Intuitionistic Fuzzy N-subgroup and Intuitionistic Fuzzy Ideal

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Abstract — In this paper we have investigated the notion of Intuitionistic fuzzy N-subgroup and Intuitionistic fuzzy ideal of an N-group E and shown a new way of defining these notions using the intuitionistic fuzzy points and a membership and non membership functions. We also define Intuitionistic fuzzy prime N-subgroup towards the end of this paper.

Keywords — Intuitionistic fuzzy N-subgroup, Intuitionistic fuzzy ideal, Intuitionistic fuzzy prime N-subgroup, Intuitionistic fuzzy point.

I. INTRODUCTION

Fuzzy set was introduced by Zadeh [12] in 1965 assigns to each element a membership degree. The notion of intuitionistic fuzzy set was introduced by Atanassov [1] in 1986 which assigns to each element a membership and a non membership degree. In last two decades many researcher paid their attention towards the application of the intuitionistic fuzzy set theory in different areas like logic programming, decision making problems, medical diagnosis etc. Using the notion of intuitionistic fuzzy sets various algebraic structures are studied. Dutta et. al. [4],Hong et. al. [5],Kim et. al. [7] carries out extensive work on fuzzy ideals and prime ideals in a nearring. After the notion of fuzzy sub-module by Mashour et. al. [8], Barthakur and Saikia [10,11] extended this concept to establish various properties of fuzzy N-subgroups,fuzzy normal N-subgroups,fuzzy prime N-subgroups of an N-group. Devi [3] extended the notion of fuzzy N-subgroups and ideals of N-groups to intuitionistic fuzzy points the notion of which is later used by Sardar et. al. [9] and Bakhadach et. al. [2] to define semigroup and prime ideal in a ring respectively.

II. PRELIMINARIES

In this section we recall some basic notions.

Definition 2.1: Let X be a non empty set. A function $\mu : X \to [0,1]$ is called a fuzzy subset of X. It is characterised by membership function μ .

Definition 2.2: The intuitionistic fuzzy sets (in short IFS) are defined on a non-empty set X as objects having the form $A = \{ \langle x, \alpha_A(x), \beta_A(x) \rangle | x \in X \}$ where the function $\alpha_A : X \to [0,1]$ and $\beta_A : X \to [0,1]$ denote the degree of membership and non-membership of each element $x \in X$ to the set A respectively and $0 \le \alpha_A(x) + \beta_A(x) \le 1, \forall x \in X$.

For the sake of simplicity , we shall use the symbol $\langle \alpha_A, \beta_A \rangle$ for the IFS $A = \{\langle x, \alpha_A(x), \beta_A(x) \rangle | x \in X \}$.

Definition 2.3:Let X be a non empty set and let $A = \langle \alpha_A, \beta_A \rangle$ and $B = \langle \alpha_B, \beta_B \rangle$ be IFSs of X. Then

 $(i) A \subset B \Leftrightarrow \alpha_{A} \leq \alpha_{B}, \beta_{A} \geq \beta_{B}$ $(ii) A = B \Leftrightarrow A \subset B, B \subset A$ $(iii) A^{c} = \langle \beta_{A}, \alpha_{A} \rangle$ $(iv) A \cap B = \langle \alpha_{A} \land \alpha_{B}, \beta_{A} \lor \beta_{B} \rangle$ $(v) A \cup B = \langle \alpha_{A} \lor \alpha_{B}, \beta_{A} \land \beta_{B} \rangle$

Definition 2.4: Let $s, t \in [0,1]$ with $s + t \le 1$. An intuitionistic fuzzy point, written as $x_{(s,t)}$ is defined to be an intuitionistic fuzzy point of N if

$$x_{(s,t)}(y) = \begin{cases} (s,t), & x = y \\ (0,1), & x \neq y \end{cases}$$

An intuitionistic fuzzy point $x_{(s,t)} \in \alpha_A, \beta_A \implies \alpha_A(x) \ge s, \beta_A(x) \le t$ for all $\forall x \in N$.

Definition 2.5: Let N be a near-ring and $A = \langle \alpha_A, \beta_A \rangle$ is an IFS in N. Then A is called intuitionistic fuzzy near-ring if

$$\begin{split} (i) \alpha_{A} (x + y) &\geq \alpha_{A} (x) \wedge \alpha_{A} (y) \\ (ii) \alpha_{A} (-x) &\geq \alpha_{A} (x) \\ (iii) \alpha_{A} (xy) &\geq \alpha_{A} (x) \wedge \alpha_{A} (y) \\ (iv) \beta_{A} (x + y) &\leq \beta_{A} (x) \vee \beta_{A} (y) \\ (v) \beta_{A} (-x) &\leq \beta_{A} (x) \\ (vi) \beta_{A} (xy) &\leq \beta_{A} (x) \vee \beta_{A} (y), \forall x, y \in N \end{split}$$

Definition 2.6: Let μ be a fuzzy near-ring in a near-ring N and E be an N-group. An IFS $A \ll \alpha_A$, $\beta_A >$ is called Intuitionistic fuzzy N-group in E if

$$\begin{split} (i)\alpha_{A}(x+y) &\geq \alpha_{A}(x) \wedge \alpha_{A}(y) \\ (ii)\alpha_{A}(nx) &\geq \mu(n) \wedge \alpha_{A}(x) \\ (iii)\beta_{A}(x+y) &\leq \beta_{A}(x) \vee \beta_{A}(y) \\ (iv)\beta_{A}(nx) &\leq \mu(n) \vee \beta_{A}(x), x, y \in E, n \in N \end{split}$$

Theorem 2.7: Let $\{(\alpha_i, \beta_i) | i \in I\}$ be a family of intuitionistic fuzzy N-groups in an N-group E over a fuzzy near-ring μ in N. Then $(\bigcap_{i \in I} \alpha_i, \bigcup_{i \in I} \beta_i)$ is also an intuitionistic fuzzy N-group in E.

Definition 2.8: An IFS $A \implies \alpha_A, \beta_A > \text{in an N-group E is called an intuitionistic fuzzy ideal of E if it satisfies the following$

 $(i) \alpha_{A} (x - y) \ge \alpha_{A} (x) \land \alpha_{A} (y)$ $(ii) \alpha_{A} (y + x - y) \ge \alpha_{A} (x)$ $(iii) \alpha_{A} (nx) \ge \alpha_{A} (x)$ $(iv) \alpha_{A} (n(x + y) - nx) \ge \alpha_{A} (y)$ $(v) \beta_{A} (x - y) \le \beta_{A} (x) \lor \beta_{A} (y)$ $(vi) \beta_{A} (y + x - y) \le \beta_{A} (x)$ $(vii) \beta_{A} (nx) \le \beta_{A} (x)$ $(viii) \beta_{A} (n(x + y) - nx) \le \beta_{A} (y), \forall x, y \in E, n \in N$

Definition 2.9: A non constant fuzzy N-subgroup μ of a near-ring N is said to be fuzzy prime N-subgroups σ, θ of N if for any fuzzy N-subgroups of N, $\sigma \circ \theta \subseteq \mu$ implies either $\sigma \subseteq \mu$ or $\theta \subseteq \mu$.

Theorem 2.10: Let μ and θ be two fuzzy prime N-subgroups of N. Then $\mu \cap \theta$ is a fuzzy prime N-subgroups of N if and only if $\mu \subseteq \theta$ or $\theta \subseteq \mu$.

III. MAIN RESULTS

Definition 3.1: Let μ be a fuzzy near-ring in a near-ring N and E be an N-group. Then an Intuitionistic fuzzy set $A = \langle \alpha_A, \beta_A \rangle$ of E is called an intuitionistic fuzzy N-subgroup of E if and only if $(i) \forall x_{(s,t)}, y_{(s',t')} \in < \alpha_{A}, \beta_{A} \implies (x+y)_{(s \land s', t \lor t')} \in < \alpha_{A}, \beta_{A} >$ $(ii) \forall x_{(s,t)} \in < \alpha_{A}, \beta_{A} > \Rightarrow (nx)_{(\mu(n) \land s, \mu(n) \lor t)} \in < \alpha_{A}, \beta_{A} >$ Where $x, y \in E, n \in N$. Proof : Suppose $A = \langle \alpha_A, \beta_A \rangle$ is an intuitionistic fuzzy N-group In E. Then for $x_{(s,t)}, y_{(s',t')} \in \alpha_A, \beta_A >$ we have $\alpha_{A}(x+y) \geq \alpha_{A}(x) \wedge \alpha_{A}(y) \geq s \wedge s';$ $\beta_A(x + y) \le \beta_A(x) \lor \beta_A(y) \le t \lor t'$ gives $(x + y)_{(x \land x', t \lor t')} \in \langle \alpha_A, \beta_A \rangle$. Again for $n \in N$ and $x_{(s,t)} \in \alpha_A$, $\beta_A > as$ $\alpha_{A}(nx) \geq \mu(n) \land \alpha_{A}(x) \geq \mu(n) \land s; \beta_{A}(nx) \leq \mu(n) \lor \beta_{A}(x) \leq \mu(n) \lor t$ so we have $(nx)_{(\mu(n)\wedge s,\mu(n)\vee t)} \in < \alpha_A, \beta_A > .$ Conversely, suppose that $x, y \in E$. Then since $x_{(\alpha_{A}(x) \land \alpha_{A}(y), \beta_{A}(x) \lor \beta_{A}(y))}, y_{(\alpha_{A}(x) \land \alpha_{A}(y), \beta_{A}(x) \lor \beta_{A}(y))} \in < \alpha_{A}, \beta_{A} >$ So by assumption we have $(x + y)_{(\alpha_A(x) \land \alpha_A(y), \beta_A(x) \lor \beta_A(y))} \in < \alpha_A, \beta_A >$ Which implies $\alpha_{A}(x+y) \geq \alpha_{A}(x) \wedge \alpha_{A}(y); \beta_{A}(x+y) \leq \beta_{A}(x) \vee \beta_{A}(y).$ Again for $n \in N, x \in E$ as by assumption $x_{(\mu(n) \land \alpha_A(x), \mu(n) \lor \beta_A(x))} \in \langle \alpha_A, \beta_A \rangle$ gives $(nx)_{(\mu(n)\wedge\alpha_A(x),\mu(n)\vee\beta_A(x))} \in < \alpha_A, \beta_A > So we have$ $\alpha_{A}(nx) \geq \mu(n) \wedge \alpha_{A}(x); \beta_{A}(nx) \leq \mu(n) \vee \beta_{A}(x)$ **Definition 3.2:** An IFS $A = \langle \alpha_A, \beta_A \rangle$ in an N-group E is called an intuitionistic fuzzy ideal of E if and only if $(i) \forall x_{(s,t)}, y_{(s',t')} \in < \alpha_A, \beta_A > \Rightarrow (x - y)_{(s \land s', t \lor t')} \in < \alpha_A, \beta_A >$ $(ii) \forall x_{(s,t)} \in \langle \alpha_A, \beta_A \rangle \Rightarrow (y + x - y)_{(s,t)} \in \langle \alpha_A, \beta_A \rangle$ $(iii) \forall x_{(s,t)} \in < \alpha_A, \beta_A > \Rightarrow (nx)_{(s,t)} \in < \alpha_A, \beta_A >$ $(iv) \forall y_{(s',t')} \in < \alpha_{A}, \beta_{A} > \Rightarrow (n(x + y) - nx)_{(s',t')} \in < \alpha_{A}, \beta_{A} >$

Proof : We shall prove (ii) and (iii) among the necessary and sufficient conditions mentioned above. Proof of (i) and (iii) are similar with the previous results.

Proof(ii): Let us assume that $A \Rightarrow \alpha_A, \beta_A > \text{is an ideal of E. Then for } x, y \in E$, we have $\alpha_A(y + x - y) \ge \alpha_A(x); \beta_A(y + x - y) \le \beta_A(x)$ Now as $x_{(s,t)} \in \alpha_A, \beta_A \Rightarrow \alpha_A(x) \ge s, \beta_A(x) \le t$ so we have $\alpha_A(y + x - y) \ge s; \beta_A(y + x - y) \le t$ and which implies that $(y + x - y)_{(s,t)} \in \langle \alpha_A, \beta_A \rangle$.

Conversely, let us choose s and t such that $\frac{\alpha_{A}(y + x - y) < s < \alpha_{A}(x)}{\beta_{A}(y + x - y) > t > \beta_{A}(x)}$

Then $\alpha_A(x) > s, \beta_A(x) < t \Rightarrow x_{(s,t)} \in \alpha_A, \beta_A > \text{ whereas } (y + x - y)_{(s,t)} \notin \alpha_A, \beta_A > \text{ which is a contradiction. Hence } \alpha_A(y + x - y) \ge \alpha_A(x); \beta_A(y + x - y) \le \beta_A(x).$

Proof (iv): Let $y_{(s',t')} \in \alpha_A, \beta_A > .$ Now if $A = \alpha_A, \beta_A > .$ is an intuitionistic fuzzy ideal of E then $\alpha_A(n(x + y) - nx) \ge \alpha_A(y) \ge s'; \beta_A(n(x + y) - nx) \le \beta_A(y) \le t'$, which implies that $(n(x + y) - nx)_{(s',t')} \le \alpha_A, \beta_A > .$

Conversely, let us choose s' and t' in such a way that

$$\alpha_{A}(n(x + y) - nx) < s' < \alpha_{A}(y),$$

$$\beta_{A}(n(x + y) - nx) > t > \beta_{A}(y)$$

Which shows that $y_{(s',t')} \in \langle \alpha_A, \beta_A \rangle$ whereas $(n(x + y) - nx)_{(s',t')} \notin \langle \alpha_A, \beta_A \rangle$

Hence $\frac{\alpha_{A}(n(x+y) - nx) \ge \alpha_{A}(y)}{\beta_{A}(n(x+y) - nx) \le \beta_{A}(y)}$

Definition 3.4: An intuitionistic fuzzy N-subgroup $P = \langle \alpha_{P}, \beta_{P} \rangle$ of a non constant near-ring N is said to be intuitionistic fuzzy prime N-subgroup of N if for any intuitionistic fuzzy N-subgroups $A = \langle \alpha_{A}, \beta_{A} \rangle$ and $B = \langle \alpha_{B}, \beta_{B} \rangle$ of N, the condition $AB \subset P$ implies $A \subset P$ or $B \subset P$.

Lemma 3.5: Let $A = \langle \alpha_A, \beta_A \rangle$ and $B = \langle \alpha_B, \beta_B \rangle$ be two prime N-subgroups of N. Then $A \cap B$ is an intuitionistic fuzzy prime N-subgroup of N if and only if $A \subset B$ or $B \subset A$.

Proof : We have if A and B be two intuitionistic fuzzy N-subgroup of N so as $A \cap B$. Now for all $x \in N$ since $\alpha_A(x) \wedge \alpha_B(x) \le \alpha_A(x)$ and $\beta_A(x) \vee \beta_B(x) \ge \beta_A(x)$ so we have $A \cap B \subset A$. Similarly we have $A \cap B \subset B$. Again $A \cap B$ is prime gives $AB \subset A \cap B$ implies $A \subset A \cap B$ or $B \subset A \cap B$. Thus we have $A \subset B$ or $B \subset A$.

Conversely, suppose that $A \subset B$ or $B \subset A$.

Now if $A \subset B$ then since $A \cap B \subset A$ and for all $x \in N$ since $\alpha_A(x) \le \alpha_B(x)$ so we have $\alpha_A(x) \le \alpha_A(x) \land \alpha_B(x)$ and since $\beta_A(x) \ge \beta_B(x)$ so $\beta_A(x) \ge \beta_A(x) \lor \beta_B(x)$, which implies $A \subset A \cap B$. Thus $A = A \cap B$. Hence for any two intuitionistic fuzzy subgroups C and D we have $CD \subset A \cap B = A$ and A is prime gives $C \subset A$ or $D \subset A \cap C \subset A \cap B$ or

 $D \subset A \cap B$. Consequently $A \cap B$ is prime.

Similarly the result can be proved for the case when $B \subset A$.

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