Zhou's Differential Transformation Method for Study of Arm Race Richardson Model

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Abstract — *The ZDTM – Zhou's differential transform method is nothing but studies of the "traditional". Taylor series which set on the same footing as Laplace, Fourier transformation which is easily adaptable and attainable to different kind of differentiation procedures. This method is very attractive to solve initial valve problems of linear and nonlinear differential equations which may homogeneous or non-homogeneous as compare to Taylor series for higher order linear differential equations. The definition and operation of Zhous differential transform method investigate particular exact solutions of system of linear differential equations.*

By considering three examples on system of linear homogeneous differential equation with initial values, the results are compared with exact solution with graphs. It is found that ZDTM solutions have very high degree of accuracy. These results show that the method introduce here for Richardson model is accurate & easy to apply by reducing lot of computational work.

Keywords — *System of Linear differential equation, ZDTM, Initial value problem, analytic continuation.*

I. INTRODUCTION

The Zhou's differential transform method as an analytic tool and not as the basis for algorithms. It is true that ZDTM implicitly involves the automatically implementing an "analytic continuation" as Euler did it Pukhov G.E. studied computational structure for solving differential equations by Taylor Series [1], Also Pukhov find differential transform of functions and equations [2], J.K. Zhou used ZDTM for application of solving electrical circuits problems [3], J. S. Chiou et.al. find solution of non-linear Vibration problems [4], C. K. Chen and S. H. Ho, applied ZDTM to eigenvalue problems [5], M. J. Jang et. al. used ZDTM for solving damped motion [6], C. L. Chen et.al. find solutions of heat conduction problems by ZDTM [7], H. Finkel used ZDTM for solving partial differential equation IVPs [8], A. Gibbon develop program for multi point ZDTM [9], D. Barton, et.al. find solution of systems of ordinary differential equations by ZDTM [10], S.S. Chen et.al. shown that non-linear conservative system can be solved by ZDTM [11], I.H.A.H. Hassan used ZDTM for solving Higher order differential equations initial value problems [12], H. Yaghoobi and M. Torabi applied ZDTM for non-linear heat transfer equation [13], M. J. Jang & Y. C. Liy et.al. solved initial value problems by ZDTM [14], A. Arikoglu et.al. used ZDTM for solving boundary value problems for integro differential equations [15], F. Kangalgil and F. Ayaz obtained solution of linear and non-linear heat equations by ZDTM [16]

II. DEFINITIONS AND PROPERTIES OF ZDTM METHOD

$$x(K) = \sum_{K=0}^{\infty} (t - t0)^k \frac{1}{K!} \left[\frac{d^k x}{dt^k}\right]_{t = t0}$$

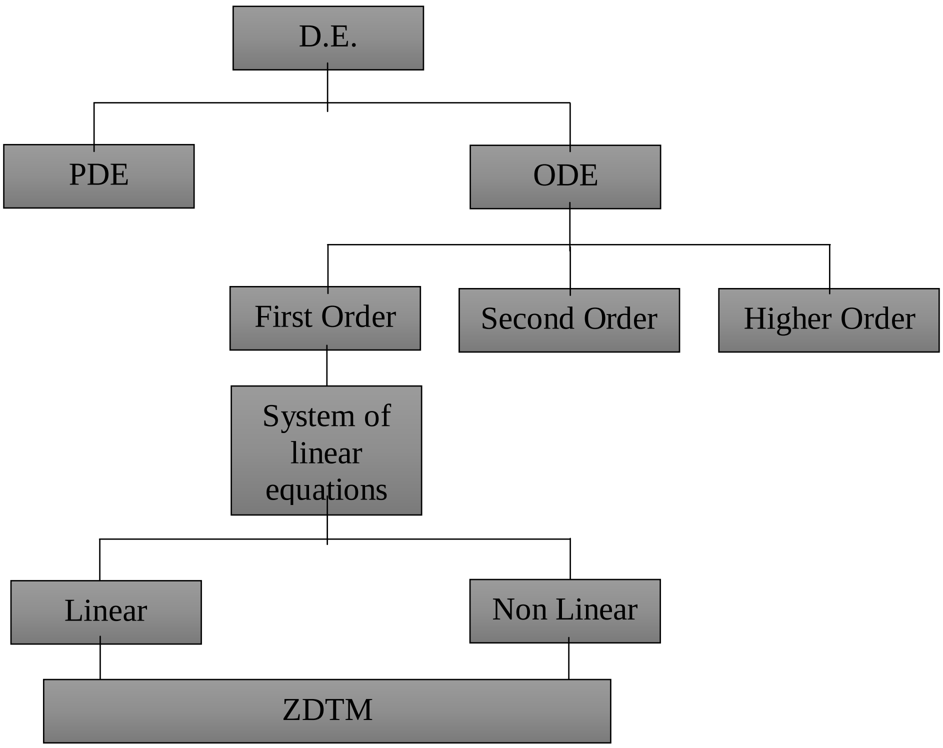
Transfer of x(t) is denoted by X(K)

III.FUNDAMENTAL THEOREMS ON ZDTM

	Original Function			**Transformation**		
1)	g(t)	=	x(t) + y(t)	G(K)	=	X (K) + Y (K)
2)	g(t)	=	$\propto$ x (t)	G(K)	=	$\propto \times$ (K)
3)	g(t)	=	$\frac{d}{dt}$ x(t)	G(K)	=	(K + 1) $\times$ (K+ 1)

4)	$g(t) = \frac{d^n}{dt^n} x(t)$	$G(K) =$	$(K + 1)(K + 2) \ldots (K + n) X (K + n)$
5)	$g(t) = t^n$	$G(K) =$	$\delta (k - n)$
		$=$	$1 \quad k = n$
		$=$	$0 \quad k \neq n$
6)	$g(t) = e^{\lambda +}$	$G(K) =$	$\frac{\lambda}{K!}$
7)	$g(t) = \mathrm{Sin}(\alpha t + \beta)$	$G(K) =$	$\frac{\alpha^k}{K!} \mathrm{Sin}\left(\frac{k\pi}{2} + \beta\right)$
8)	$g(t) = \mathrm{Cos}(\alpha t + \beta)$	$G(K) =$	$\frac{\alpha^k}{K!} \mathrm{Cos}\left(\frac{k\pi}{2} + \beta\right)$
9)	$g(t) = x(t) . y(t)$	$G(K) =$	$\sum_{m=0}^{K} Y(n) \times (k - n)$
10)	$g(t) = (1 + t)^n$	$G(K) =$	$\frac{n(n-1) \ldots (n-k+1)}{K!}$

IV. FLOW CHART OF ZDTM (FOR RELATED RESEARCH PAPER)



V. Experimentation Of ZDTM Results To Arm Rale Problems

Example : 1

It X and Y denote the expenditure incurred by two countries on armament by

$$\frac{dx}{dt} = 4y - 3x + 2$$

$$\frac{dy}{dt} = 2x - y + 2$$

with initial conditions $x(0) = 4$

$y(0) = 1$

$\rightarrow$ Exact Solution of above system of linear equations is given by

$$\left.\begin{aligned} x(t) &= 4e^{t} + 2e^{-5t} - 2 \\ y(t) &= 4e^{t} - e^{-5t} - 2 \end{aligned}\right\} \quad \text{.........(i)}$$

By ZDTM of given Richardson model

$$(K + 1)\, X(K + 1) = 4\, Y(K) - 3X(K) + 2\,\delta\,(K - 0)$$

$$(K + 1)\, Y(K + 1) = 2\, X(K) - Y(K) + 2\,\delta\,(K - 0)$$

Put **K = 0, 1, 2, 3, 4 …………………**

$X(0) = 4$ and $Y(0) = 1$

$X(1) = -6$ $\quad Y(1) = 9$

$X(2) = 27$ $\quad Y(2) = -\frac{13}{2}$

$X(3) = \frac{-107}{3}$ $\quad Y(3) = \frac{121}{6}$

$X(4) = \frac{563}{3}$ $\quad Y(4) = \frac{-549}{24}$

Solution is given by

$$X(+) = X(0) + X(1)t + X(2)t^2 + X(3)t^3 + \ldots\ldots\ldots\ldots$$

$$= 4 - 6t + 27t^2 - \frac{107}{3}t^3 + \frac{563}{12}t^4 + \ldots\ldots\ldots\ldots$$

$$Y(+) = Y(0) + Y(1)t + Y(2)t^2 + Y(3)t^3 + \ldots\ldots\ldots\ldots$$

$$= 1 - 9t - \frac{13}{2}t^2 + \frac{121}{6}t^3 - \frac{549}{24}t^4 + \ldots\ldots\ldots\ldots$$

Table 1

t	**Exact Sol^n**		**DTM Sol^n**	
	x(t)	**y(t)**	**x(t)**	**y(t)**
0.0	4	1	4	1
0.1	3.63374499	1.814153013	3.639025	1.852879167
0.2	3.6213699	2.5177315	3.6697333	2.66473333
0.3	3.84569555	3.1763050	4.047025	3.4742125
0.4	4.2379693	3.83196350	4.8384	4.26506666
0.5	4.75905508	4.5128000	6.2239	4.966145833

x(t) & y(t) are unbounded time increases a situation may result in runway arm race by solving Richard son model.

Example : 2

Discuss arm race model wore D.E. are in Richarson model.

$$\frac{dx}{dt} = 4y - 3x - 2$$

$$\frac{dy}{dt} = 2x - y - 2$$

With x(0) = 1, y(0) = $\frac{1}{2}$

$\rightarrow$ Exact solution of above Richard son model is

$x(t) = -e^{t} + e^{-5t} + 2$

$y(t) = -et - \frac{1}{2} e^{-5t} + 2$

By ZDTM

$(K + 1) X(K + 1) = 4 Y(K) – 3X(K) - 2 \delta (K – 0)$

$(K + 1) Y(K + 1) = 2 X(K) – Y(K) - 2 \delta (K – 0)$

Put K = 0, 1, 2, 3, **4 …………………**

X (0) = 2 and Y (0) = $\frac{1}{2}$

X (1) = -6 Y (1) = $\frac{3}{2}$

X (2) = 12 Y (2) = $-\frac{27}{4}$

X (3) = -21 Y (3) = $\frac{123}{12}$

X (4) = 26 Y (4) = $\frac{-627}{12}$

Solution is given by

$X(+) = X(0) + t\, X(1) + t^2 \times (2)\, t^3 \times (3) + \ldots\ldots\ldots\ldots\ldots\ldots$

$= 2 – 6t + 12t^2 - 21t^3 + 26t^4 + \ldots\ldots\ldots\ldots\ldots\ldots$

$Y(+) = Y(0) + t\, Y(1) + t^2 \times Y(2)\, t^3 \times Y(3) + \ldots\ldots\ldots\ldots\ldots\ldots$

$= \frac{1}{2} + \frac{3}{2}t - \frac{27}{4}t^2 + \frac{123}{12}t^3 - \frac{627}{12}t^4 + \ldots\ldots\ldots\ldots\ldots\ldots$

Table 2

t	**Exact**		**ZDTM**	
	x(t)	**y(t)**	**x(t)**	**y(t)**
0.0	2	0.5	2	0.5

0.1	1.5013597	0.59156375	1.5016	0.587525
0.2	1.14647666	0.5946575	1.1536	0.5284
0.3	0.87327135	0.53857611	0.9236	0.196025
0.4	0.64351058	0.44050766	0.8416	-0.6616
0.5	0.43336372	0.31023623	1	-2.4218

Each country will eventually its expenditure for arms to zero a condition o disarmament.

Example : 3

Solve Richardson model of arm race

$$\frac{dx}{dt} = 4x + 3y + 6$$

$$\frac{dy}{dt} = x - 2y + 1$$

Subject to x(0) = 0

y(0) = 0

→ Exact Solution of above model is given by

$$x(t) = -\frac{9}{4}e^{t} - \frac{3}{4}e^{-5t} + 3$$

$$y(t) = -\frac{9}{4}e^{t} - \frac{1}{4}e^{-5t} + 2$$

By ZDTM of given Richard son model is

$$(K + 1)\, X(K + 1) = -4 \times (K) + 3Y(K) + 6\,\delta\,(K - 0)$$

$$(K + 1)\, Y(K + 1) = X(K) - 2Y(K) + \delta\,(K - 0)$$

Put K = 0, 1, 2, 3, 4 ………………….

X (0) = 0 and Y (0) = 0

X (1) = 6 Y (1) = 2

X (2) = -9 Y (2) = 1

X (3) = 13 Y (3) = $-\frac{11}{3}$

X (4) = Y (4) =

Solution is given by

$X(t) = X(0) + X(1)t + X(2)\,t^2 + X(3)\,t^3 + \ldots\ldots\ldots\ldots$

$Y(t) = Y(0) + Y(1)t + Y(2)\,t^2 + Y(3)\,t^3 + \ldots\ldots\ldots\ldots$

$X(t) = 6t - 9t^2 + 13t^2 - \frac{85}{4}t^4 + \ldots\ldots\ldots\ldots$

$Y(t) = 2t - t^2 + -\frac{11}{3}t^3 - \frac{61}{12}t^4 + \ldots\ldots\ldots\ldots$

At X = 3, Y = 2 stablised arm race

Expenditure tends to zero

VI. VALIDATION AND COMPARISON

The ZDTM has very promising approach for many applications in various field of science and engineering. In this research paper arm race Richardson model are discussed by using ZDTM method with assumptions are made in this model.

i) The expenditure for armament of each country will increase at a rate proportional to the other country's expenditure.

ii) The expenditure for armament of each country will decrease at a rate. Which is proportional to its own expenditure.

iii) The rate of change of arms expenditure for a country has a constant component that measures antagonism towards other.

By company Exact solution & ZDTM solution the fact is ZDTM applicable to many linear & non-linear systems of linear differential equations with initial value problems. This method is reliable than other different method.

VII. CONCLUSION

In this work ZDTM is applied to solve arm race problems in form Richardson model which reduces the computational work of the other traditional method like Laplace transform method. ZDTM solution is compare with exact solution without any kind of any strong assumptions. By increasing the order of approximate more accuracy can be obtained ZDTM is powerful tool.

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