One Point Union Families and Invariance under Cordial Labeling

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Abstract

We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling.We take G as tail graph. A tail graph (orantenna graph) is obtained by attaching a path P_m to a vertex of given graph. It is denoted by tail(G,P_m) where G is given graph. We take G as C_3 and restrict our attention to m = 2,3,4 in P_m.Further we consider all possible structures of $G^{(k)}$ by changing the common point and obtain non-isomorphic structures. We show all these structures as cordial graphs.This is called as invariance of different structures of $G^{(k)}$under cordial labeling.

Key words

cordial,one point union, tail graph, cycle,labeling, path.

Subject Classification: 05C78

I.Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e$v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling.Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if tis not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian[8].

Our focus of attention is onone point unions on different graphs. For a given graph there are different one point unions (upto isomorphism) structures possible. It depends on which point on G is used to fuse to obtain one point union. We have shown that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union $P_m(G)$ are cordial[4].It is called as invariance under cordial labeling. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are.The graph whose cordial labeling is available is called as cordial graph. In this paper we define tail graph and obtain one point union graphs on it.

II.Preliminaries

3.1Tail Graph: A (p,q)graph G to which a path P_m is fused at some vertex.This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1.It is denoted by tail(G,P_m).In this paper we fix G as C_3 and take P_m for m =2,3,4,5.

3.2 Fusion of vertices. Let $u \neq v$ be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.

3.3$G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies.If G is a (p, q) graph then $|V(G_{(k)}|$ = k(p-1)+1and |E(G)|= k.q

4. Theorems Proved

4.1 Theorem: Let G =tail(C_3,P_2).Then $G^{(k)}$ is cordial.

Proof: Under the function f:V(G)→{0,1} we define different types of units as shown below.

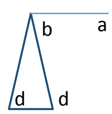

Fig 4.1 G = tail (C_3,P_2)

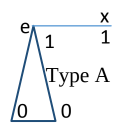

Fig 4.2 $v_f(0,1)$ =(2,2), $e_f(0,1)$ =(2,2)

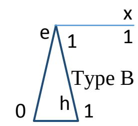

Fig 4.3 $v_f(0,1)$=(1,3) , $e_f(0,1)$ =(2,2)

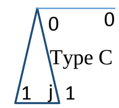

Fig 4.4 $v_f(0,1)$,$e_f(0,1)$ =(2,2)

Fromfig 4.1 It follows that we can take one point union at points ‘a’ or ‘b’ or ‘d’ and all will be different.(up toisomorphism).We call them as three different structures.

Structure1 :Pendent vertex ‘a’ on G is used as common vertex on $G^{(k)}$ .The i^{th} copy on $G^{(k)}$ is **type A** with vertex ‘x’ – a common vertex on it when i≡1 (mod 2) and **type B** label with vertex ‘x’ – a common vertex on it when i≡0 (mod 2).

Structure 2 : vertex ‘b’ on G is used as common vertex on $G^{(k)}$ .The i^{th} copy on $G^{(k)}$ is **type A** with vertex ‘e’ on it when i≡1 (mod 2) and **type B** label with vertex ‘e ’ on it when i≡0 (mod 2).

Structure 3 : vertex ‘d’ on G is used as common vertex on $G^{(k)}$ .The i^{th} copy on $G^{(k)}$ is **type C** with vertex ‘j’ on it when i≡1 (mod 2) and **type B** label with vertex ‘h ’ on it when i≡0 (mod 2).The label number distribution for all structures above is as follows: $v_f(0,1)$=(3x+2,3x+2) and $e_f(0,1)$ = (2k,2k) when k is an odd number given by 2x+1,x = 0,1,2..

$v_f(0,1)$=(3x,3x+1) and $e_f(0,1)$ = (2k,2k) when k is an odd number given by 2x,x = 1,2..

Thus all possible 3 structures on $G^{(k)}$ arecordial. This is also called as $G^{(k)}$ is invariant under cordial labeling when G = tail(C_3,P_2).

Thus the graph is cordial. #

4.2 Theorem: Let G =tail(C_3,P_3).Then $G^{(k)}$ is cordial.

Proof: Define a function f:V(G)→{0,1} as follows: We define four types of labels as above, Type A, Type B, Type C and Type D. All are cordial but pairwise non isomorphic. They differ in label numbers or label of vertex are not same.

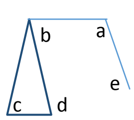

Fig 4.5 G = tail (C_3,P_3)

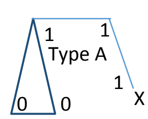

Fig 4.6 G = tail (C_3,P_3) $v_f(0,1)$ = (2,3),$e_f(0,1)$=(3,2)

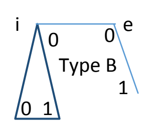

Fig 4.7 G = tail (C_3,P_3) $v_f(0,1)$ = (3,2),$e_f(0,1)$=(2,3)

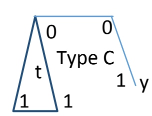

Fig 4.8 G = tail (C_3,P_3) $v_f(0,1)$ = (2,3),$e_f(0,1)$=(2,3)

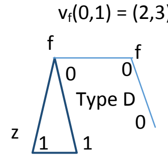

Fig 4.9 G = tail (C_3,P_3) $v_f(0,1)$ = (3,2),$e_f(0,1)$=(3,2)

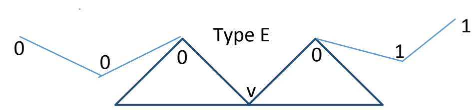

Fig 4.10 $G^{(2)}$; $v_f(0,1)$ = (4,5), $e_f(0,1)$=(5,5)

Structure 1: vertex ‘x’ from Type A and vertex ‘y’ from Type C are fused together.In $G^{(k)}$ the i^{th} copy is of type A if i≡1 (mod2)i.e.i is of type 2x+1.x= 0,1,2,..and of type C if i≡0 (mod 2).i.e. i is of type 2x, x=1,2,..The label numbers are as follows:

if k is of type 2x+1, x= 0,1,2,..then $v_f(0,1)$ =(2+4x, 3+4x) and $e_f(0,1)$=(3+5x,2+5x).

If k is of type 2x,x = 1,2,3…then $v_f(0,1)$ =(4x, 1+4x) and $e_f(0,1)$=(5x, 5x).

Thus structure 1 is cordial.

Structure 2 and structure 3: These are obtained by fusing vertex ‘a’ on G and vertex ‘b’ on G respectively for Structure 2 and structure 3.

For **structure 2** vertex ‘e’ from type B and vertex ‘f’ from type D are fused together. To obtain a labeled copy of $G^{(k)}$ we use type B label as each i^{th} copy if i≡1 (mod2) i.e. i is of type 2x+1.x= 0,1,2,.. and copy type D if i≡0 (mod 2).i.e. i is of type 2x, x=1,2,.. (i= 1, 2, ..)

To obtain **structure 3** fuse vertex ‘i’ from type B and ‘f’ from type D. To obtain a labeled copy of $G^{(k)}$ we use type B label as each i^{th} copy if i≡1 (mod2) i.e. i is of type 2x+1.x= 0,1,2,.. and copy type D if i≡0 (mod 2).i.e. i is of type 2x, x=1,2,..

The resultant label numbers for both structures are as follows:

if k is of type 2x+1,x= 0,1,2,..then $v_f(0,1)$ =(3+4x, 2+4x) and $e_f(0,1)$=(2+5x,3+5x).
If k is of type 2x,x = 1,2,3…then $v_f(0,1)$ =(1+4x, 4x) and $e_f(0,1)$=(5x, 5x).
Thus Structure 2 and structure 3 are cordial graphs.

Structure 4: When k = 2x, x= 1, 2,.. we use Type E label .We take xcopies of Type E label and fuse it at vertex v to get labeled copy of $G^{(2x)}$ .The label numbers are $v_f(0,1)$ = (4x,4x+1), $e_f(0,1)$=(5x,5x). To obtain a labeled copy of $G^{(k)}$ for k = 2x+1 we start with labeled copy of $G^{(2x)}$.Choose type C label and fuse it at vertex ‘t’ on it with $G^{(2x)}$ with vertex v on $G^{(2x)}$. The resultant graph has label distribution given by $v_f(0,1)$ = (4x+2,4x+3), $e_f(0,1)$=(5x+2,5x+3).

Thus the graph is cordial. #.

4.3 Theorem: One point union of k copies of G=tail(C_3,p_4) is cordial.

Proof. Define a function f:V(G)→{0,1} as follows. Under f we get labeled copies Type A, type B and Type C. Only type A is cordial labeling and not type B and type C .We use these copies suitably to obtain labeled copy of $G^{(k)}$.

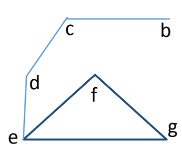

Fig 4.11 ordinary labeling of tail(C_3,P_4)

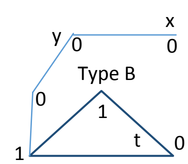

Fig 4.12 $v_f(0,1)$ = (4,2) , $e_f(0,1)$ = (3,3)

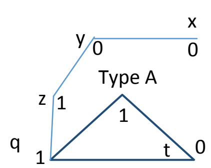

Fig 4.13$v_f(0,1)$ = (3,3) , $e_f(0,1)$ = (3,3)

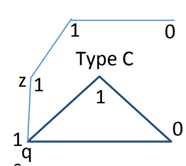

Fig 4.14$v_f(0,1)$ = (2,4) , $e_f(0,1)$ = (3,3)

If we take one point union at point ‘b ‘ **structure 1** is obtained.
If we take one point union at point ‘c ‘ **structure 2** is obtained.
If we take one point union at point ‘d ‘ **structure 3** is obtained.

If we take one point union at point ‘e ‘ **structure 4** is obtained.
If we take one point union at point ‘f ‘ or ‘g’ **structure 5** is obtained.

To obtain labeled copy of **structure 1** we take labeled copy of type A and Type B and fuse it at vertex ‘x’ on it. For i = 1, 2use Type A label. For i>2, thei^{th} copy on $G^{(k)}$ will be type B label if i≡1 (mod 2) and Type A, if i≡ 0(mod 2).

To obtain labeled copy of **structure 2** we take labeled copy of type A and Type B and fuse it at vertex ‘y’ on it. For i = 1, 2use Type A label. For i>2, thei^{th} copy on $G^{(k)}$ will be type B label if i≡1 (mod 2) and Type A, if i≡ 0(mod 2).

To obtain labeled copy of **structure 5** we take labeled copy of type A and Type B and fuse it at vertex ‘t’ on it. For i = 1, 2 use Type A label. For i>2,the i^{th} copy on $G^{(k)}$ will be type B label if i≡1 (mod 2) and Type A, if i≡ 0(mod 2).

The resultant label numbers for above **three structures** are as follows :

$v_f(0,1) = (3+5x,3+5x)$, $e_f(0,1) = (3k,3k)$ when m is of the type 2x+1, x= 0,1,2..

$v_f(0,1) = (5x,1+5x)$, $e_f(0,1) = (3k,3k)$ when m can be written as 2x, x= 0,1,2…

To obtain labeled copy of **structure 3** we take labeled copy of type A and Type C and fuse it at vertex ‘z’ on it. For i = 1,2 use Type A label. For i>2, thei^{th} copy on $G^{(k)}$ will be type C label if i≡1 (mod 2) and Type A, if i≡ 0(mod 2).

To obtain labeled copy of **structure 4** we take labeled copy of type A and Type C and fuse it at vertex ‘q’ on it. For i = 1,2 use Type A label. For i>2, thei^{th} copy on $G^{(k)}$ will be type C label if i≡1 (mod 2) and Type A, if i≡ 0(mod 2).

The resultant label numbers for above **two structures** are as follows :

$v_f(0,1) = (3+5x,3+5x)$, $e_f(0,1) = (3k,3k)$ when k is of the type 2x+1, x= 0,1,2..

$v_f(0,1) = (6+5x,5+5x)$, $e_f(0,1) = (3k,3k)$ when k can be written as 2x, x= 0,1,2…

Thus the graph is cordial. #.

Theorem 4.4 $G^{(k)}$ is cordial where G = tail(C_3,P_5)

Proof:From fig 4.15 it follows that there are 6 non- isomorphic structures possible on $G^{(k)}$.

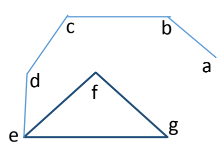

Fig 4.15; tail(C_3,P_5)

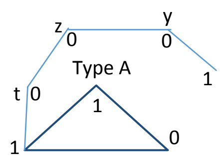

Fig 4.16 $v_f(0,1) = (4,3)$ $e_f(0,1) = (3,4)$

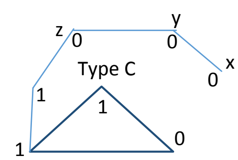

Fig 4.17 $v_f(0,1) = (4,3)$ $e_f(0,1) = (4,3)$

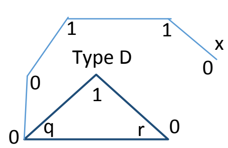

Fig 4.18 $v_f(0,1) = (4,3)$, $e_f(0,1) = (3,4)$

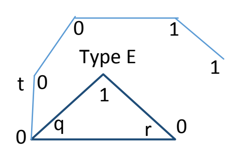

Fig 4.19 $v_f(0,1) = (4,3)$, $e_f(0,1) = (4,3)$

If we take one point union at point ‘a ‘ structure 1is obtained.If we take one point union at point ‘b ‘ structure 2 is obtained. If we take one point union at point ‘c‘ structure 3 is obtained.If we take one point union at point ‘d ‘ structure 4 is obtained.If we take one point union at point ‘e‘ structure 5 is obtained.If we take one point union at point ‘f‘or‘g’ structure 6 is obtained.

Define a function f:V(G)→{0,1} then different types of labeling are obtained as shown in figure.
To obtain labeled copy of **structure 1** we take labeled copy of type C and Type D and fuse it at vertex 'x' on it.The i^{th} copy on $G^{(k)}$ will be type C label if i≡1 (mod 2). And Type D if i≡0(mod 2).

To obtain labeled copy of **structure 2** we take labeled copy of type C and Type A and fuse it at vertex 'y' on it. The i^{th} copy on $G^{(k)}$ will be type C label if i≡1 (mod 2). And Type A if i≡0(mod 2).

To obtain labeled copy of **structure 3** we take labeled copy of type C and Type A and fuse it at vertex 'z' on it. The i^{th} copy on $G^{(k)}$ will be type C label if i≡1 (mod 2). And Type A if i≡0(mod 2).

To obtain labeled copy of **structure 4** we take labeled copy of type A and Type E and fuse it at vertex 't' on it. The i^{th} copy on $G^{(k)}$ will be type E label if i≡1 (mod 2). And Type A if i≡0(mod 2).

To obtain labeled copy of **structure 5** we take labeled copy of type D and Type E and fuse it at vertex 'q' on it. The i^{th} copy on $G^{(k)}$ will be type E label if i≡1 (mod 2). And Type D if i≡0(mod 2).

To obtain labeled copy of **structure 6** we take labeled copy of type D and Type E and fuse it at vertex 'r' on it. The i^{th} copy on $G^{(k)}$ will be type E label if i≡1 (mod 2). And Type D if i≡0(mod 2).

The resultant label numbers are as follows and same for all structures.

$v_f(0,1) = (4+6x,3+6x)$, $e_f(0,1) = (4+7x,3+7x)$ for k =2x+1, x=0,1,2…

And$v_f(0,1) = (1+6x,6x)$, $e_f(0,1) = (7x,7x)$ for k =2x, x=1,2…

Thus the graph is cordial #

Theorem 4.5: $G^{(k)}$ is cordial where G = tail(C_3,P_6)

Proof. Define a function f: V(G)→{0,1] as follows. Under f we get labeled copies Type A, type B and Type C. Type A is cordial but Type B and Type C are not .We use these copies suitably to obtain labeled copy of $G^{(k)}$

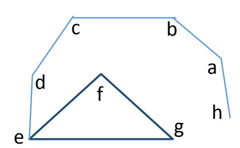

Fig 4.20; tail(C_3,P_6)

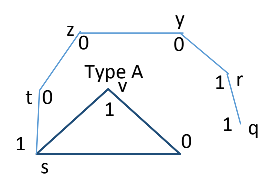

Fig 4.21$v_f(0,1) = (4,4)$, $e_f(0,1) = (4,4)$

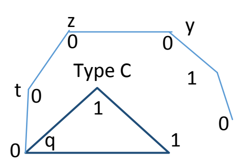

Fig 4.22 $v_f(0,1) = (5,3)$, $e_f(0,1) = (4,4)$

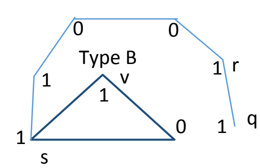

Fig 4.23 $v_f(0,1) = (3,5)$ $e_f(0,1) = (4,4)$

We take one point union at point 'h' **structure 1** is obtained.

If we take one point union at point 'a '**structure 2** is obtained.
If we take one point union at point 'b ' **structure 3** is obtained.
If we take one point union at point 'c' **structure 4** is obtained.
If we take one point union at point 'd ' **structure 5** is obtained.
If we take one point union at point 'e' **structure6** is obtained .
If we take one point union at point 'f 'or 'g' **structure 7** is obtained.

To obtain labeled copy of **structure 1** we take labeled copy of type A and Type B and fuse it at vertex 'q' on it. For i = 1,2 use Type A label. For i>2 ,the i^{th} copy on $G^{(k)}$ will be type B label if $i \equiv 1 \pmod 2$ and Type A, if $i \equiv 0 \pmod 2$.

To obtain labeled copy of **structure 2** we take labeled copy of type A and Type B and fuse it at vertex 'r' on it. For i = 1,2 use Type A label. For i>2 ,the i^{th} copy on $G^{(k)}$ will be type B label if $i \equiv 1 \pmod 2$ and Type A, if $i \equiv 0 \pmod 2$.

To obtain labeled copy of **structure 6** we take labeled copy of type A and Type B and fuse it at vertex 's' on it. For i = 1,2 use Type A label. For i>2 ,the i^{th} copy on $G^{(k)}$ will be type B label if $i \equiv 1 \pmod 2$ and Type A, if $i \equiv 0 \pmod 2$.

To obtain labeled copy of **structure 7** we take labeled copy of type A and Type B and fuse it at vertex 'v' on it. For i = 1,2 use Type A label. For i>2 ,the i^{th} copy on $G^{(k)}$ will be type B label if $i \equiv 1 \pmod 2$ and Type A, if $i \equiv 0 \pmod 2$.

The resultant label numbers for above **four structures** are as follows :

$v_f(0,1) = (4+7x,4+7x)$, $e_f(0,1) = (4k,4k)$ when m is of the type 2x+1, x= 0,1,2..

$v_f(0,1) = (7x+1,7x)$, $e_f(0,1) = (4k,4k)$ when m can be written as 2x, x= 1,2…

To obtain labeled copy of **structure 3** we take labeled copy of type A and Type C and fuse it at vertex 'y' on it. For i = 1, 2 use Type A label. For i>2,the i^{th} copy on $G^{(k)}$ will be type A label if $i \equiv 1 \pmod 2$ and Type C, if $i \equiv 0 \pmod 2$.

To obtain labeled copy of **structure 4** we take labeled copy of type A and Type C and fuse it at vertex 'z' on it. For i = 1,2 use Type A label. For i>2,the i^{th} copy on $G^{(k)}$ will be type A label if $i \equiv 1 \pmod 2$ and Type C, if $i \equiv 0 \pmod 2$.

To obtain labeled copy of **structure 5** we take labeled copy of type A and Type C and fuse it at vertex 't' on it. For i = 1,2 use Type A label. For i>2,the i^{th} copy on $G^{(k)}$ will be type A label if $i \equiv 1 \pmod 2$ and Type C, if $i \equiv 0 \pmod 2$.

The resultant label numbers for above **three structures** are as follows :

$v_f(0,1) = (4+7x,4+7x)$, $e_f(0,1) = (4k,4k)$ when m is of the type 2x+1, x= 0,1,2..

$v_f(0,1) = (7x,7x+1)$, $e_f(0,1) = (4k,4k)$ when m can be written as 2x, x= 0,1,2…

Thus the graph is cordial #.

Conclusions: A tail graph is obtained by attaching a path P_m to a vertex on G.We have investigated and shown that for G = C_3 and m = 2 to 6. The one point union of k copiestaken at different vertices on copies of tail(G,P_m) are shown to be cordial.We expect that if G=tail(C_3,P_m)then $G^{(k)}$ is cordial for all m and given k. We have proved that:

1) Let G =tail(C_3,P_2).Then $G^{(k)}$ is cordial.
2) Let G =tail(C_3,P_3).Then $G^{(k)}$ is cordial.
3) One point union of k copies of G=tail(C_3,p_4) is cordial.
4.) $G^{(k)}$ is cordial where G = tail(C_3,P_5)
5.) $G^{(k)}$ is cordial where G = tail(C_3,P_6).

It is necessary to investigate such type of graphs further for cordiality.

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