= [[min(\frac{1}{2}(N_{AL}(x)^{1/3} + N_{CL}(x)^{1/3}), \frac{1}{2}(N_{BL}(x)^{1/3} + N_{CL}(x)^{1/3}))]^3, [min(\frac{1}{2}(N_{AU}(x)^{1/3} + N_{CU}(x)^{1/3}), \frac{1}{2}(N_{BU}(x)^{1/3} + N_{CU}(x)^{1/3}))]^3]$$

$$= N_{(A@C)\cup(B@C)}(x)$$

Proof is complete.Proof of (ii) is similiar to that of (i)

$(iii) M_{(A\cup B)\$C} = [\sqrt{max(M_{AL}(x), M_{BL}(x)).M_{CL}(x)}, \sqrt{max(M_{AU}(x), M_{BU}(x)).M_{CU}(x)}]$

$$= [\sqrt{max(M_{AL}(x)M_{CL}(x), M_{BL}(x)M_{CL}(x))}, \sqrt{max(M_{AU}(x)M_{CU}(x), M_{BU}(x)M_{CU}(x))}]$$

$$= M_{(A\$C)\cup(B\$C)}$$

And $N_{(A\cup B)\$C} = [\sqrt{min(N_{AL}(x), N_{BL}(x)).N_{CL}(x)}, \sqrt{min(N_{AU}(x), N_{BU}(x)).N_{CU}(x)}]$

$= [\sqrt{min(N_{AL}(x)N_{CL}(x), N_{BL}(x)N_{CL}(x))}, \sqrt{min(N_{AU}(x)N_{CU}(x), N_{BU}(x)N_{CU}(x))}]$

$= N_{(A\$C)\cup(B\$C)}$

Proof is complete.Proof of (iv) is similiar to that of (iii)

Theorem 3.9 For every two IVIFSCRTs A and B we have (inclusion laws)

(i) If $A \subset B$ then $A@B \subset B$

(ii) If $A \subset B$ then $A\$B \subset B$

Proof: (i) If $a \leq b$ then $a \leq (\frac{1}{2}(a^{1/3} + b^{1/3}))^3 \leq b$

In this case,proof is clear

(ii)If $a \leq b$ then $a \leq \sqrt{ab} \leq b$

In this case,proof is clear

Corollary 3.10 For every two IVIFSCRTs A and B we have (Absorption laws)

(i)$A@(A \cup B) \subset A \cup B$

(ii)$A\$(A \cup B) \subset A \cup B$

(iii)$A@(A \cap B) \subset A$

(iv)$A\$(A \cap B) \subset A$

Proof: since $A \subset A \cup B$, $A \cap B \subset A$,proof are clear using theorem 3.9

Theorem 3.11 For every three IVIFSCRTs A,B and C

(i)$\Box(A@B) = \Box A@\Box B$

(ii)$\Box(A\$B) \subset \Box A\$\Box B$

(iii)$\Diamond(A@B) = \Diamond A@\Diamond B$

(iv)$\Diamond(A\$B) \supset \Diamond A\$\Diamond B$

Proof

(i)By using Definition 3.1 and definition of Neccesity measure we have

$\Box(A@B) = \{\langle x, M_{\Box(A@B)}(x), N_{\Box(A@B)}(x)\rangle : x \in X\}$

since $M_{\Box(A@B)} = [(\frac{1}{2}(M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3}))^3, (\frac{1}{2}(M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}))^3]$

$= M_{\Box A@\Box B}$

$N_{\Box(A@B)} = [(1 - \frac{1}{2}(M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}))^3, (1 - \frac{1}{2}(M_{AL}(x)^{1/3} + M_{BL}(x)^{1/3}))^3]$

$= [(\frac{1}{2}((1 - M_{AU}(x)^{1/3}) + (1 - M_{BU}(x)^{1/3})))^3, (\frac{1}{2}((1 - M_{AL}(x)^{1/3}) + (1 - M_{BL}(x)^{1/3})))^3]$

$= N_{\Box A@\Box B}$

$\Box(A@B) = \Box A@\Box B$. Proof is complete

(ii)By using Definition 3.1 and defintion of Necessity measure, we have

$\Box(A\$B) = \{\langle x, M_{\Box(A\$B)}(x), N_{\Box(A\$B)}(x)\rangle : x \in X\}$

$$= \{\langle x, [\sqrt{M_{AL}(x)M_{BL}(x)}, \sqrt{M_{AU}(x)M_{BU}(x)}], \\ [(1 - (\sqrt{M_{AU}(x)M_{BU}(x)})^{1/3})^3, (1 - (\sqrt{M_{AL}(x)M_{BL}(x)})^{1/3})^3]\rangle : x \in X\}$$

$= \{\langle x, M_{\Box A\$\Box B}(x), N_{\Box A\$\Box B}(x)\rangle : x \in X\}$

$$= \{\langle x, [\sqrt{M_{AL}(x)M_{BL}(x)}, \sqrt{M_{AU}(x)M_{BU}(x)}], \\ [\sqrt{(1 - M_{AU}(x)^{1/3})^3(1 - M_{BU}(x)^{1/3})^3}, \sqrt{(1 - M_{AL}(x)^{1/3})^3(1 - M_{BL}(x)^{1/3})^3}]\rangle : x \in X\}$$

To prove $\Box A\$B \subset \Box A\$\Box B$, it is enough to prove $N_{\Box(A\$B)} \geq N_{\Box A\$\Box B}$. It is clear

$0 \leq (\sqrt{a^{1/3}} + \sqrt{b^{1/3}})^2$,therefore

$0 \leq a^{1/3} + b^{1/3} - 2(\sqrt{ab})^{1/3}$

$0 \leq a^{1/3} + b^{1/3} + [1 + (ab)^{1/3}] - [1 + (ab)^{1/3}] - 2(\sqrt{ab})^{1/3}$

$0 \leq a^{1/3} + b^{1/3} - 1 - (ab)^{1/3} + 1 + (ab)^{1/3} - 2(\sqrt{ab})^{1/3}$

$0 \leq -1(1 - a^{1/3}) + b^{1/3}(1 - a^{1/3}) + [1 - (\sqrt{ab})^{1/3}]^2$

$0 \leq -(1 - a^{1/3})(1 - b^{1/3}) + [1 - (\sqrt{ab})^{1/3}]^2$

$\sqrt{(1 - a^{1/3})(1 - b^{1/3})} \leq [1 - (\sqrt{ab})^{1/3}]$

this completes the proof

(iii) By using the definition of possibility measure,we have

$\Diamond(A@B) = \{\langle x, M_{\Diamond(A@B)}(x), N_{\Diamond(A@B)}(x)\rangle : x \in X\}$

since $M_{\Diamond(A@B)} = [(1 - \frac{1}{2}(N_{AU}(x)^{1/3} + N_{BU}(x)^{1/3}))^3, (1 - \frac{1}{2}(N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3}))^3]$

$= [(\frac{1}{2}((1 - N_{AU}(x)^{1/3}) + (1 - N_{BU}(x)^{1/3})))^3, (\frac{1}{2}((1 - N_{AL}(x)^{1/3}) + (1 - N_{BL}(x)^{1/3})))^3]$

$= M_{\Diamond A@\Diamond B}$

$\Diamond A = \{\langle x, [(1 - N_{AU}(x)^{1/3})^3, (1 - N_{AL}(x)^{1/3})^3], [N_{AL}(x), N_{AU}(x)]\rangle : x \in X\}$

$N_{\Diamond(A@B)} = [(\frac{1}{2}(N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3}))^3, (\frac{1}{2}(N_{AL}(x)^{1/3} + N_{BL}(x)^{1/3}))^3]$

$= N_{\Diamond A@\Diamond B}$

$\Diamond(A@B) = \Diamond A@\Diamond B$

proof is complete.proof (iv) are similiar to that of (ii)

Theorem 3.12 For every two IVIFSCRTs A and B(Distribution laws)

$(i)(A@B) + C = (A + C)@(B + C)$

$(ii)(A@B).C = (A.C)@(B.C)$

Proof

$$(i)M_{(A@B)+C}(x) = [\frac{1}{2}(M_{AL}(x)^{1/3}+M_{BL}(x)^{1/3})+M_{CL}(x)^{1/3}-\frac{1}{2}(M_{AL}(x)^{1/3}+M_{BL}(x)^{1/3})M_{CL}(x)^{1/3},$$
$$\frac{1}{2}(M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3}) + M_{CU}(x)^{1/3} - \frac{1}{2}(M_{AU}(x)^{1/3} + M_{BU}(x)^{1/3})M_{CU}(x)^{1/3}]$$

Since $f_{(A@B)+C} = \frac{1}{2}(a^{1/3} + b^{1/3}) + c^{1/3} - \frac{1}{2}(a^{1/3} + b^{1/3}).c^{1/3}$

$= \frac{1}{2}((a^{1/3} + c^{1/3}) + (b^{1/3} + c^{1/3})) - \frac{1}{2}(a^{1/3}c^{1/3} + b^{1/3}.c^{1/3})$

$= \frac{1}{2}((a^{1/3} + c^{1/3} - a^{1/3}c^{1/3}) + (b^{1/3} + c^{1/3} - b^{1/3}.c^{1/3}))$

$= f_{(A+C)@(B+C)}$

therefore $M_{(A@B)+C} = M_{(A+C)@(B+C)}$

$N_{(A@B)+C}(x) = [\frac{1}{2}(N_{AL}(x)^{1/3}+N_{BL}(x)^{1/3})N_{CL}(x)^{1/3}, \frac{1}{2}(N_{AU}(x)^{1/3}+N_{BU}(x)^{1/3})N_{CU}(x)^{1/3}]$

since $g_{(A@B)+C} = \frac{1}{2}(a^{1/3} + b^{1/3})c^{1/3}$

$= \frac{1}{2}(a^{1/3}c^{1/3} + b^{1/3}c^{1/3})$

$= g_{(A+C)@(B+C)}$

therefore $N_{(A@B)+C} = N_{(A+C)@(B+C)}$

Proof (ii) is similiar to that of (i)

4 Conclusion

In this paper, we have defined two new operations over Interval Valued Intuitionistic Fuzzy Sets of Cube Root Type and their relationships are proved. We have studied some desirable properties of the proposed operations, such as idempotent laws, complementary law, commutative laws, distributive laws and etc.

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