A Game Theory Approach on Winning Probability of N Person Game

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Abstract

An n-player game is a game which is well defined for any number of players. This is usually used in contrast to standard 2 player game that are only specified for two players. In defining n-player games**.** When there are more than two players, fundamental differences arise. For example, 2-person zero-sum games which says that all equilibrium solutions have the same payoff cannot be generalized to three players. Once there are more than two players, a zero-sum game can have equilibrium solutions that do not have the same payoffs. *n*-person games in which the players are not allowed to communicate and make binding agreements are not fundamentally different from two-person noncooperative games. This study explores the topic of N-person cooperative game theory. The following paper begins with an introduction to the basic definitions and example of game theory. These results are then applied to the prisoners and puzzle.

Key Words

Game theory, Player, Payoff, Strategy, n-person cooperative game, n person noncooperative game, Nash equilibrium.
MSC: 91A60, 60G40

I. Introduction

Game Theory is the mathematical theory of interactive decision situations characterized by a group of agents |each of whom has to make a decision|, the set of possible outcomes and the preferences that each agent has on that set of outcomes. These situations are called games, agents are players and decisions are strategies. The concepts of game theory provide a common language to formulate, structure, analyse and eventually understand diff erent strategical scenarios. Generally, game theory investigates conflict situations, the interaction between the agents and their decisions.

A game in the sense of game theory is given by a (mostly finite) number of players, who interact according to given rules. Those players might be individuals, groups, companies, associations and so on. Their interactions will have an impact on each of the players and on the whole group of players, i.e. they are interdependent. As we have seen in the previous section, game theory is a branch of mathematics. Mathematics provide a common language to describe these games. We have also seen that game theory was already applied to economics by von Neumann. When there is competition for a resource to be analysed, game theory can be used either to explain existing behaviour or to improve strategies.

Game Theory is concerned with both cooperative and noncooperative models, with the latter being the most studied of the two branches. Although Noncooperative Game Theory is able to include cooperation within its reach, the complexity of the description of some situations with a mathematical model has led game theorists to regard the necessity of building cooperative models as an imperative one. The theory of the general n-person game, in contrast to that of the zero-sum two-person game, remains in an unsettled state. The chief problem seems to be that of determining the proper definition of a solution for such games. The efforts in this direction divide themselves into two groups, the cooperative theory in which the players are expected to form coalitions, and the non-cooperative in which such coalitions are forbidden**.**

The concept of n-person game, a game in characteristic function form, coalitions in n-person games. Sequential pairwise voting: sincere and sophisticated voting; voting method non-vulnerable to strategic manipulation; defects of democracy. The n-person Prisoner's Dilemma, examples from economics and sport. The Divide- the-Dollar game: individual and collective rationality. A solutions theories for n-person games: theory of stable sets and the Shapley value.

II. Historical Background

Game theory was conceived in the seventeenth century by mathematicians attempting to solve the gambling problems of the idle French nobility, evidenced for example by the correspondence of Pascal and Fermat concerning the amusement of an aristocrat called de Mere (Colman, 1982; David, 1962). Game theory in the modern era was ushered in with the publication in 1913, by the German mathematician Ernst Zermelo, of Uber eine Anwendung der Mengenlehre auf die Theorie des Schachspiels, in which he proved that every competitive two-person game possesses a best strategy for both players, provided both players have complete information about each other's intentions and preferences. In 1921, the eminent French academician Emile Borel began publishing on gaming strategies, building on the work of Zermelo and others. Over the course of the next six years, he published Wve papers on the subject, including the Wrst modern formulation of a mixed strategy game. The minimax theorem was proved for the general case in December 1926, by the Hungarian mathematician, John von Neumann. The complicated proof, published in 1928, was subsequently modiWed by von Neumann himself (1937), Jean Ville (1938), Hermann Weyl (1950) and others. Its predictions were later veriWed by experiment to be accurate to within one per cent and it remains a keystone in game theoretic constructions. The verdict of history is probably that they did not give each other much credit.

Von Neumann, tongue Wrmly in cheek, wrote that he considered it an honour 'to have labored on ground over which Borel had passed' (Frechet, 1953), but the natural competition that can sometimes exist between intellectuals of this stature, allied to some local Franco–German rivalry, seems to have got the better of common sense. In 1938, the economist Oskar Morgenstern, unable to return to his native Vienna, joined von Neumann at Princeton. He was to provide game theory with a link to a bygone era, having met the aging Edgeworth in Oxford some 13 years previously with a view to convincing him to republish Mathematical Psychics. Morgenstern's research interests were pretty eclectic, but centred mainly on the treatment of time in economic theory.

By 1940, von Neumann was synthesising his work to date on game theory (Leonard, 1992). Morgenstern, meanwhile, in his work on maxims of behaviour, was developing the thesis that, since individuals make decisions whose outcomes depend on corresponding decisions being made by others, social interaction is by dewnition performed against a backdrop of incomplete information. Their writing styles contrasted starkly: von Neumann's was precise; Morgenstern's eloquent. Nonetheless, they decided in 1941, to combine their eVorts in a book, and three years later they published what was to become the most famous book on game theory, Theory of Games and Economic Behaviour. In 1957, von Neumann died of cancer. Morgenstern was to live foranother 20 years, but he never came close to producing work of a similar calibre again. His appreciation of von Neumann grew in awe with the passing years and was undimmed at the time of his death in 1977.

John Nash (1951) succeeded in generalising the minimax theorem by proving that every competitive game possesses at least one equilibrium point in both mixed and pure strategies. In the process, he gave his name to the equilibrium points that represent these solutions and with various rewnements, such as Reinhard Selten's (1975) trembling hand equilibrium, it remains the most widely used game theoretic concept to this day. von Neumann was the founding father of game theory, Nash was its prodigal son. Born in 1928 in West Virginia, the precocious son of an engineer, he was proving theorems by Gauss and Fermat by the time he was 15. Five years later, he joined the star-studded mathematics department at Princeton – which included Einstein, Oppenheimer and von Neumann – and within a year had made the discovery that was to earn him a share (with Harsanyi and Selten) of the 1994 Nobel Prize for Economics. Nash's solution established game theory as a glamorous academic pursuit – if there was ever such a thing – and made Nash a celebrity. Sadly, by 1959, his eccentricity and self-conWdence had turned to paranoia and delusion, and Nash – one of the most brilliant mathematicians of his generation – abandoned himself to mysticism and numerology (Nasar, 1998).

The triad formed by these three works – von Neumann–Morgenstern, Luce–RaiVa and Nash – was hugely inXuential. It encouraged a community of game theorists to communicate with each other and many important concepts followed as a result: the notion of cooperative games, which Harsanyi (1966) was later to deWne as ones in which promises and threats were enforceable; the study of repeated games, in which players are allowed to learn from previous interactions (Milnor & Shapley, 1957; Rosenthal, 1979; Rosenthal & Rubinstein, 1984; Shubik, 1959); and bargaining games where, instead of players simply bidding, they are allowed to make oVers, counteroVers and side payments (Aumann, 1975; Aumann & Peleg, 1960; Champsaur, 1975; Hart, 1977; Mas-Colell 1977; Peleg, 1963; Shapley & Shubik, 1969).

It was a period of great activity at Rand from which a new rising star, Lloyd Shapley, emerged. Shapley, who was a student with Nash at Princeton and was considered for the same Nobel Prize in 1994, made numerous important contributions to game theory: with Shubik, he developed an index of power (Shapley & Shubik, 1954 & 1969); with Donald Gillies, he invented the concept of the core of a game (Gale & Shapley, 1962; Gillies, 1959; Scarf, 1967); and in 1964, he deWned his 'value' for multi-person games. Sadly, by this time, the Rand Corporation had acquired something of a 'Dr Strangelove' image, reXecting a growing popular cynicism during the Vietnam war. The mad wheelchair-bound strategist in the movie of the same name was even thought by some to be modelled on von Neumann. In 1969, Robin Farquharson used the game theoretic concept of strategic choice to propose that, in reality, voters exercised their franchise not sincerely, according to their true preferences, but tactically, to bring about a preferred outcome. game theory expanded dramatically. Important centres of research were established in many countries and at many universities. It was successfully applied to many new Welds, most notably evolutionary biology (Maynard Smith, 1982; Selten, 1980) and computer science, where system failures are modelled as competing players in a destructive game designed to model worst-case scenarios.

III. Definitions

game theory deals with interactive situations where two or more individuals, called players, make decisions that jointly determine the final outcome. The object of study in game theory is the game, which is a formal model of an interactive situation. It typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. The object of study in game theory is the game, which is a formal model of an interactive situation. It typically involves several players; a game with only one player is usually called a decision problem. The formal definition lays out the players, their preferences, their information, the strategic actions available to them, and how these influence the outcome. Games can be described formally at various levels of detail.

A coalitional (or cooperative) game is a high-level description, specifying only what payoffs each potential group, or coalition, can obtain by the cooperation of its members. What is not made explicit is the process by which the coalition forms. As an example, the players may be several parties in parliament. Each party has a different strength, based upon the number of seats occupied by party members. The game describes which coalitions of parties can form a majority, but does not delineate, for example, the negotiation process through which an agreement to vote en bloc is achieved. Cooperative game theory investigates such coalitional games with respect to the relative amounts of power held by various players, or how a successful coalition should divide its proceeds.

This is most naturally applied to situations arising in political science or international relations, where concepts like power are most important. For example, Nash proposed a solution for the division of gains from agreement in a bargaining problem which depends solely on the relative strengths of the two parties' bargaining position. The amount of power a side has is determined by the usually inefficient outcome that results when negotiations break down. Nash's model fits within the cooperative frame work in that it does not delineate a specific timeline of offers and counteroffers, but rather focuses solely on the outcome of the bargaining process.

Games and Solutions

A game is a description of strategic interaction that includes the constraints on the actions that the players can take and the players' interests, but does not specify the actions that the players do take. A solution is a systematic description of the outcomes that may emerge in a family of games. Game theory suggests reasonable solutions for classes of games and examines their properties.

Noncooperative and Cooperative Games

In all game theoretic models the basic entity is a player. A player may be interpreted as an individual or as a group of individuals making a decision. Once we define the set of players, we may distinguish between two types of models: those in which the sets of possible actions of individual players are primitives and those in which the sets of possible joint actions of groups of players are primitives. Sometimes models of the first type are referred to as "noncooperative", while those of the second type are referred to as "cooperative".

Strategic Games and Extensive Games

A strategic game is a model of a situation in which each player chooses his plan of action once and for all, and all players' decisions are made simultaneously (that is, when choosing a plan of action each player is not informed of the plan of action chosen by any other player). By contrast, the model of an extensive game specifies the possible orders of events; each player can consider his plan of action not only at the beginning of the game but also whenever he has to make a decision.

Some definition-

Game
A description of the strategic interaction between opposing, or co-operating, interests where the constraints and payoff for actions are taken into consideration**.**

Game theory
Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players.

Player
A basic entity in a game that is tasked with making choices for actions. A player can represent a person, machine, or group of persons within a game.

Action
An action constitutes a move in the given game.

Payoff
The positive or negative reward to a player for a given action within the game.

Strategy
Plan of action within the game that a given player can take during game play.

Mixed strategy
A mixed strategy is an active randomization, with given probabilities, that determines the
player's decision. As a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies.

Nash equilibrium
Nash equilibrium, also called strategic equilibrium, is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff.

Rationality
A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. It is often assumed that the rationality of all players is common knowledge.

Perfect Information Game
A game in which each player is aware of the moves of all other players that have already taken place. Examples of perfect information games are: chess, tictac-toe, and go. A game where at least one player is not aware of the moves of at least one other player that have taken place is called an imperfect information game.

Complete Information Game
This is a game in which every player knows both the strategies and payoffs of all players in the game, but not necessarily the actions. This term is often confused with that of perfect information games but is distinct in the fact that it does not take into account the actions each player have already taken. Incomplete information games are those in which at least one player is unaware of the possible strategies and payoffs for at least one of the other players.

Bayesian Game
A game in which information about the strategies and payoff for other players is incomplete and a player assigns a 'type' to other players at the onset of the game. Such games are labeled Bayesian games due to the use of Bayesian analysis in predicting the outcome.

Static/Strategic Game
A one-shot game in which each player chooseshis plan of action and all players' decisions are made simultaneously. This means when choosing a plan of action each player is not informed of the plan of action chosen by any other player. In the rest of this paper, this class of game is referred to as 'static game'.

Dynamic/Extensive Game
A game with more than one stages in each of which the players can consider their action. It can be considered as a sequential structure of the decision making problems encountered by the players in a static game. The sequences of the game can be either finite, or infinite. In the rest of this paper, this class of game is referred to as 'dynamic game.

Stochastic Game
A game that involves probabilistic transitions through several states of the system. The game progresses as a sequence of states. The game begins with a start state; the players choose actions and receives a payoff that depend on the current state of the game, and then the game transitions into a new state with a probability based upon players' actions and the current state.

n- person game
n-person games in which the players are not allowed to communicate and make binding agreements are not fundamentally different from two-person noncooperative games. when there are more than two players, fundamental differences arise. For example, the theorem about 2-person zero-sum games which says that all equilibrium solutions have the same payoff cannot be generalized to three players. Once there are more than two players, a zero-sum game can have equilibrium solutions that do not have the same payoffs**.**

IV. Repeated games

The theory of repeated games explores how mutual help and cooperation are sustained through repeated interaction, even when economic agents are completely self-interested beings. This thesis analyzes two models that involve repeated interaction in an environment where some information is private. we characterize the equilibrium set of the following game. Two players interact repeatedly over an infinite horizon and occasionally, one of the players has an opportunity to do a favor to the other player.

The ability to do a favor is private information and only one of the players is in a position to do a favor at a time. The cost of doing a favor is less than the benefit to the receiver so that, always doing a favor is the socially optimal outcome. Intuitively, a player who develops the ability to do a favor in some period might have an incentive to reveal this information and do a favor if she has reason to expect future favors in return.

We show that the equilibrium set expands monotonically in the likelihood that someone is in a position to do a favor. It also expands with the discount factor. How-ever, there are no fully efficient equilibrium for any discount factor less than unity. We find sufficient conditions under which equilibrium on the Pareto frontier of the equilibrium set are supported by efficient payoffs. We also provide a partial characterization of payoffs on the frontier in terms of the action profiles that support them. These inner and outer monotone approximations are found by looking for boundary points of the relevant sets and then connecting these to form convex sets. Working with eight boundary points gives us estimates that are coarse but still capture the comparative statics of the equilibrium set with respect to the discount factor and the other parameters.

By increasing the number of boundary points from eight to twelve, we obtain very precise estimates of the equilibrium set. With this tightly approximated equilibrium set, the properties of its inner approximation provide good indications of the properties of the equilibrium set itself. We find a very specific shape of the equilibrium set and see that payoffs on the Pareto frontier of the equilibrium set are supported by current actions of full favors. This is true so long as there is room for full favors, that is, away from the two ends of the frontier.

In n-person game a player can increase his winning probability by follow the following steps: first he should fix a principal amount. For example a player choose principal amount 100 ru. For example of n person game we are taking a board game. This game name is "Langur Burja" (also called "Jhanda Burja" and "Khor Khore"/"Khod Khode" as well as "Crown and Anchor") is popular game played during festivals like Dashain and Tihar in chhatisgarh (india) . This game is called as "Jhandi Munda" in India.

Jhandi Munda is played with six, six sided dice. There are six different symbols(image) on each of the sides of the dice (these are a heart, a spade, a diamond, a club, a face and a flag). Players bet on which symbol will appear face up the most often. The dice are then rolled and the symbol that appears most often wins.

In this game there are one organizer who arrange the game and another are players. According to this game organizer has six same type of small closed box in his basket. In which every small box has six same image in all outside cover. And they have also a chart which shows all six image. Organizer mix his all six same type of small box and without showing the box he covered all boxes by the basket. Then after player choose any image for play. According to this game organizer will pay that player who has same type of minimum 2 image shows.

In the box after the open if a player have two same image then he will find double money of their principal amount. If a player have 3 same image then that player will find triple amount of their principal amount. Same if a player have four same image then that player will find 4*multiple amont. same organizer will pay for 5th and 6th image. but if a player have a single image or no image then organizer will not pay and organizer get player's money. And organizer wins.

I. If a player want to profit in this game then he should repeat that image for next play and he should also increase his principal amount. In the last step whenever minimum 2 image will come then he will earn his lost money and get profit. For winning probability a player should follow the given chart by using $N=2^{n-1}*p$ rule:

TABLE

No. (N= number)	Princi-pal amount ($2^{n-1}*p$)	If loss (no image or single image)	profit				
			2 image	3 image	4 image	5 image	6 image
1.	100	100	200	300	400	500	600
2.	200	100+200 =300	400-300 =100	600-300 =300	800-300 =500	1000-300 =700	1200-300 =900
3.	400	300+400 =700	800-700 =100	1200-700 =500	1600-700 =900	2000-700 =1300	2400-700 =1700
4.	800	700+800 =1500	1600-1500 =100	2400-1500 =900	3200-1500 =1700	4000-1500=2500	4800-1500=3300
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V. Conclusion

With the help of the mathematical formula a player can increase the probability of winning, but it required a lot of money, passion, continuity, and patience. And a player can always get same profit of their principal amount. In a two person model, the meaning of exchanging favors is very clear. With more than two players, when in a position to do a favor, it is not so clear whom a player will provide a favor to.

This will require careful modelling with respect to the values of favors from different opponents and the cost of doing favors to different opponents. If we assume that the benefit and cost are identical for all players, we will still have to incorporate in the strategies some rules on how favors are done. For example, a player might do one favor for each opponent before doing any second favors.

With an appropriate generalization to the n-player case, it is reasonable to still expect the comparative statics results for the equilibrium set that we see in the current model. It is harder to say what the equilibrium strategies for the Pareto frontier will look like.

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