Edge Natural Product Cordial (enpc) Labeling: A new graph labeling method.

Mukund V. Bapat[1]

Abstract — *We obtain a new type of graph labeling called as Edge Natural product cordial labeling.For a graph G= (p, q) the edges takes value from Nq and the vertices labels are derived by taking product of incident edge labels over (modulo 2).We show that C_n is enpc iff n is odd number,path P_m, Snake $S(C_3,m)$, $S(C_4,m)$,$P_m(C_4)$ under respective conditions are families of enpc graphs.*

I. INTRODUCTION:

The graphs we consider are finite, simple, undirected and connected. The word cordial was used by I. Cahit in defining " cordial labeling" [3].That assignment of numbers to edges (to vertices) that results in labels of vertices (of edges) in '0' and '1' is generally referred as cordial labeling. Here we propose a new type of graph labeling. LetG be a (p>1, q) graph. Define a bijective function f: E(G) → N_q= {1, 2, ..q} .Further the vertex label is derived as: label of vertex u =$\prod_{(uv)\epsilon E(G)} f(uv)(mod2)$for every u$\epsilon$ V(G).Further the condition is satisfied that $|v_f(0)-v_f(1)|\leq 1$ where $v_f(0)$ and $v_f(1)$ are vertices with label 0 and vertices with label 1 respectively. Then the function f is called as edge natural product cordial (enpc) function. And the graph G as enpc graph. We use $v_f(0,1)$ = (x,y) to indicate number of vertices with label '0' are x in number and the number of vertices with label '1' are y in number.

II. PRELIMINARIES:

A. A path: is alternate sequence of vertices and edges given by (v_1, e_1, v_2, e_2, v_n).It has n vertices and n-1 edges.

B. Cycle: It is a closed path given by (v_1, e_1, v_2, e_2, v_n, e_n, v_1).It has n vertices and as many edges.

C.Fusion of vertex.[4] Let G be a (p,q) graph. let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges.[].If uϵG_1 and vϵG_2, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and q_1 + q_2 edges. Sometimes this is referred as u is identified with v.

D. Path union of G i.e.$P_m(G)$ is obtained by taking a path P_m and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p,q) graph. If we change the vertex on G that is fused with vertex of P_m then we generally get a path union non-isomorphic to earlier structure of path union.

E. A snake S(G,m) is obtained by taking a path P_m and m copies of graph G.Each of path edge is shared by a copy of graph G.This commonedge is fixed and same for all copies of G. It has (m-1)q edges and (m-1)(p-1) +1 vertices where G is a (p,q) graph. 3.6 When a product of natural numbers is taken and we are interested whether the product leave remainder 0 or 1 when divided by 2 depends on if the product is a even number or an odd number. When we multiply any even number by any odd or even number the result is a even number. Further the product of any number of odd numbers is an odd number and as such leaves remainder 1 on division by 2.

III. THEOREMSPROVED:

Theorem 3.1 ***Path P_m is enpc graph iff m>2.***

Proof: When p =2 the graph will contain only one edge and it's labeled as 1. The both end vertices of G will be 1.This will give $v_f(0)=0$ and $v_f(1) = 2$.Thus $|v_f(0)-v_f(1)|> 1$.And the graph will not be encp graph.

Let the $P_m = (v_1, e_1, v_2, e_2, v_3, e_3, .. v_{n-1}, e_n, v_n)$.

Define $f:E(P_m)\rightarrow\{1, 2, .., m-1\}$ given by:

case 1:m is even number given by 2x, x=2, 3 ,.. then |EG)| = 2x-1 an odd number.

$f(e_j)$ =2j-1 for j = 1 to x ;

f(ej) = 2y where j = x+y, y = 1, 2, .., (x-1).

Vertex number distribution is vf(0,1) = (x,x).

case 2: m= 2x+1; x= 1, 2, ..;

f(ej) = 2j-1 for j = 1, 2, ,,x. and

f(ej)= 2y where j = x+y, y = 1, 2, ..x; vertex number distribution is vf(0,1) = (x+1, x).

Theorem 3.2 ***G= C_n is enpc iff n is odd number.***

Proof: Let The cycle be defined as $(v_1, c_1, v_2, c_2, .., c_n, v_n)$.

Define $f:E(G)\rightarrow\{1, 2, .., m\}$ given by:
case 1: m = 2x+1; x= 1, 2, 3, 4, ..
$f(c_j)$ = 2j-1 for j = 1, 2, ..x+1
$f(c_j)$ = 2y where j = x+1+y, y= 1, 2, ..x.Vertex number distribution is $v_f(0,1) = (x+1,x)$.
case 2 : Multiplication of two odd numbers only can produce an odd number. Therefore we cannot label an edge with even number which is adjacent at both ends with edges of odd label number. As a result we have to label consecutive edges with odd numbers and remaining edges all consecutive with en even number each. At two places an edge with odd number label will be adjacent with an edge with even number label. This will produce vertex label as even number .The total number of vertices with label '0' will be greater than at least by 2 than the total number of vertices with label '1'. Therefore the condition $|v_f(0)-v_f(1)|\leq 1$ cannot be satisfied. And the graph is not enpc when n is an even number.

Theorem 3.3 Let G = $(C_n)^{(k)}$, the one point union of k copies of C_n is enpc graph when n is odd number and if n is even number then k is also even number.

Proof: Let the t^{th} copy of $C_n$in G be given by $(v_{t,1}, c_{t,1}, v_{t,2}, c_{t,2}, v_{t,3}, .., c_{t,n}, v_{t,1})$. The common vertex be $v_1 = v_{t,1}$.
Define $f:E(G)\rightarrow\{1, 2, .., , km\}$ given by:

case 1: n is odd number 2x+1, x= 1, 2, 3..

When k =1 use the label as given in theorem 4.2 .

Subcase k = 2p:There will be numbers 1, 2, .. , 4px+2 available to label as many edges.

$f(c_{t,j})$ = (t-1)2n+2j-1; j = 1, 2, ..n.; t = 1, 2, .. p

$f(c_{t,j})$ = (t-1-p)2n+2j; j = 1, 2, ..n.; t = p+1, p+2, .., 2p. Vertex number distribution is $v_f(0,1)$ = (np-p+1,np-p).

For k = 2p+1 first label consecutive 2p copies of C_n as shown in case 1, subcase k = 2p. The edges on last copy of $C_n$are labeled as $f(c_{t,j})$ = 2np + (2j-1); j = 1, 2, ..x+1; t = 2p+1 and $f(c_{t,j})$ = 2np +2y for j = x+1+y; y = 1, 2, ..x.

$v_f(0,1)$ = (np-p+x+1, np-p+x).

Case 2: n is even number given by 2x; x= 2, 3, .. Subcase k = 2p.The numbers available for labeling are 1, 2, ..4px for as many edges.$f(e_{t,j})$ = (t-1)2x+2j-1; for j= 1, 2, n and t = 1, 2, ..x.

$f(e_{t,j})$ = (t-p-1)+2j; for j= 1, 2, n and t = x+1, x+2,, .., 2x.
Vertex number distribution is $v_f(0,1)$ = (np-p+1, np-p).

Theorem 3.4 ***$S(C_3,m)$ is enpc graph.***

Proof: We define a snake $S(C_3,m)$ by taking a path P_{m+1} = (v_1, e_1, v_2, e_2, ..,e_m, v_{m+1}) .take new vertices u_1,u_2, .., u_m and new edges $c_{i,1}$=(v_iu_i) and $c_{i,2}$= (u_iv_{i+1}); i = 1, 2, ..m

Define f:E(G)→{1, 2, .., m} given by:

Case m is even number given by 2x. Where x= 1, 2, ..

We have 1 through 6xnatural numbers available for as many edges to be labeled and there are 3x odd numbers and as many even numbers.

$f(e_i)$= 1+6(i-1); for i = 1, 2, 3,.., x.;

$f(c_{i,j})$ = $f(e_i)$+2j,j = 1, 2..; i = 1, 2, ..x-1;
$f(c_{x,1})$ = $f(e_x)$+2,
$f(c_{x,2})$ = 2,

$f(e_{x+y})$= 4+6(y-1) y = 1,2,3, .., x-1;
$f(c_{i,j})$ = $f(e_i)$+2j,i = x+1, x+2..,2x-1; j = 1, 2;

The label distribution vertices is given by $v_f(0,1)$ = (2x+1, 2x).

Case m is odd number given by 2x+1:
$f(e_i)$= 1+6(i-1); for i = 1, 2, 3,.., x;

$f(c_{i,j})$ = $f(e_i)$+2j,j = 1, 2..; i = 1, 2, ..x;
$f(e_{x+j})$= 2+6(j-1) j =1, 2,3, .., x-1;
$f(c_{i,j})$= f(ei)+2j; j= 1,2; i = x+1, x+2,..2x-1.;The label distribution vertices is given by $v_f(0,1)$ = (2x, 2x+1).

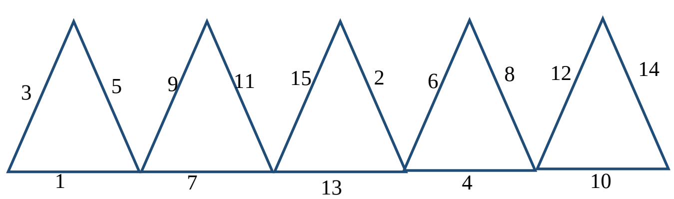

Fig 4.1 Labeled copy of $S(C_3,5)$; edge labels are shown.

Thus $S(C_3,m)$ is enpc graph.

Theorem 3.5 ***A $S(C_4,m)$ is encp graph iff $m \equiv 0 (mod\ 2)$.***

Proof:Define a C4 snake on m blocks as Take a path Pm_{+1} = (v_1, e_1, v_2, e_2, ..,e_m, v_{m+1}). New 2m vertices $u_{i,j}$ for i = 1, 2, ..,m and j = 1, 2 are taken. Theedges that forms i^{th} C_4 cycle are $c_{i,1}$=($v_iu_{i,1}$),$c_{i,2}$= ($u_{i,1}u_{i,2}$) and $c_{i,3}$= ($u_{i,2}v_{i+1}$) .The graph has 4m edges and 3m+1 vertices.

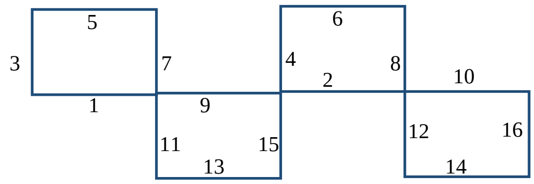

Fig 4.2 : labeled copy of $S(C_4,4)$; edge labels are shown.

Define $f:E(G)\rightarrow\{1, 2, .., m\}$ given by:

Case m = 2x, x= 1, 2, ..

$f(e_i)= 1+8(i-1)$; i = 1, 2, ..x;

$f(e_{x+j})=2+8(j-1)$; j = 1, 2, ..x.

$f(c_{i,j})= f(e_i)+2j$; j =1, 2, 3. i = 1, 2, ..2x;

The label distribution on vertices is given by $v_f(0,1) = (3x+1, 3x)$.

Theorem 3.6 ***Path union $P_m(C_3)$ is enpc for all m.***

Proof: We design $P_m(C_3)$ by taking a path Pm = (v_1, e_1, v_2, e_2, ..,e_m, v_m).Take 2m new vertices, two for each node, are given by $u_{i,j}$ for i = 1, 2, ..m and j = 1, 2. The three edges are $c_{i,1} = (v_iu_{i,1})$, $c_{i,2} = (u_{i,1}u_{i,2})$, $c_{i,3} = (u_{i,2}v_{i+1})$ forms an C_3 cycle at i^{th} vertex of path P_m.

Define $f:E(G)\rightarrow\{1, 2, .., m\}$ given by:

case m = 2x:

$f(e_i) =7 + 8(i-1)$, i = 1, 2, ..,x;

$f(c_{i,j})= 8(i-1) + 2j-1$; j = 1, 2, 3
$f(e_i)= 4+ 8(j-1)$; where x+j = i. j = 1, 2, ..,x.

$f(c_{i,j}) = f(e_i)+ 2j$.; j =1, 2, 3.; i = x + 1, x + 2, .., x + x = m.

The label distribution on vertices is given by $v_f(0,1) = (3x, 3x)$.

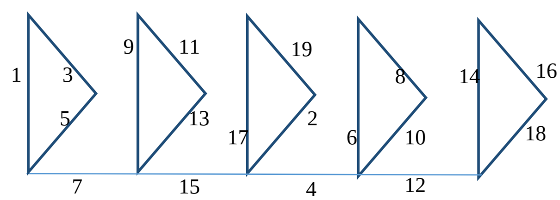

Fig 4.3 : labeled copy of $P_4(C_4)$; edge labels are shown.

Case m = 2x+1.

$f(e_i) =7 + 8(i-1)$, i = 1, 2, ..,x;
$f(c_{i,j})= 8(i-1) + 2j-1$; j = 1, 2, 3 for i = 1, 2, ..x.

$f(c_{i,j}) =8(i-1) + 2j-1$; j = 1, 2 and i = x+1 and

$f(c_{i,3})=2$; i = x+1,

$f(e_i) = 4+8(t-1)$, x+t=i t = 1, 2, 3, ..,x;

$f(c_{i,j}) = f(e_i)+ 2j$, j = 1, 2, 3; Ii = x=1, x+2,.. x+x.

The label distribution on vertices is given by $v_f(0,1) = (\frac{3m+1}{2},\frac{3m-1}{2})$

Theorem 3.6 Path union $P_m(C_4)$ is enpc iff m is a even number.

Proof: : We design Pm(C_3) by taking a path $P_m = (v_1, e_1, v_2, e_2, .., e_m, v_m)$. Take 3m new vertices, three for each node, are given by $u_{i,j}$ for i = 1, 2, ..m and j = 1, 2, 3.The four edges are $c_{i,1} = (v_iu_{i,1})$, $c_{i,2} = (u_{i,1}u_{i,2})$, $c_{i,3} = (u_{i,2}u_{i,3})$, $c_{i,4} = (u_{i,3}v_{i+1})$ forms an C_4 cycle at i^{th} vertex of path P_m.

Define f:E(G)→{1, 2, .., ,5m-1} given by:

Case m = 2x an even number.

$f(c_{i,j})= 10(i-1) + 2j-1$; j = 1, 2, 3, 4;
$f(e_i) = 9 + 10(i-1)$, i = 1, 2, ..,x;
$f(c_{i,j}) = 10(i-x-1) + 2j$.; j =1, 2, 3, 4.; i = x + 1, x + 2, .., x + x-1.
$f(e_i) = 10(i-x)$, i = x+1,x+2, .., 2x-1;The label distribution on vertices is given by $v_f(0,1)$ =(2m, 2m).

When m is an odd number the number of vertices with label '1' are greater than number vertices with label '0' by 2.This difference cannot be reduced further. Therefore when m is odd number there is no encp labeling.

CONCLUSION

We have presented a new labeling in which edges are allowed to take values from first q natural numbers. The vertices label is determined by the product of labels of all incident edges.If this number is even number the vertexlabel is '0' and '1' otherwise. This is achieved by taking (modulo 2) on the product of edges.We have shon some families to follow enpc labeling. These families are

1) Path P_m is enpc graph iff m>2.
2) C_n is enpc iff n is odd number.
3) $(C_n)^{(k)}$, the one point union of k copies of C_n is enpc graph when n is odd number and if n is even number then k is also even number.
4) snake S(C_3,m) is enpc graph.
5) snake S(C_4,m) is encp graph iff m ≡ 0(mod 2).
6) Path union $P_m(C_4)$ is enpc iff m is a even number.

It is necessary to investigate this labeling further.

REFERENCES

[1] Bapat Mukund ,Ph.D. thesis submitted to university of Mumbai.India 2004.
[2] Bapat Mukund V. Some Path Unions Invariance Under Cordial labeling,IJSAM feb.2018 issue.
[3] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987) 201-207.
[4] J. Clark and D. A. Holton, A first look at graph theory; world scientific
[5] Harary,Graph Theory,Narosa publishing ,New Delhi
[6] Yilmaz,Cahit ,E-cordial graphs,Ars combina,46,251-256.
[7] J.Gallian, Dynamic survey of graph labeling, E.J.C 2017
[8] D. WEST,Introduction to Graph Theory ,Pearson Education Asia.