On Non Bondage Number of a Jump Graph

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Abstract

For a jump graph J(G) a set D ⊂ V(J(G) is a dominating set if every vertex in V(J(G))-D is adjacent to at least one vertex in D. The domination number √(J(G)) of J(G) is the minimum cardinality of a total dominating set. The non bondage number b_n(J(G)) of J(G) is the maximum cardinality among all sets of edges X⊆ E(J(G)) such that √(J(G) –X) =√(J(G)). A set D⊆ V(J(G)) is a strong dominating set if every vertex in V(J(G))-D has a neighbor u in D such that the degree of u is not smaller than the degree of v, The strong domination number $√_s$(J(G)) og J(G) is minimum cardinality of a strong dominating set. The non bondage number b_{sn}(J(G)) of a non empty jump graph J(G) is the maximum cardinality among all sets of edges X ⊆ E(J(G)) such that $√_s$ (J(G) –X) = $√_s$(J(G)). In this paper some results on the non bondage number, exact values of b_n(J(G)) for some standard graphs are obtained. Also some result on the strong non bondage number and bondage number are established. Also Nordhaus-Gaddum type results are found.

Key words

Bondage number, Non bondage number strong non bondage number connectivity.

Mathematics subject classification: 05C

Introduction

All graphs considered here are finite, undirected without loops or multiple edges and isolated vertices and hence p vertices and q edges. Any undefined terms here may be found in Harary[1].

A set D of vertices in a jump graph J(G) is a dominating set of J(G) if vertex in V(J(G))-D is adjacent to some vertex in d. The dominating set of J(G). a recent survey if √(G) can be found in Kulli [2]. Among the various applications of the theory of domination that have been consider3e the one that is perhaps most often discussed concerns a communication network. To minimize the direct communication links in the network. In[4] kulli and janakiram introduced the concept of non bondage number as follows.
The non bondage number b_n(G) of a graph G is the maximum cardinality of all sets of edges X⊆ E(G) for which √(G - X) = √(G). let uv be an edge of g Then u and v dominate each other, u strongly dominate v if deg u ≥ deg v. A set D⊆ V is strongly dominating set if every vertex in V – d is strongly dominating by some u in D. The strong domination number $√_s$(G) of a graph G is the minimum cardinality of a strong dominating set see[5]. The strong non bondage number $√_{sn}$(G) of a non empty graph g is the cardinality among all sets of edges X⊆ E for which $√_s$(G – X)=$√_s$(G) see[6]. The strong bondage number b_s(G) of G is minimum cardinality among all sets of edges X⊆ E for which $√_s$(G – X) > $√_s$(G) see[8].

Analogously we define non bondage number of jump graph. the non bondage number b_n(J(G)) of a jump graph J(G) is maximum cardinality of all sets of edges
X ⊆ E(J(G)) for which √ (J(G) – X) = √ (J(G)). Let uv be an edge of J(G). then u and v dominate each other.

Further u strongly dominates v if deg u ≥ deg v. A set D⊆V(J(G)) is strongly dominating set,if every vertex v in V(J(G) –D is strongly dominating by some u in D.

The strong dominating number $\surd_s$(J(G)) of a jump graph J(G) is he minimum cardinality of a strong dominating set. The strong non bondage number b_{sn}(J(G)) of a non empty jump graph J(G) is the maximum cardinality among all sets of edges X ⊆ E(J(G)) for which √sn(J(G)) = $\surd_{sn}$(J(G)). The strong bondage number b_s(J(G))of J(G) is the maximum cardinality among all sets of edges X ⊆ E(J(G)) for which $\surd_s$(J(G) - X) > $\surd_s$(J(G))

II Exact values of b_n(J(G)) for some standard jump gaphs:

Poposition 1; If p_p is path with $p \geq 4$ vertices then $b_n(J(p_p)) = [\frac{p}{3}] - 1$.

Proposition 2; If C_p is a cycle with $p \geq 3$ vertices then $b_n(J(C_p)) = [\frac{p}{3}]$

Proposition 3; If K_p is a complete graph with $p \geq 3$ vertices then
$b_n(J(K_p)) = \frac{(p-1)(p-2)}{2}$

Proposition 4; If W_p is wheel with $p \geq 3$ vertices then $b_n(JW_p)) = p - 1$

III Non bondage number.

Theorem A[4] For any graph J(G) b_n (J(G)) = q – p +√(J(G)) ……(1)

Theorem 1; For any tee T, b_n (J(T)) = √ (J(T)) – 1.
Proof: This follows from (1) and for a tree T, q=p-1

Theorem 2; For any unicycle jump graph b_n(J(G))= √(J(G))
Proof: This follows from (1) and for any unicyclic jump graph J(G) p= q

TheoremB[6] For any graph G $b_n(G) \leq b_{sn}(G)$.

Theorem 3. For any jump graph J(G) $b_n(J(G)) \leq b_{sn}(J(G))$

Theorem C[6]: For any Tree T $b_{sn}(T) \leq \frac{4(p-2)}{2}$

Theorem 4; For any tree **T** $b_{sn}(T) \leq \frac{4(p-2)}{2}$
Proof; This follows from theorem b, theorem C and Theorem D [9]. For any jump graph J(G). .√(J(G) + √(J($\bar{G}$)) ≤ p+1
In the following theorems, we established Nordhaus-Gaddum type results.

Teorem 5; for a jump graph J(G) and iots complement J($\bar{G}$)
$b_n J(G)) + b_n J(\bar{G}) \leq \frac{(p-1)(p-2)}{2}$
By theorem A[4] $b_n(J(G)) + b_n (J(\bar{G}))$ = q - p +√(J(G)) + $\bar{q}$ - p √(J($\bar{G}$))
= (q + $\bar{q}$) – 2p + √(J(G)) + √ ((J($\bar{G}$))
$\leq \frac{p^2-5p}{2} + p + 1$

$b_n(J(G)) + b_n (J(\bar{G})) \leq \frac{(p-1)(p-2)}{2}$
For any jump graph j(g) $b_n(J(G)) \leq q - \Delta(J(G))$

We now give another proof of theorem1.

By theorem E [9]

$b_n(J(G)) \leq q - \Delta(J(G)) \leq q - \delta(J(G))$

$b_n(J\bar{G}) \leq \bar{q} - \Delta(J(\bar{G}))$

then $b_n(J(G)) + b_n(J\bar{G}) = q - \delta(J(G)) + \bar{q} - \Delta(J(\bar{G}))$

$= q + \bar{q} - \Delta(J(\bar{G})) + \delta(J(G))$

$\leq \frac{p(p-1)}{2} - (p-1)$

$\leq \frac{(p-1)(p-2)}{2}$

Theorem 6,; If J(G) and J($\bar{G}$) are connected then

$$b_n(J(G)) + b_n(J(\bar{G})) \leq \frac{p(p-3)}{2}$$

Proof; By theorem(1)

$b_n(J(G)) + b_n(J(\bar{G})) = (q + \bar{q}) - 2p + \surd(J(G)) + \surd(J(J(\bar{G}))$

$= \frac{p^2-5p}{2} + \surd(J(G)) + \surd(J(J(\bar{G}))$

By theorem [F]

$b_n(J(G)) + b_n(J(\bar{G})) \leq \frac{p^2-5p}{2} + p = \frac{p^2-3p}{2}$

Or $b_n(J(G)) + b_n(J(\bar{G})) \leq \frac{p(p-3)}{2}$

Theorem 7; if J(T) and J($\bar{T}$) are connected then

$$b_n(J(T)) + b_n(J(\bar{T})) \leq p - 2$$

Prtoof; By theorem (1) $b_n(J(T)) = \surd(J(T)) - 1$

$b_n(J\bar{T})) = \surd(J(\bar{T})) - 1$

$\therefore \quad b_n(J(G)) + b_n(J(\bar{G})) = \surd(J(T)) + \surd(J(\bar{T})) - 2$

By theorem 1.

$\surd(J(T)) + \surd(J(\bar{T})) \leq p$

$b_n(J(G)) + b_n(J(\bar{G})) \leq p - 2$

By Theorem H[4] Let g be a unicyclic graph if $\surd(G) \leq \frac{p}{2}$ then $b_n(G) \leq \Delta(G)$

Similarly L:et J(G) be a unicyclic jump graph if $\surd(J(G)) \leq \frac{p}{2}$ then

$b_n(J(G)) \leq \Delta(J(G))$

IV strongly non bondage number;

Theorem J[4] For any connected graph G

$\frac{diam(G) - 2}{3} \leq b_n(G)$

Theorem 8 ; For any connected jump graph J(G)

$\frac{diam(J(G)) - 2}{3} \leq b_n(J(G))$

Proof; This follows from theorem 2 and theorem 1.
We give a simple proof of the following theorem
Theorem 11[6] let G be a unicyclic graph if $\sqrt{}(G) \leq \frac{p}{2}$ then $b_n(G) \leq \Delta(G)$
Let J(G) be a unicyclic jump graph if $\sqrt{}(J(G)) \leq \frac{p}{2}$ then

$$b_n(J(G)) \leq \Delta(J(G))$$

Proof; This follows from Theorem H[4] and Theorem(2)
Theorem K[4]; For any graph G $b(G) \leq b_n(G) + 1$
The following result involving the bondage number gives a lower bound for $b_{sn}(G)$.

Theorem;9. For any jump graph J(G) $b(J(G)) - 1 \leq b_n(J(G))$

Proof; By theorem B[6] $b_n(J(G)) \leq b_{sn}(J(G))$ and by Theorem K $b(G)) \leq b_n(G)) + 1$
$b_n(J(G)) \leq b_{sn}(J(G)) + 1$.

Bondage number;
We establish Nordhaus-Gaddum type result.
Theorem For any jump graph J(G) and its complement $J(\bar{G})$
$b(J(G)) + b(J(\bar{G})) \leq b_n(J(G)) + b_n(J(\bar{G})) + 2$
By theorem 1
$b(G) + b(\bar{G}) \leq b_n(G) + b_n(\bar{G}) + 2$
$\therefore\ b(J(G)) + b(J(\bar{G})) \leq b_n(J(G)) + b_n(J(\bar{G})) + 2$

$$\leq \frac{(p-1)(p-2)}{2} + 2 \text{ by theorem 4}$$

Theorem10: If J(G) and $J(\bar{G})$ are connected then $b(J(G)) + b(J(\bar{G})) \leq \frac{p\,(p-3)}{2} + 2$ and this bound is sharp.

Proof; By theorem K.
$b(J(G)) + b(J(\bar{G})) \leq b_n(J(G)) + b_n(J(\bar{G})) + 2$
then by theorem 4

$$\therefore\ b(J(G)) + b(J(\bar{G})) \leq \frac{(p-1)(p-2)}{2} + 2$$

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