A New Form of Nano Generalized Closed Sets in Nano Ideal Topological Spaces

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ABSTRACT

This paper focuses on $\mathcal{NI}_{g^\star}$-Closed sets (nano $\mathcal{I}_{g^\star}$-closed sets) and $\mathcal{NI}_{g^\star}$-open sets (nano $\mathcal{I}_{g^\star}$-open sets) in nano ideal topological spaces and certain properties of these are investigated. We also investigate the concept of $\mathcal{NI}_{g^\star}$-closed sets and discussed their relationships with other forms of nano ideal sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

Keywords: *$\mathcal{N}g$-closed sets, $\mathcal{N}g^\star$-closed sets, $\mathcal{NI}_g$-closed sets, $\mathcal{NI}_{g^\star}$-closed sets and $\mathcal{NI}_{g^\star}$-open sets.*

1. INTRODUCTION

In 1970, Levine [10] introduced the concept of generalized closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 1991, Balachandran et.al [1] introduced and investigated the notion of generalized continuous functions in topological spaces. In 2000, Veerakumar [14] introduced $g^\star$-closed sets in topological spaces. The concept of ideal topological space was introduced by kuratowski [5]. Also he defined the local functions in ideal topological spaces. Further, Jankovic and Hamlett [4] investigated further properties of ideal topological spaces. In 2014, Ravi et.al [13] introduced $\mathcal{I}_{g^\star}$-closed sets in ideal topological spaces.

The notion of nano topology was introduced by LellisThivagar [7,8] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed the nano forms of weakly open sets such as nano α-open sets, nano semi-open sets and nano pre-open sets. Bhuvaneswari et.al [2], introduced and studied the concept of Nano generalized-closed sets. LellisThivagar et.al [9] defined nano ideal topological spaces.

The structure of this manuscript is as follows. In section 2, we recall some fundamental definitions and results which are useful to prove our main results. In section 3, we define and study the notion of $\mathcal{NI}_{g^\star}$-closed sets and $\mathcal{NI}_{g^\star}$-open sets in nano ideal topological spaces. We also discuss the concept of $\mathcal{NI}_{g^\star}$-closed sets and discussed the relationships between the other existing nano ideal sets.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or U) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $\mathcal{N}cl(A)$ and $\mathcal{N}int(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definition which are useful in the sequel.

Definition 2.1. [7] *Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}\}$, where $R(x)$ denotes the equivalence class determined by x.*
(2) *The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}\}$*
(3) *The Boundary region of X with respect to R is the set of all objects, which can be classified as neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X)$-$L_R(X)$*

Definition 2.2. [7] *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:*

(1) *U and $\phi \in \tau_R(X)$*
(2) *The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*
(3) *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$*

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call $\{U,\tau_R(X)\}$ is called the nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. The complement of the nano-open sets are called nano-closed sets.

Definition 2.3. [3] *An ideal $\mathcal{I}$ on a topological space is a non-empty collection of subsets of X which satisfies*

(1) *$A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$.*
(2) *$A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$.*

Definition 2.4. *A nano topological space $\{U, \tau_R(X)\}$ with an ideal $\mathcal{I}$ on U is called a nano ideal topological space or nano ideal space and denoted as $\{U, \tau_R(X), \mathcal{I}\}$.*

Definition 2.5. *Let $\{U, \tau_R(X), \mathcal{I}\}$ be a nano ideal topological space. A set operator $(A)^{\star\mathcal{N}} : P(U) \to P(U)$ is called the nano local function of $\mathcal{I}$ on U with respect to $\mathcal{I}$ on $\tau_R(X)$ is defined as $(A)^{\star\mathcal{N}} = \{x \in U : U \cap A \notin \mathcal{I}$; for every $U \in \tau_R(X)\}$ and is denoted by $(A)^{\star\mathcal{N}}$, where nano closure operator is defined as $\mathcal{N}cl^{\star}(A) = A \cup (A)^{\star\mathcal{N}}$.*

Result 2.6. [9] *Let $\{U, \tau_R(X), \mathcal{I}\}$ be a nano ideal topological space and let A and B be subsets of U, then*

(1) $(\phi)^{\star\mathcal{N}} = \phi$.

(2) $A \subset B \to (A)^{\star\mathcal{N}} \subset (B)^{\star\mathcal{N}}$.

(3) *For another* $J \supseteq \mathcal{I}$ *on* U, $(A)^{\star\mathcal{N}}(J) \subset (A)^{\star\mathcal{N}}(\mathcal{I})$.

(4) $(A)^{\star\mathcal{N}} \subset \mathcal{N}cl^{\star}(A)$.

(5) $(A)^{\star\mathcal{N}}$ *is a nano closed set.*

(6) $((A)^{\star\mathcal{N}})^{\star\mathcal{N}} \subset (A)^{\star\mathcal{N}}$.

(7) $(A)^{\star\mathcal{N}} \cup (B)^{\star\mathcal{N}} = (A \cup B)^{\star\mathcal{N}}$.

(8) $(A \cap B)^{\star\mathcal{N}} = (A)^{\star\mathcal{N}} \cap (B)^{\star\mathcal{N}}$.

(9) *For every nano open set* V, $V \cap (V \cap A)^{\star\mathcal{N}} \subset (V \cap A)^{\star\mathcal{N}}$.

(10) *For* $\mathcal{I} \in \mathcal{I}, (A \cup \mathcal{I})^{\star\mathcal{N}} = (A)^{\star\mathcal{N}} = (A - \mathcal{I})^{\star\mathcal{N}}$.

Result 2.7. [9] *Let $\{U, \tau_R(X), \mathcal{I}\}$ be a nano ideal topological space and A be a subset of U, If $A \subset (A)^{\star\mathcal{N}}$, then $(A)^{\star\mathcal{N}} = \mathcal{N}cl(A)^{\star\mathcal{N}} = \mathcal{N}cl(A) = \mathcal{N}cl^{\star}(A)$.*

Definition 2.8. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*

(1) *Nano semi-closed* [7], *if* $\mathcal{N}cl(\mathcal{N}int(A)) \subseteq A$.

(2) $\mathcal{N}g$*-closed* [2], *if* $\mathcal{N}cl(A) \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano open.*

(3) $\mathcal{N}\widehat{g}$*-closed* [6], *if* $\mathcal{N}cl(A) \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano semi-open.*

(4) $\mathcal{N}g^{\star}$*-closed*[12], *if* $\mathcal{N}cl(A) \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano g-open.*

Definition 2.9. *A subset A of a nano ideal space. Let $(U, \tau_R(X), \mathcal{I})$ is said to be*

(1) $\star^{\mathcal{N}}$*-closed* [11], *if* $(A)^{\star\mathcal{N}} \subseteq A$.

(2) $\star^{\mathcal{N}}$*-dense* [11], *if* $A \subseteq (A)^{\star\mathcal{N}}$.

(3) $\mathcal{N}\mathcal{I}_g$*-closed* [11], *if* $(A)^{\star\mathcal{N}} \subseteq G$ *whenever* $A \subseteq G$ *and* G *is nano open.*

3. $\mathcal{NI}_{g^\star}$-CLOSED SETS

In this section we define and study the notion of $\mathcal{NI}_{g^\star}$-closed sets and $\mathcal{NI}_{g^\star}$-open sets in nano ideal topological spaces. Also we discuss their basic properties and study the relationship between other existing nano closed sets in nano ideal topological spaces.

Definition 3.1. *A subset A of a nano ideal space $(U, \tau_R(X), \mathcal{I})$ is said to be $\mathcal{NI}_{g^\star}$-closed if $(A)^{\star\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. The complement of $\mathcal{NI}_{g^\star}$-closed set is $\mathcal{NI}_{g^\star}$-open set.*

Theorem 3.2. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space, then every $\mathcal{NI}_{g^\star}$-closed set is $\mathcal{NI}_g$-closed but not conversely.*

Example 3.3. *Let $U = \{a, b, c, d\}$, $U \backslash R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $\tau_R(X) = \{U, \phi, \{a, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}\}$. Then $\mathcal{NI}_{g^\star}$ closed sets are $\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\mathcal{NI}_g$-closed sets $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{b\}$ is $\mathcal{NI}_g$-closed but it is not in $\mathcal{NI}_{g^\star}$-closed.*

Theorem 3.4. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space and $A \subseteq U$, then the following are equivalent.*

(1) *A is $\mathcal{NI}_{g^\star}$-closed.*

(2) *$\mathcal{N}cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open in U.*

(3) *For all $x \in \mathcal{N}cl^\star(A)$, $\mathcal{N}gcl(\{x\}) \cap A \neq \phi$.*

(4) *$\mathcal{N}cl^\star(A) - A$ contains no nonempty nano g-closed set.*

(5) *$A^{\star\mathcal{N}} - A$ contains no nonempty nano g-closed set.*

Proof: (1) $\Rightarrow$ (2) *If A is $\mathcal{NI}_{g^\star}$-closed, then $(A)^{\star\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano g-open in U and so $\mathcal{N}cl^\star(A) = A \cup (A)^{\star\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano g-open in U. This proves (2).*

(2) $\Rightarrow$ (3) *Suppose $x \in \mathcal{N}cl^\star(A)$. If $\mathcal{N}gcl(\{x\}) \cap A = \phi$, then $A \subseteq U - \mathcal{N}gcl(\{x\})$. By (2), $\mathcal{N}cl^\star(A) \subseteq U - \mathcal{N}gcl(\{x\})$, a contradiction, since $x \in \mathcal{N}cl^\star(A)$.*

(3) $\Rightarrow$ (4) *Suppose $F \subseteq \mathcal{N}cl^\star(A) - A$, F is nano g-closed and $x \in F$. Since $F \subseteq U - A$ and F is nano g-closed, then $A \subseteq U - F$ and F is nano g-closed, $\mathcal{N}gcl(\{x\}) \cap A = \phi$. Since $x \in \mathcal{N}cl^\star(A)$ by (3), $\mathcal{N}gcl(\{x\}) \cap A \neq \phi$. Therefore $\mathcal{N}cl^\star(A) - A$ contains no nonempty nano g-closed set.*

(4) $\Rightarrow$ (5) *Since $\mathcal{N}cl^\star(A) - A = (A \cup (A)^{\star\mathcal{N}}) - A = (A \cup (A)^{\star\mathcal{N}}) \cap A^c = (A \cap A^c) \cup ((A)^{\star\mathcal{N}} \cap A^c) = (A)^{\star\mathcal{N}} \cap A^c = (A)^{\star\mathcal{N}} - A$. Therefore $(A)^{\star\mathcal{N}} - A$ contains no nonempty nano g-closed set.*

(5) $\Rightarrow$ (1) *Let $A \subseteq G$ where G is nano g-open set. Therefore $U - G \subseteq U - A$ and so $(A)^{\star\mathcal{N}} \cap (U - G) \subseteq (A)^{\star\mathcal{N}} \cap (U - A) = (A)^{\star\mathcal{N}} - A$. Therefore $(A)^{\star\mathcal{N}} \cap (U - G) \subseteq (A)^{\star\mathcal{N}} - A$. Since $(A)^{\star\mathcal{N}}$ is always nano closed set, so $(A)^{\star\mathcal{N}} \cap (U - G)$ is a nano g-closed set contained in $(A)^{\star\mathcal{N}} - A$. Therefore $(A)^{\star\mathcal{N}} \cap (U - G) = \phi$ and hence $(A)^{\star\mathcal{N}} \subseteq G$. Therefore A is $\mathcal{NI}_{g^\star}$-closed.*

Theorem 3.5. *Every $\star^{\mathcal{N}}$ closed set is $\mathcal{N}\mathcal{I}_{g^\star}$-closed but not conversely.*

Proof: *Let A be a $\star^{\mathcal{N}}$-closed, then $(A)^{\star\mathcal{N}} \subseteq A$. Let $A \subseteq G$ where G is nano g-open. Hence $(A)^{\star\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. Therefore A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*

Example 3.6. *Let $U = \{a, b, c, d\}$, $U\backslash R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$.Let the nano ideal space $\tau_R(X)$=$\{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}, \{a, b, d\}\}$. Then $\mathcal{N}\mathcal{I}_{g^\star}$-closed sets are $\{U, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $\star^{\mathcal{N}}$-closed sets are $\{U, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$. It is clear that $\{b, c\}$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed set but it is not in $\star^{\mathcal{N}}$-closed.*

Theorem 3.7. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space, then every nano closed set is a $\mathcal{N}\mathcal{I}_{g^\star}$-closed but not conversely.*

Proof: *Let A be a nano closed set in $\tau_R(X)$ and $A \subseteq G$, where G is nano g-open. Since A is nano closed, $(A)^{\star\mathcal{N}} = \mathcal{N}cl(A) = A \subseteq G$. That is $(A)^{\star\mathcal{N}} \subseteq A \subseteq G$. Hence A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*

Example 3.8. *Let $U = \{a, b, c, d\}$, $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{c\}$. Let the nano ideal space $\tau_R(X)$=$\{U, \phi, \{c, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{b\}, \{a, b\}\}$. Then $\mathcal{N}\mathcal{I}_{g^\star}$-closed sets are $\{U, \phi, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ and nano closed sets are $\{U, \phi, \{a, b\}\}$. It is clear that $\{b\}$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed set but it is not in nano closed.*

Theorem 3.9. *If A and B are $\mathcal{N}\mathcal{I}_{g^\star}$ - closed, then $A \cup B$ is $\mathcal{N}\mathcal{I}_{g^\star}$ - closed.*

Proof: *Let A and B are $\mathcal{N}\mathcal{I}_{g^\star}$ - closed sets. Then $(A)^{\star\mathcal{N}}$ and $(B)^{\star\mathcal{N}}$ are subsets of G where A and B are subsets of G and G is nano g-open. Since A and B are subsets of G. $(A \cup B)$ is also subset of G and G is nano g-open. Then $(A)^{\star\mathcal{N}} \cup (B)^{\star\mathcal{N}} \subseteq G$, which implies that $(A \cup B)$ is $\mathcal{N}\mathcal{I}_{g^\star}$ - closed.*

Theorem 3.10. *If A and B are $\mathcal{N}\mathcal{I}_{g^\star}$ - closed, then $A \cap B$ is $\mathcal{N}\mathcal{I}_{g^\star}$ - closed.*

Proof: *Let A and B are $\mathcal{N}\mathcal{I}_{g^\star}$ - closed sets. Then $(A)^{\star\mathcal{N}}$ and $(B)^{\star\mathcal{N}}$ are subsets of G where A and B are subsets of G and G is nano g-open. Since A and B are subsets of G. $(A \cap B)$ is a subset of G and G is nano g-open. Then $(A)^{\star\mathcal{N}} \cap (B)^{\star\mathcal{N}} \subseteq G$, which implies that $(A \cap B)$ is $\mathcal{N}\mathcal{I}_{g^\star}$ - closed.*

Example 3.11. *Let $U = \{a, b, c, d\}$, $U\backslash R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{c\}$. Let the nano ideal space $\tau_R(X)$=$\{U, \phi, \{c, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{b\}, \{a, b\}\}$. Then $\mathcal{N}\mathcal{I}_{g^\star}$-closed sets are $\{U, \phi, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Let $A = \{a, b\}$ and $B = \{a, b, c\}$ and $A \cup B = \{a, b, c\}$ is also $\mathcal{N}\mathcal{I}_{g^\star}$-closed set and $A \cap B = \{a, b\}$ is also $\mathcal{N}\mathcal{I}_{g^\star}$-closed set.*

Theorem 3.12. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. For every $A \in \mathcal{I}$, A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*

Proof: *Let $A \subseteq G$ where G is nano g-open set. Since $(A)^{\star\mathcal{N}} = \phi$ for every $A \in \mathcal{I}$, then $\mathcal{N}cl^\star(A) = A \cup (A)^{\star\mathcal{N}} = A \subseteq G$. Therefore, by Theorem 3.4, A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*

Theorem 3.13. *If $(U, \tau_R(X), \mathcal{I})$ is a nano ideal space, then $(A)^{\star\mathcal{N}}$ is always $\mathcal{N}\mathcal{I}_{g^\star}$-closed for every subset A of U.*

Proof: *Let $(A)^{\star\mathcal{N}} \subseteq G$ where G is nano g-open. Since $((A)^{\star\mathcal{N}})^{\star\mathcal{N}} \subseteq (A)^{\star\mathcal{N}}$, we have $((A)^{\star\mathcal{N}})^{\star\mathcal{N}} \subseteq G$ whenever $(A)^{\star\mathcal{N}} \subseteq G$ and G is nano g-open. Hence $(A)^{\star\mathcal{N}}$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*

Theorem 3.14. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. Then every $\mathcal{N}\mathcal{I}_{g^\star}$-closed, nano g-open set is $\star^{\mathcal{N}}$- closed set.*

Proof: *Since A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed and g-open. Then $(A)^{\star\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. Hence A is $\star^{\mathcal{N}}$ -closed.*

Corollary 3.15. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and A be a $\mathcal{N}\mathcal{I}_{g^\star}$- closed set. Then the following are equivalent.*

(1) *A is a $\star^{\mathcal{N}}$-closed set.*

(2) *$\mathcal{N}cl^\star(A) - A$ is a nano g-closed set.*

(3) *$(A)^{\star\mathcal{N}} - A$ is a nano g-closed set.*

Proof:

(1) $\Rightarrow$ (2) *If A is $\star^{\mathcal{N}}$ - closed, then $(A)^{\star\mathcal{N}} \subseteq A$ and so $\mathcal{N}cl^\star(A) - A = (A \cup (A)^{\star\mathcal{N}}) - A = \phi$. Hence $\mathcal{N}cl^\star(A) - A$ is a nano g- closed set.*

(2) $\Rightarrow$ (3) *Since $\mathcal{N}cl^\star(A) - A = A^{\star\mathcal{N}} - A$ and so $A^{\star\mathcal{N}} - A$ is nano g-closed set.*

(3) $\Rightarrow$ (1) *If $A^{\star\mathcal{N}} - A$ is a nano g - closed set, since A is $\mathcal{N}\mathcal{I}_{g^\star}$- closed set, by Theorem 3.4, $A^{\star\mathcal{N}} - A = \phi$ and so A is $\star^{\mathcal{N}}$ - closed.*

Theorem 3.16. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. Then every $\mathcal{N}g^\star$ -closed set is a $\mathcal{N}\mathcal{I}_{g^\star}$-closed set but not conversely.*

Proof: *Let A be a $\mathcal{N}g^\star$-closed set. Then $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. We have $\mathcal{N}cl^\star(A) \subseteq \mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. Hence A is $\mathcal{N}\mathcal{I}_{g^\star}$ -closed.*

Example 3.17. *Let $U = \{a, b, c, d\}, U\backslash R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $\tau_R(X) = \{U, \phi, \{a, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}\}$. Then $\mathcal{N}\mathcal{I}_{g^\star}$- closed sets are $\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\mathcal{N}g^\star$-closed sets are $\{U, \phi, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. It is clear that $\{a\}$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed set but it is not in $\mathcal{N}g^\star$-closed.*

Theorem 3.18. *If $(U, \tau_R(X), \mathcal{I})$ is a nano ideal space and A is a $\star^{\mathcal{N}}$-dense in itself, $\mathcal{N}\mathcal{I}_{g^\star}$-closed subset of U then A is $\mathcal{N}g^\star$-closed.*

Proof: *Suppose A is a $\star^{\mathcal{N}}$-dense in itself, $\mathcal{N}\mathcal{I}_{g^\star}$-closed subset of U. Let $A \subseteq G$ where G is nano g-open. Then by Theorem 3.4 (2), $\mathcal{N}cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. Since A is $\star^{\mathcal{N}}$-dense in itself, by Result 2.7, $\mathcal{N}cl(A) = \mathcal{N}cl^\star(A)$. Therefore $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano g-open. Hence A is $\mathcal{N}g^\star$-closed.*

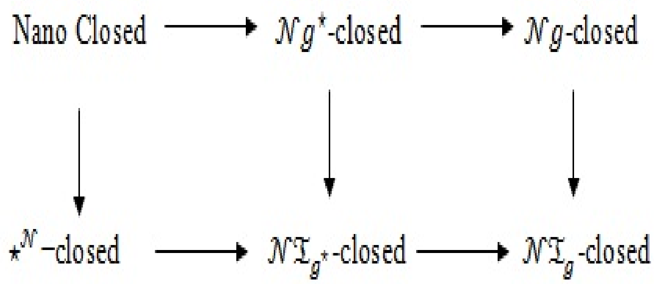
Corollary 3.19. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space where $\mathcal{I} = \{\phi\}$, then A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed if and only if A is $\mathcal{N}g^\star$-closed.*

Proof: *From the fact that for* $\mathcal{I} = \{\phi\}$, $(A)^{\star\mathcal{N}} = \mathcal{N}cl(A) \supseteq A$*. Therefore* A *is* $\star^{\mathcal{N}}$*-dense in itself. Since* A *is* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed, by Theorem 3.18,* A *is* $\mathcal{N}g^\star$*-closed. Conversely, by Theorem 3.16, every* $\mathcal{N}g^\star$*-closed. set is* $\mathcal{N}\mathcal{I}_{g^\star}$ *-closed set.*

Example 3.20. *Let* $U = \{a, b, c, d\}$, $U \backslash R = \{\{a\}, \{d\}, \{b, c\}\}$ *and* $X = \{a, d\}$*. Then in the nano ideal space* $\tau_R(X) = \{U, \phi, \{a, d\}\}$ *with a nano ideal* $\mathcal{I} = \{\phi, \{a\}\}$*. Then* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed sets are* $\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ *and* $\mathcal{N}g$*-closed sets are* $\{U, \phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$*. It is clear that* $\{b\}$ *is* $\mathcal{N}g$*-closed set but it is not in* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed and* $\{a\}$ *is a* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed set but it is not in* $\mathcal{N}g$*-closed.*

Remark 3.21. *From the above Example, nano g-closed sets and* $\mathcal{N}\mathcal{I}_{g^\star}$ *-closed sets are independent of each other.*

Remark 3.22. *The following figure shows that the relationship of* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed sets with some of the sets existing,which we have discussed in this section*



These implications are not reversible.

Theorem 3.23. *Let* $(U, \tau_R(X), \mathcal{I})$ *be a nano ideal space and* $A \subseteq U$*. Then* A *is* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed if and only if* $A = F - N$ *where* F *is* $\star^{\mathcal{N}}$*-closed and* N *contains no nonempty nano g-closed set.*

Proof: *If* A *is* $\mathcal{N}\mathcal{I}_{g^\star}$*-closed, then by Theorem 3.4(5),* $N = (A)^{\star\mathcal{N}} - A$ *contains no nonempty nano g-closed set. If* $F = \mathcal{N}cl^\star(A)$*, then* F *is* $\star^{\mathcal{N}}$*-closed such that* $F - N = (A \cup (A)^{\star\mathcal{N}}) - ((A)^{\star\mathcal{N}} - A) = (A \cup (A)^{\star\mathcal{N}}) \cap ((A)^{\star\mathcal{N}} \cap A^c)^c = (A \cup (A)^{\star\mathcal{N}}) \cap (((A)^{\star\mathcal{N}})^c \cup A) = (A \cup (A)^{\star\mathcal{N}}) \cap (A \cup ((A)^{\star\mathcal{N}})^c) = A \cup ((A)^{\star\mathcal{N}} \cap ((A)^{\star\mathcal{N}})^c) = A$*.*

Conversely, suppose $A = F - N$ *where* F *is* $\star^{\mathcal{N}}$*-closed and* N *contains no nonempty nano g-closed set. Let* G *be a nano g-open set such that* $A \subseteq G$*. Then* $F - N \subseteq G \Rightarrow F \cap (U - G) \subseteq N$*. Now* $A \subseteq F$ *and* $(F)^{\star\mathcal{N}} \subseteq F$ *then* $(A)^{\star\mathcal{N}} \subseteq (F)^{\star\mathcal{N}}$ *and so* $(A)^{\star\mathcal{N}} \cap (U - G) \subseteq (F)^{\star\mathcal{N}} \cap (U - G) \subseteq F \cap (U - G) \subseteq N$*. By hypothesis, since* $(A)^{\star\mathcal{N}} \cap (U - G)$ *is nano g-closed,* $(A)^{\star\mathcal{N}} \cap (U - G) = \phi$ *and so* $(A)^{\star\mathcal{N}} \subseteq G$*. Hence* A *is* $\mathcal{N}\mathcal{I}_{g^\star}$ *-closed.*

Theorem 3.24. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. If $A \subseteq B \subseteq (A)^{\star\mathcal{N}}$ then $(A)^{\star\mathcal{N}} = (B)^{\star\mathcal{N}}$ and B is $\star^{\mathcal{N}}$-dense in itself.*

Proof: *Since $A \subseteq B$, then $(A)^{\star\mathcal{N}} \subseteq (B)^{\star\mathcal{N}}$ and since $B \subseteq (A)^{\star\mathcal{N}}$, then $(B)^{\star\mathcal{N}} \subseteq ((A)^{\star\mathcal{N}})^{\star\mathcal{N}} \subseteq (A)^{\star\mathcal{N}}$. Therefore $(A)^{\star\mathcal{N}} = (B)^{\star\mathcal{N}}$ and $B \subseteq (A)^{\star\mathcal{N}} \subseteq (B)^{\star\mathcal{N}}$.*

Theorem 3.25. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. If A and B are subsets of U such that $A \subseteq B \subseteq \mathcal{N}cl^\star(A)$ and A is $\mathcal{N}\mathcal{I}_{g^\star}$ -closed then B is $\mathcal{N}\mathcal{I}_{g^\star}$ -closed.*

Proof: *Since A is $\mathcal{N}\mathcal{I}_{g^\star}$ -closed then by Theorem 3.4(4), $\mathcal{N}cl^\star(A) - A$ contains no nonempty nano g-closed set. Since $\mathcal{N}cl^\star(B) - B \subseteq \mathcal{N}cl^\star(A) - A$ and so $\mathcal{N}cl^\star(B) - B$ contains no nonempty nano g-closed set. Hence B is $\mathcal{N}\mathcal{I}_{g^\star}$ -closed.*

Corollary 3.26. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. If A and B are subsets of U such that $A \subseteq B \subseteq (A)^{\star\mathcal{N}}$ and A is $\mathcal{N}\mathcal{I}_{g^\star}$ -closed then A and B are $\mathcal{N}g^\star$-closed sets.*

Proof: *Let A and B be subsets of U such that $A \subseteq B \subseteq (A)^{\star\mathcal{N}} \Rightarrow A \subseteq B \subseteq (A)^{\star\mathcal{N}} \subseteq \mathcal{N}cl^\star(A)$and A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed. By the above Theorem, B is $\mathcal{N}\mathcal{I}_{g^\star}$-closed. Since $A \subseteq B \subseteq (A)^{\star\mathcal{N}}$, then $(A)^{\star\mathcal{N}} = (B)^{\star\mathcal{N}}$ and so A and B are $\star^{\mathcal{N}}$-dense in itself. By Theorem 3.18, A and B are $\mathcal{N}g^\star$-closed.*

Theorem 3.27. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. Then A is $\mathcal{N}\mathcal{I}_{g^\star}$-open if and only if $F \subseteq \mathcal{N}int^\star(A)$ whenever F is nano g-closed and $F \subseteq A$.*

Proof: *Suppose A is $\mathcal{N}\mathcal{I}_{g^\star}$-open. If F is nano g-closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so $\mathcal{N}cl^\star(U - A) \subseteq U - F$ by Theorem 3.4(2). Therefore $F \subseteq U - \mathcal{N}cl^\star(U - A) = \mathcal{N}int^\star(A)$. Hence $F \subseteq \mathcal{N}int^\star(A)$.*

Conversely, suppose the condition holds. Let G be a nano g-open set such that $U - A \subseteq G$. Then $U - G \subseteq A$ and so $U - G \subseteq \mathcal{N}int^\star(A)$. Therefore $\mathcal{N}cl^\star(U - A) \subseteq G$. By Theorem 3.4 (2), $U - A$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed. Hence A is $\mathcal{N}\mathcal{I}_{g^\star}$-open.

Corollary 3.28. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. If A is $\mathcal{N}\mathcal{I}_{g^\star}$-open then $F \subseteq \mathcal{N}int^\star(A)$ whenever F is closed and $F \subseteq A$.*

Theorem 3.29. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. If A is $\mathcal{N}\mathcal{I}_{g^\star}$-open and $\mathcal{N}int^\star(A) \subseteq B \subseteq A$, then B is $\mathcal{N}\mathcal{I}_{g^\star}$-open.*

Proof: *Since A is $\mathcal{N}\mathcal{I}_{g^\star}$-open, then $U - A$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed. By Theorem 3.4(4), $\mathcal{N}cl^\star(U - A) - (U - A)$ contains no nonempty nano g-closed set. Since $\mathcal{N}int^\star(A) \subseteq \mathcal{N}int^\star(B)$ which implies that $\mathcal{N}cl^\star(U - B) \subseteq \mathcal{N}cl^\star(U - A)$ and so $\mathcal{N}cl^\star(U - B) - (U - B) \subseteq \mathcal{N}cl^\star(U - A) - (U - A)$. Hence B is $\mathcal{N}\mathcal{I}_{g^\star}$-open.*

Theorem 3.30. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. Then following are equivalent.*

(1) *A is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*
(2) *$A \cup (U - (A)^{\star N})$ is $\mathcal{N}\mathcal{I}_{g^\star}$-closed.*
(3) *$(A)^{\star\mathcal{N}} - A$ is $\mathcal{N}\mathcal{I}_{g^\star}$-open.*

Proof: (1) $\Rightarrow$ (2) *Suppose* A *is* $\mathcal{NI}_{g^\star}$*-closed. If* G *is any nano g-open set such that* $A\cup(U-(A)^{\star\mathcal{N}}) \subseteq G$*, then* $U - G \subseteq U - (A \cup (U - (A)^{\star\mathcal{N}})) = U \cap (A \cup ((A)^{\star\mathcal{N}})^c)^c = (A)^{\star\mathcal{N}} \cap A^c = (A)^{\star\mathcal{N}} - A$*. Since* A *is* $\mathcal{NI}_{g^\star}$*-closed, by Theorem 3.4(5), it follows that* $U - G = \phi$ *and so* $U = G$*. Therefore* $A\cup(U-(A)^{\star\mathcal{N}}) \subseteq G \Rightarrow A \cup (U - (A)^{\star\mathcal{N}}) \subseteq U$ *and so* $(A \cup (U - (A)^{\star\mathcal{N}}))^{\star\mathcal{N}} \subseteq U^\star \subseteq U = G$*. Hence* $A \cup (U - (A)^{\star\mathcal{N}})$ *is* $\mathcal{NI}_{g^\star}$*-closed.*

(2) $\Rightarrow$ (1) *Suppose* $A \cup (U - (A)^{\star\mathcal{N}})$ *is* $\mathcal{NI}_{g^\star}$*-closed. If* F *is any nano g-closed set such that* $F \subseteq (A)^{\star\mathcal{N}} - A$*, then* $F \subseteq (A)^{\star\mathcal{N}}$ *and* $F \subseteq X/A \Rightarrow U - (A)^{\star\mathcal{N}} \subseteq U - F$ *and* $A \subseteq U - F$*. Therefore* $A \cup (U - (A)^{\star\mathcal{N}}) \subseteq A \cup (U - F) = U - F$ *and* $U - F$ *is nano g-open. Since* $(A \cup (U - (A)^{\star\mathcal{N}}))^{\star\mathcal{N}} \subseteq U - F \Rightarrow (A)^{\star\mathcal{N}} \cup (U - (A)^{\star\mathcal{N}})^{\star\mathcal{N}} \subseteq U - F$ *and so* $(A)^{\star\mathcal{N}} \subseteq U - F \Rightarrow F \subseteq U - (A)^{\star\mathcal{N}}$*. Since* $F \subseteq (A)^{\star\mathcal{N}}$*, it follows that* $F = \phi$*. Hence* A *is* $\mathcal{NI}_{g^\star}$*-closed.*

(2) $\Rightarrow$ (3) *Since* $U-((A)^{\star\mathcal{N}}-A) = U\cap((A)^{\star\mathcal{N}}\cap A^c)^c = U\cap((A)^{\star\mathcal{N}})^c\cup A) = (U\cap((A)^{\star\mathcal{N}})^c)\cup(U\cap A) = A \cup (U - (A)^{\star\mathcal{N}})$.

Theorem 3.31. *Let* $(U, \tau_R(X), \mathcal{I})$ *be a nano ideal space. Then every subset of* U *is* $\mathcal{NI}_{g^\star}$*-closed if and only if every nano g-open set is* $\star^{\mathcal{N}}$*-closed.*

Proof: *Suppose every subset of* U *is* $\mathcal{NI}_{g^\star}$*-closed. If* $G \subseteq U$ *is nano g-open then* G *is* $\mathcal{NI}_{g^\star}$*-closed and so* $(G)^{\star\mathcal{N}} \subseteq G$*. Hence* G *is* $\star^{\mathcal{N}}$*-closed. Conversely, suppose that every nano g-open set is* $\star^{\mathcal{N}}$*-closed. If* G *is nano g-open set such that* $A \subseteq G \subseteq U$ *then* $(A)^{\star\mathcal{N}} \subseteq (G)^{\star\mathcal{N}} \subseteq G$ *and so* A *is* $\mathcal{NI}_{g^\star}$*-closed.*

References

[1] Balachandran. K, Sundaram. P, and Maki. H, On generalized continuous maps in topological spaces, Mem.Fac.sci. Kochi. Univ.Ser.A. Maths., 12 (1991), 5-13.

[2] Bhuvaneshwari. K and Mythili Gnanapriya. K, Nano Generalized closed sets, International Journal of Scientific and Research Publications. 2014;14(5):1-3.

[3] Hamlett. T.R and Jonkovic. D, " Ideals in General Topology", Lecture notes in pure and Appl. Math. , 123, 115-125 (1990).

[4] Jankovic. D and Hamlett. T.R, New topological from oldvia ideals, Amer.Math. Monthly 97 (1990), no. 4, 295-310.

[5] Kuratowski. K, Topology, Vol. 1, Academic Press,New York, 1966.

[6] Lalitha. R and Francina Shalini A, $\mathcal{N}\widehat{g}$-closed and open sets in nano topological spaces, International Journal of Applied Research 2017; 3(5): 368-371.

[7] Lellis Thivagar. M and Carmel Richard, On Nano Forms of Weakly Open sets International Journal of Mathematics and Statistics Invention. 1 (1)(2013) 31 - 37.

[8] Lellis Thivagar. M and Carmel Richard, On Nano Forms of continuity, Mathematical Theory and Modeling, Vol. *3*, No. 7(2013), 32-37.

[9] Lellis Thivagar. M and Sutha Devi. V, " New sort of operators in Nano Ideal Topology", Ultra Scientist Vol.28(1)A,51-64(2016).

[10] Levine. N, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2) 19 (1970), 89-96.

[11] Parimala. M, Jafari. S, and Murali. S, Nano ideal generalized closed sets in nano ideal topological spaces August 2016(communicate)

[12] Rajendran. V, Sathishmohan. P, and Indirani. K, On Nano generalized star closed sets in nano topological spaces, International Journal of Applied Research 2015; 1(9):04-07.

[13] Ravi. O, Paranjothi. M, Murugesan. and S, Meharin. M, $g^{\star}$ -closed sets in ideal topological spaces, South Asian Journal of Mathematics 2014, Vol.4(6):252 264.

[14] Veerakumar. M.K.R.S., Between $g^{\star}$-closed sets and g-closed sets, Antartica J. Math., Reprint.