Neutrosophic Vague Generalized Pre Connectedness in Neutrosophic Vague Topological Space

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Abstract

The purpose of this paper is to initiate and study some of the different types of connected spaces in neutrosophic vague topological space such as neutrosophic vague C_5-connected space, neutrosophic vague generalized connected space and neutrosophic vague generalized pre connected space. Spaces such as neutrosophic vague generalized pre super connected space and neutrosophic vague generalized pre extremally disconnected space are also introduced. We also obtain several properties and characterizations concerning connectedness in these spaces.

Keywords: *NVC_5-connected space, NVG-connected space and NVGP-connected space, NVGP super connected space and NVGP extremally disconnected space.*

I. INTRODUCTION

The fuzzy concept has invaded almost all branches of mathematics, this fuzzy sets was introduced by Zadeh[9] in 1965. The concept of fuzzy topology was introduced by C.L.Chang[2] in 1967. In 1970, Levine [4] initiated the study of generalized closed sets. Atanassov[1] in 1986 introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations. Gau and Buehrer[3] in 1993 defined the vague sets as an extension of fuzzy sets. Then Smarandache[8] in1995 introduced the neutrosophic sets. Each element of a neutrosophic set has three membership degrees including a truth membership degree, an indeterminacy membership degree, and a falsity membership degree which are within the real standard or non standard unit interval]−0, 1+[. As a combination of neutrosophic set and vague set Shawkat Alkhazaleh[7] in 2015 initiated the concept of neutrosophic vague set. In this paper neutrosophic vague C_5-connected space, neutrosophic vague generalized connected space and neutrosophic vague generalized pre connected space, neutrosophic vague generalized pre super connected space and neutrosophic vague generalized pre extremally disconnected space are introduced and their properties and characterizations are studied.

II. PRELIMINARIES

Definition 2.1:[7] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV}=\left\{\left\langle x;\hat{T}_{A_{NV}}(x);\hat{I}_{A_{NV}}(x);\hat{F}_{A_{NV}}(x)\right\rangle;x\in X\right\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x)=\left[T^-,T^+\right],\hat{I}_{A_{NV}}(x)=\left[I^-,I^+\right],\hat{F}_{A_{NV}}(x)=\left[F^-,F^+\right]$$

where,

1) $T^+=1-F^-$
2) $F^+=1-T^-$ and
3) $^-0\le T^-+I^-+F^-\le 2^+$.

Definition 2.2:[7] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i\in U$, $\hat{T}_{A_{NV}}(u_i)\le\hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i)\ge\hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i)\ge\hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV}\subseteq B_{NV}$, where $1\le i\le n$.

Definition 2.3:[7] The complement of NVS A_{NV} is denoted by A^c_{NV} and is defined by

$$\hat{T}^{c}_{A_{NV}}(x)=\left[1-T^{+},1-T^{-}\right],\hat{I}^{c}_{A_{NV}}(x)=\left[1-I^{+},1-I^{-}\right],\hat{F}^{c}_{A_{NV}}(x)=\left[1-F^{+},1-F^{-}\right].$$

Definition 2.4:[7] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x)=[1,1]$; $\hat{I}_{A_{NV}}(x)=[0,0]$; $\hat{F}_{A_{NV}}(x)=[0,0]$. Then A_{NV} is called unit NVS(1_{NV} in short), where $1\le i\le n$.

Definition 2.5:[7] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x)=[0,0]$; $\hat{I}_{A_{NV}}(x)=[1,1]$; $\hat{F}_{A_{NV}}(x)=[1,1]$. Then A_{NV} is called zero NVS(0_{NV} in short), where $1\le i\le n$.

Definition 2.6:[7] The union of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV}=A_{NV}\cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x)=\left[\max\left(T^{-}_{A_{NV_x}},T^{-}_{B_{NV_x}}\right),\max\left(T^{+}_{A_{NV_x}},T^{+}_{B_{NV_x}}\right)\right]$$
$$\hat{I}_{C_{NV}}(x)=\left[\min\left(I^{-}_{A_{NV_x}},I^{-}_{B_{NV_x}}\right),\min\left(I^{+}_{A_{NV_x}},I^{+}_{B_{NV_x}}\right)\right]$$
$$\hat{F}_{C_{NV}}(x)=\left[\min\left(F^{-}_{A_{NV_x}},F^{-}_{B_{NV_x}}\right),\min\left(F^{+}_{A_{NV_x}},F^{+}_{B_{NV_x}}\right)\right].$$

Definition 2.7:[7] The intersection of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV}=A_{NV}\cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x)=\left[\min\left(T^{-}_{A_{NV_x}},T^{-}_{B_{NV_x}}\right),\min\left(T^{+}_{A_{NV_x}},T^{+}_{B_{NV_x}}\right)\right]$$
$$\hat{I}_{C_{NV}}(x)=\left[\max\left(I^{-}_{A_{NV_x}},I^{-}_{B_{NV_x}}\right),\max\left(I^{+}_{A_{NV_x}},I^{+}_{B_{NV_x}}\right)\right]$$
$$\hat{F}_{C_{NV}}(x)=\left[\max\left(F^{-}_{A_{NV_x}},F^{-}_{B_{NV_x}}\right),\max\left(F^{+}_{A_{NV_x}},F^{+}_{B_{NV_x}}\right)\right].$$

Definition 2.8:[7] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i)=\hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i)=\hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i)=\hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV}, are called equal, where $1\le i\le n$.

Definition 2.9:[5] A neutrosophic vague topology (NVT in short) on X is a family τ of neutrosophic vague sets (NVS in short) in X satisfying the following axioms:

- $0_{NV},1_{NV}\in\tau$
- $G_1\cap G_2\in\tau$ for any $G_1,G_2\in\tau$
- $\cup G_i\in\tau,\forall\{G_i : i\in J\}\subseteq\tau$

In this case the pair (X,τ) is called a neutrosophic vague topological space (NVTS in short) and any NVS in τ is known as a neutrosophic vague open set (NVOS in short) in X. The complement A^c of NVOS in a NVTS (X,τ) is called neutrosophic vague closed set (NVCS in short) in X.

Definition 2.10:[5] A NVS $A=\left\{\left\langle x,\left[\hat{T}_A,\hat{I}_A,\hat{F}_A\right]\right\rangle\right\}$ in NVTS (X,τ) is said to be

i) Neutrosophic Vague pre- closed set (NVPCS in short) if $NVcl(NV\operatorname{int}(A))\subseteq A$,

ii) Neutrosophic Vague pre-open set (NVPOS in short) if $A\subseteq NV\operatorname{int}(NVcl(A))$,

Definition 2.11:[5] Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague pre interior of A ($NVp\operatorname{int}(A)$ in short) and neutrosophic vague pre closure of A ($NVpcl(A)$ in short) are defined by

- $NVp\operatorname{int}(A)=\cup\{G / G \text{ is a NVPOS in } X \text{ and } G\subseteq A\}$,
- $NVpcl(A)=\cap\{K / K \text{ is a NVPCS in } X \text{ and } A\subseteq K\}$.

Definition 2.12:[5] A NVS A of a NVTS (X,τ) is said to be neutrosophic vague generalized closed set (NVGCS in short) if $NVcl(A)\subseteq U$ whenever $A\subseteq U$ and U is NVOS in X.

Definition 2.13:[5] A NVS A is said to be neutrosophic vague generalized pre-closed set (NVGPCS in short) in (X,τ) if $NVpcl(A)\subseteq U$ whenever $A\subseteq U$ and U is NVOS in X.

Definition 2.14:[6] Let (X,τ) be a NVTS. The neutrosophic vague generalized pre closure ($NVgpcl(A)$ in short) for any NVS A is defined as follows,

- $NVgpcl(A)=\cap\{K / K \text{ is a } NVGPCS \text{ in } X \text{ and } A\subseteq K\}$,
- $NVgp\operatorname{int}(A)=\cup\{G / G \text{ is a } NVGPOS \text{ in } X \text{ and } G\subseteq A\}$.

Definition 2.15:[5] A NVTS (X,τ) is said to be neutrosophic vague ${}_{gp}T_{1/2}$ space (NV ${}_{gp}T_{1/2}$ in short) if every NVGPCS in X is NVCS in X.

Definition 2.16:[6] A map $f:(X,\tau)\to(Y,\sigma)$ is said to be neutrosophic vague generalized pre-continuous (NVGP continuous in short) mapping if $f^{-1}(A)$ is NVGPCS in (X,τ) for every NVGPCS A of (Y,σ).

Definition 2.17:[6] A map $f:(X,\tau)\to(Y,\sigma)$ is said to be neutrosophic vague generalized pre irresolute (NVGP irresolute in short) mapping if $f^{-1}(A)$ is NVGPCS in (X,τ) for every NVGPCS A in (Y,σ).

III. NEUTROSOPHIC VAGUE GENERALIZED PRE-CONNECTED SPACES

Definition 3.1: A NVTS (X,τ) is said to be neutrosophic vague C_5-connected (NV C_5-connected for short) space if the only NVSs which are both neutrosophic vague open and neutrosophic vague closed are 0_{NV} and 1_{NV}.

Definition 3.2: A NVTS (X,τ) is said to be neutrosophic vague generalized connected (NVG-connected for short) space if the only NVSs which are both neutrosophic vague generalized open and neutrosophic vague generalized closed are 0_{NV} and 1_{NV}.

Definition 3.3: A NVTS (X,τ) is said to be neutrosophic vague generalized pre-connected (NVGP-connected for short) space if the only NVSs which are both neutrosophic vague generalized pre-open and neutrosophic vague generalized pre-closed are 0_{NV} and 1_{NV}.

Example 3.4: Let $X=\{a,b,c\}$ and let $\tau=\{0_{NV},G_1,G_2,1_{NV}\}$ be NVT on X, where

$$G_1=\left\{x,\frac{a}{\langle[0.4,0.7];[0.4,0.5];[0.3,0.6]\rangle},\frac{b}{\langle[0.6,0.8];[0.2,0.5];[0.2,0.4]\rangle},\frac{c}{\langle[0.5,0.7];[0.2,0.6];[0.3,0.5]\rangle}\right\}$$

$$G_2=\left\{x,\frac{a}{\langle[0.2,0.5];[0.6,0.8];[0.5,0.8]\rangle},\frac{b}{\langle[0.1,0.4];[0.7,0.9];[0.6,0.9]\rangle},\frac{c}{\langle[0,0.2];[0.7,0.9];[0.8,1]\rangle}\right\}.$$

Then (X,τ) is NVGP-connected space.

Theorem 3.5: Every NVGP-connected space is NV C_5-connected.

Proof: Let (X,τ) be NVGP-connected space. Suppose (X,τ) is not NV C_5-connected space, then there exists a proper NVS A which is both neutrosophic vague open and neutrosophic vague closed in (X,τ). That is A is both neutrosophic vague generalized pre open and neutrosophic vague generalized pre closed in (X,τ). This implies that (X,τ) is not NVGP-connected space which is a contradiction to the fact that (X,τ) is NVGP-connected space. Therefore (X,τ) is NV C_5-connected space.

Theorem 3.6: Every NVGP-connected space is NVG-connected.

Proof: Let (X,τ) be NVGP-connected space. Suppose $(X,\tau)$is not NVG-connected space, then there exists a proper NVS A which is both neutrosophic vague generalized open and neutrosophic vague generalized closed in (X,τ). This implies that A is both neutrosophic vague generalized pre open and neutrosophic vague generalized pre closed in (X,τ). This implies that (X,τ) is not NVGP-connected space which is a contradiction to the fact that (X,τ) is NVGP-connected space. Therefore (X,τ) is NV C_5 -connected space.

Theorem 3.7: A NVTS (X,τ) is NVGP-connected space if and only if there exists no non-zero NVGPOSs A and B in (X,τ) such that $A=B^c$.

Proof: Necessity: Let A and B be two NVGPOSs in (X,τ) such that $A\neq 0_{NV}\neq B$ and $A=B^c$. Since $A=B^c$, B is NVGPOS which implies that $B^c=A$ is NVGPCS. Since $B\neq 0$ this implies that $B^c\neq 1_{NV}$ (ie.,) $A\neq 1_{NV}$. Hence there exists a proper NVS $A\left(A\neq 0_{NV}, A\neq 1_{NV}\right)$ which is both NVGPOS and NVGPCS in (X,τ). Hence (X,τ) is not NVGP-connected space. But it is contradiction to our hypothesis. Thus there exists no non-zero NVGPOSs A and B in (X,τ) such that $A=B^c$.

Sufficiency: Let (X,τ) be NVTS and A is both NVGPO and NVGPC in (X,τ) such that $0_{NV}\neq A\neq 1_{NV}$. Now let $B=A^c$. In this case, B is NVGPOS and $A\neq 1_{NV}$ this implies that $B=A^c\neq 0_{NV}$. Hence $B\neq 0_{NV}$ which is a contradiction to our hypothesis. Therefore there is a proper NVS of (X,τ) which is both NVGPO and NVGPC in (X,τ). Hence (X,τ) is NVGP-connected space.

Theorem 3.8: A NVTS (X,τ) is NVGP-connected space if and only if there exists no non-zero NVGPOSs A and B in (X,τ) such that $A=B^c$, $B=\left(NVgpcl(A)\right)^c$ and $A=\left(NVgpcl(B)\right)^c$.

Proof: Necessity: Assume that there exists NVSs A and B in (X,τ) such that $A\neq 0_{NV}\neq B$, $B=A^c$, $B=\left(NVgpcl(A)\right)^c$ and $A=\left(NVgpcl(B)\right)^c$. Since $\left(NVgpcl(A)\right)^c$ and $\left(NVgpcl(B)\right)^c$ are NVGPOSs in (X,τ), A and B are NVGPOSs in (X,τ). This implies (X,τ) is not NVGP-connected space, which is a contradiction to the statement that (X,τ) is NVGP-connected space. Therefore there exists no non-zero NVGPOSs A and B in (X,τ) such that $A=B^c$, $B=\left(NVgpcl(A)\right)^c$ and $A=\left(NVgpcl(B)\right)^c$.

Sufficiency: Let A be both NVGPO and NVGPC in (X,τ) such that $1_{NV}\neq A\neq 0_{NV}$. Now by taking $B=A^c$ we obtain a contradictory to our hypothesis. Hence (X,τ) is NVGP-connected space.

Theorem 3.9: Let (X,τ) be $\mathrm{NV}_{gp}\mathrm{T}_{1/2}$ space. Then the following statements are equivalent.

i) (X,τ) is NVGP-connected space.

ii) (X,τ) is NVG-connected space.

iii) $(X,\tau)$is NV C_5 -connected space.

Proof: $(\mathrm{i})\Rightarrow(\mathrm{ii})$ It is obvious from the Theorem 3.6.

$(\mathrm{ii})\Rightarrow(\mathrm{iii})$ It is obvious.

$(\mathrm{iii})\Rightarrow(\mathrm{i})$ Let (X,τ) be NV C_5 -connected space. Suppose (X,τ) is not NVGP-connected space, then there exists a proper NVS A in (X,τ)which is both NVGPO and NVGPC in(X,τ). But since (X,τ) is $\mathrm{NV}_{gp}\mathrm{T}_{1/2}$ space, A is both NVO and NVC in(X,τ). This implies that (X,τ) is not NV C_5 -connected space, which is a contradiction to our hypothesis. Therefore (X,τ) must be NVGP-connected space.

Theorem 3.10: If $f:(X,\tau)\rightarrow(Y,\sigma)$ is NVGP continuous mapping and (X,τ) is NVGP-connected space, then (Y,σ) be NV C_5 -connected space.

Proof: Let(X,τ) be NVGP-connected space. Suppose (Y,σ) is not NV C_5 -connected space, then there exists a proper NVS A which is both NVO and NVC in(Y,σ). Since f is NVGP continuous mapping, $f^{-1}(A)$ is a proper NVS of (X,τ) which is both NVGPO and NVGPC in (X,τ). But this is a contradiction to our hypothesis. Hence (Y,σ) is NV C_5-connected space.

Theorem 3.11: If $f:(X,\tau)\to(Y,\sigma)$ is NVGP irresolute mapping and (X,τ) is NVGP-connected space, then (Y,σ) be NVGP-connected space.

Proof: Let (X,τ) be NVGP-connected space. Suppose (Y,σ) is not NVGP-connected space, then there exists a proper NVS A which is both NVGPO and NVGPC in (Y,σ). Since f is NVGP irresolute mapping, $f^{-1}(A)$ is a proper NVS of (X,τ) which is both NVGPO and NVGPC in (X,τ). But this is a contradiction to our hypothesis. Hence (Y,σ) is NVGP-connected space.

Definition 3.12: Two NVSs A and B in (X,τ) are said to be q-coincident (AqB for short) if and only if there exists an element $x\in X$ such that $\hat{\mathrm{T}}_{A_{NV}}(x)>\hat{\mathrm{T}}^{c}_{B_{NV}}(x), \hat{\mathrm{I}}_{A_{NV}}(x)<\hat{\mathrm{I}}^{c}_{B_{NV}}(x)$ and $\hat{\mathrm{F}}_{A_{NV}}(x)<\hat{\mathrm{F}}^{c}_{B_{NV}}(x)$.

Definition 3.13: Two NVSs A and B in (X,τ) are said to be not q-coincident (Aq^cB for short) if and only if $A\subseteq B^c$.

Definition 3.14: A NVTS (X,τ) is called NV C_5-connected between two NVSs A and B if there is no neutrosophic vague open set D in (X,τ) such that $A\subseteq D$ and Dq^cB.

Definition 3.15: A NVTS (X,τ) is called NVGP-connected between two NVSs A and B if there is no neutrosophic vague generalized pre open set D in (X,τ) such that $A\subseteq D$ and Dq^cB.

Example 3.16: Let $X=\{a,b,c\}$ and let $\tau=\{0_{NV},G,1_{NV}\}$ be NVT on X, where

$$G=\left\{x,\frac{a}{\langle[0.3,0.5];[0.8,1];[0.5,0.7]\rangle},\frac{b}{\langle[0.2,0.6];[0.5,0.7];[0.4,0.8]\rangle},\frac{c}{\langle[0.1,0.4];[0.3,0.8];[0.6,0.9]\rangle}\right\}.$$

Then (X,τ) is NVGP-connected between two NVSs

$$A=\left\{x,\frac{a}{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle},\frac{b}{\langle[0.5,0.6];[0.1,0.3];[0.4,0.5]\rangle},\frac{c}{\langle[0.3,0.8];[0.4,0.9];[0.2,0.7]\rangle}\right\}\&$$

$$B=\left\{x,\frac{a}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle},\frac{b}{\langle[0.6,0.9];[0.2,0.5];[0.1,0.4]\rangle},\frac{c}{\langle[0.4,0.7];[0.3,0.8];[0.3,0.6]\rangle}\right\}.$$

Theorem 3.17: If a NVTS (X,τ) is NVGP connected between two NVSs A and B, then it is NV C_5-connected between two NVSs A and B but the converse may not be true in general.

Proof: Suppose (X,τ) is not NV C_5-connected between two NVSs A and B, then there exists a NVOS D in (X,τ) such that $A\subseteq D$ and Dq^cB. Since every NVOS is NVGPOS, there exists a NVGPOS D in (X,τ) such that $A\subseteq D$ and Dq^cB. This implies (X,τ) is not NVGP connected between A and B, which is a contradiction to our hypothesis. Therefore (X,τ) is NV C_5-connected between two NVSs A and B.

Example 3.18: Let $X=\{a,b\}$ and let $\tau=\{0_{NV},G,1_{NV}\}$ be NVT on X, where

$$G=\left\{x,\frac{a}{\langle[0.6,0.8];[0.3,0.5];[0.2,0.4]\rangle},\frac{b}{\langle[0.2,0.7];[0.1,0.4];[0.3,0.8]\rangle}\right\}.$$

Then (X,τ) is NV C_5-connected between two NVSs

$$A=\left\{x,\frac{a}{\langle[0.3,0.5];[0.8,0.9];[0.5,0.7]\rangle},\frac{b}{\langle[0.2,0.4];[0.6,0.7];[0.6,0.8]\rangle}\right\}$$ and

$B=\left\{x,\frac{a}{\langle[0.2,0.4];[0.5,0.8];[0.6,0.8]\rangle},\frac{b}{\langle[0,0.2];[0.6,0.7];[0.8,1]\rangle}\right\}$. But (X,τ) is not NVGP connected between A and B, since the NVS $D=\left\{x,\frac{a}{\langle[0.4,0.7];[0.3,0.8];[0.3,0.6]\rangle},\frac{b}{\langle[0.5,0.8];[0.4,0.6];[0.2,0.5]\rangle}\right\}$ is NVGPOS such that $A\subseteq D$ and $D\subseteq B^c$.

Theorem 3.19: A NVTS (X,τ) is NVGP connected between two NVSs A and B if and only if there is no NVGP open and NVGP closed set D in (X,τ) such that $A\subseteq D\subseteq B^c$.

Proof: Necessity: Let (X,τ) be NVGP connected between two NVSs A and B. Suppose that there exists NVGP open and NVGP closed set D in (X,τ) such that $A\subseteq D\subseteq B^c$, then Dq^cB and $A\subseteq D$. This implies (X,τ) is not NVGP connected between two NVSs A and B, by Definition 3.15. It is a contradiction to our hypothesis. Therefore there is no NVGPO and NVGPC set D in (X,τ) such that $A\subseteq D\subseteq B^c$.

Sufficiency: Suppose that (X,τ) is not NVGP connected between two NVSs A and B. Then there exists a NVGPOS D in (X,τ) such that $A\subseteq D$ and Dq^cB. This implies that there is NVGPOS D in (X,τ) such that $A\subseteq D\subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X,τ) is NVGP connected between two NVSs A and B.

Theorem 3.20: If a NVTS (X,τ) is NVGP connected between two NVSs A and B, $A\subseteq A_1$ and $B\subseteq B_1$, then (X,τ) is NVGP connected between A_1 and B_1.

Proof: Suppose that (X,τ) is not NVGP connected between A_1 and B_1, then by Definition 3.15, there exists a NVGPOS D in (X,τ) such that $A_1\subseteq D$ and Dq^cB_1. This implies $D\subseteq B_1^c$ and $A_1\subseteq D$ implies $A\subseteq A_1\subseteq D$. That is $A\subseteq D$. Now let us prove that $D\subseteq B^c$, that is, to prove that Dq^cB. Suppose that DqB, then by Definition 3.12, there exists an element $x\in X$ such that $\hat{T}_{D_{NV}}(x)>\hat{T}^c_{B_{NV}}(x)$, $\hat{I}_{D_{NV}}(x)<\hat{I}^c_{B_{NV}}(x)$ and $\hat{F}_{D_{NV}}(x)<\hat{F}^c_{B_{NV}}(x)$. Therefore $\hat{T}_{D_{NV}}(x)>\hat{T}^c_{B_{NV}}(x)>\hat{T}^c_{B_1\,NV}(x)$, $\hat{I}_{D_{NV}}(x)<\hat{I}^c_{B_{NV}}(x)<\hat{I}^c_{B_1\,NV}(x)$ and $\hat{F}_{D_{NV}}(x)<\hat{F}^c_{B_{NV}}(x)<\hat{F}^c_{B_1\,NV}(x)$, since $B\subseteq B_1$. Thus DqB_1. But $D\subseteq B_1$. That is Dq^cB_1, which is a contradiction. Therefore Dq^cB. That is $D\subseteq B^c$. Hence (X,τ) is not NVGP connected between A and B, which is a contradiction to our hypothesis. Thus (X,τ) is NVGP connected between A_1 and B_1.

Theorem 3.21: Let (X,τ) be a NVTS and A and B be NVSs in (X,τ). If AqB then (X,τ) is NVGP connected between A and B.

Proof: Suppose (X,τ) is not NVGP connected between A and B. Then there exists a NVGPOS D in (X,τ) such that $A\subseteq D$ and $D\subseteq B^c$. This implies that $A\subseteq B^c$. That is Aq^cB. But this is a contradiction to our hypothesis. Therefore (X,τ) is NVGP connected between A and B.

Remark 3.22: The converse of the above theorem may not be true in general.

Example 3.23: Let $X=\{a,b\}$ and let $\tau=\{0_{NV},G,1_{NV}\}$ be NVT on X, where $G=\left\{x,\frac{a}{\langle[0.2,0.5];[0.1,0.4];[0.5,0.8]\rangle},\frac{b}{\langle[0.7,0.9];[0.3,0.6];[0.1,0.3]\rangle}\right\}$. Then (X,τ) is NVGP-connected between two NVSs

$A=\left\{x,\frac{a}{\langle[0.3,0.7];[0.7,0.8];[0.3,0.7]\rangle},\frac{b}{\langle[0.3,0.6];[0.3,0.5];[0.4,0.7]\rangle}\right\}$ and

$B=\left\{x,\frac{a}{\langle[0.1,0.8];[0.6,0.7];[0.2,0.9]\rangle},\frac{b}{\langle[0.8,0.9];[0.5,0.7];[0.1,0.2]\rangle}\right\}$, but not q-coincident with B.

Definition 3.24: A NVGP open set A is called neutrosophic vague regular generalized pre open set (NVRGPOS in short) if $A=NVgp\operatorname{int}(NVgpcl(A))$. The complement of a NVRGPOS is called NVRGPCS.

Definition 3.25: A NVTS (X,τ) is called neutrosophic vague generalized pre super connected space (NVGP super connected space) if there exists no NVRGPOS in (X,τ).

Theorem 3.26: Let (X,τ) be NVTS, then the following are equivalent.

i) (X,τ) is NVGP super connected space.

ii) For every non-zero NVRGPOS A, $NVgpcl(A)=1_{NV}$.

iii) For every NVRGPCS A with $A\neq 1$, $NVgp\operatorname{int}(A)=0_{NV}$.

iv) There exists no NVRGPOS A and B in (X,τ) such that $A\neq 0_{NV}\neq B$, $A\subseteq B^c$.

v) There exists no NVRGPOSs A and B in (X,τ) such that $A\neq 0_{NV}\neq B$, $B=(NVgpcl(A))^c$, $A=(NVgpcl(B))^c$.

vi) There exists no NVRGPCSs A and B in (X,τ) such that $A\neq 1_{NV}\neq B$, $B=(NVgp\operatorname{int}(A))^c$, $A=(NVgp\operatorname{int}(B))^c$.

Proof: (i) $\Rightarrow$ (ii) Assume that there exists a NVRGPOS A in (X,τ) such that $A\neq 0_{NV}$ and $NVgpcl(A)\neq 1_{NV}$. Now let $B=NVgp\operatorname{int}(NVgpcl(B))$, then B is a proper NVRGPOS in (X,τ). But this is a contradiction to the fact that (X,τ) is NVGP super connected space. Therefore $NVgpcl(A)=1_{NV}$.

(ii) $\Rightarrow$ (iii) Let $A\neq 1_{NV}$ be NVRGPCS in (X,τ). If $B=A^c$ then B is a NVRGPOS in (X,τ) with $B\neq 0_{NV}$. Hence $NVgpcl(B)=1_{NV}$. This implies $(NVgpcl(B))^c=0_{NV}$. That is $NVgp\operatorname{int}(B^c)=0_{NV}$. Hence $NVgp\operatorname{int}(A)=0_{NV}$.

(iii) $\Rightarrow$ (iv) Let A and B be two NVRGPOSs in (X,τ) such that $A\neq 0_{NV}\neq B$, $A\subseteq B^c$. Since B^c is NVRGPCS in (X,τ) and $B\neq 0_{NV}$ implies $B^c\neq 1_{NV}$, $B^c=NVgpcl(NVgp\operatorname{int}(B^c))$ and we have $NVgp\operatorname{int}(B^c)=0_{NV}$. But $A\subseteq B^c$. Therefore $0_{NV}\neq A=NVgp\operatorname{int}(NVgpcl(A))\subset NVgp\operatorname{int}(NVgpcl(B^c))=NVgp\operatorname{int}(NVgpcl(NVgpcl(NVgp\operatorname{int}(B^c))))=NVgp\operatorname{int}((NVgpcl(NVgp\operatorname{int}(B^c))))=0_{NV}$ which is a contradiction. Therefore (iv) is true.

(iv) $\Rightarrow$ (i) Let $0_{NV}\neq A\neq 1_{NV}$ be a NVRGPOS in (X,τ). If we take $B=(NVgpcl(A))^c$, since $NVgp\operatorname{int}(NVgpcl(B))=NVgp\operatorname{int}(NVgpcl(NVgpcl(A))^c)=NVgp\operatorname{int}(NVgp\operatorname{int}(NVgpcl(A)))^c=NVgp\operatorname{int}(A^c)=(NVgpcl(A))^c=B$. Also we get $B\neq 0_{NV}$, since otherwise, we have $B=0_{NV}$ and this implies $(NVgpcl(A))^c=0_{NV}$. That is $NVgpcl(A)=1_{NV}$. Hence $A=NVgp\operatorname{int}(NVgpcl(A))=NVgp\operatorname{int}(1)=1_{NV}$. That is $A=1_{NV}$, which is a contradiction. Therefore $B\neq 0_{NV}$ and $A\subseteq B^c$. But this is a contradiction to (iv). Therefore (X,τ) is NVGP super connected space.

(i) $\Rightarrow$ (v) Let A and B be two NVRGPOSs in (X,τ) such that $A\neq 0_{NV}\neq B$, $B=(NVgpcl(A))^c$, $A=(NVgpcl(B))^c$. Now we have $NVgp\operatorname{int}(NVgpcl(A))=NVgp\operatorname{int}(B^c)=(NVgpcl(B))^c=A$, $A\neq 0_{NV}$ and $A\neq 1_{NV}$, since if $A=1_{NV}$, then $1_{NV}=(NVgpcl(B))^c\Rightarrow NVgpcl(B)=0_{NV}\Rightarrow$

$B = 0_{NV}$. But $B \neq 0_{NV}$. Therefore $A \neq 1_{NV}$ implies that A is proper NVRGPOS in (X,τ), which is a contradiction to (i). Hence (v) is true.

(v) $\Rightarrow$ (i) Let A be NVRGPOS in (X,τ) such that $A = NVgp\,\text{int}(NVgpcl(A))$ and $0_{NV} \neq A \neq 1_{NV}$. Now take $B = (NVgpcl(A))^c$. In this case we get $B \neq 0_{NV}$ and B is a NVRGPOS in (X,τ), $B = (NVgpcl(A))^c$ and $(NVgpcl(B))^c = (NVgpcl(NVgpcl(A))^c)^c = NVgp\,\text{int}(NVgpcl(A)^c)^c$ $= NVgp\,\text{int}(NVgpcl(A)) = A$. But this is a contradiction to (v). Therefore (X,τ) is NVGP super connected space.

(v) $\Rightarrow$ (vi) Let A and B be two NVRGPCSs in (X,τ) such that $A \neq 1_{NV} \neq B$, $B = (NVgp\,\text{int}(A))^c$, $A = (NVgp\,\text{int}(B))^c$. Taking $C = A^c$ and $D = B^c$, C and D becomes NVRGPOSs in (X,τ) with $C \neq 0_{NV} \neq D$, $D = (NVgp\,\text{int}(C))^c$, $C = (NVgp\,\text{int}(D))^c$, Which is a contradiction to (v). Hence (vi) is true.

(vi) $\Rightarrow$ (v) Can be easily proved by the similar way as in (v) $\Rightarrow$ (vi).

Definition 3.27: A NVTS (X,τ) is said to be a neutrosophic vague generalized pre extremally disconnected space (NVGP extremally disconnected space in short) if the neutrosophic vague generalized pre-closure of every NVGPOS in (X,τ) is NVGPOS.

Theorem 3.28: Let (X,τ) be NVTS, then the following are equivalent:

i) (X,τ) is a NVGP extremally disconnected space.

ii) For each NVGPCS A, $NVgp\,\text{int}(A)$ is NVGPCS.

iii) For each NVGPOS A, $NVgpcl(A) = (NVgpcl(NVgpcl(A))^c)^c$.

iv) For each NVGPOSs A and B with $NVgpcl(A) = B^c$, $NVgpcl(A) = (NVgpcl(B))^c$.

Proof: (i) $\Rightarrow$ (ii) Let A be any NVGPCS. Then A^c is NVGPOS. So (i) implies that $NVgpcl(A^c) = (NVgp\,\text{int}(A))^c$ is NVGPOS. Thus $NVgp\,\text{int}(A)$ is NVGPCS in (X,τ).

(ii) $\Rightarrow$ (iii) Let A be NVGPOS. Then we have $NVgpcl(NVgpcl(A))^c = NVgpcl(NVgp\,\text{int}(A^c))$. Therefore $(NVgpcl(NVgpcl(A))^c)^c = (NVgpcl(NVgp\,\text{int}(A^c)))^c$. Since A is NVGPOS. Then A^c is NVGPCS. So by (ii) $NVgp\,\text{int}(A^c)$ is NVGPCS. That is $NVgpcl(NVgp\,\text{int}(A^c)) = NVgp\,\text{int}(A^c)$. Hence $(NVgpcl(NVgp\,\text{int}(A^c)))^c = (NVgp\,\text{int}(A^c))^c = NVgpcl(A)$.

(iii) $\Rightarrow$ (iv) Let A and B be any two NVGPOS in (X,τ) such that $NVgpcl(A) = B^c$. (iii) implies $NVgpcl(A) = (NVgpcl(NVgpcl(A))^c)^c = (NVgpcl(B^c)^c)^c = (NVgpcl(B))^c$.

(iv) $\Rightarrow$ (i) Let A and B be any two NVGPOS in (X,τ) with $NVgpcl(A) = B^c$ and $NVgpcl(A) = (NVgpcl(B))^c$. From $NVgpcl(A) = B^c \Rightarrow B = (NVgpcl(A))^c$. Since $NVgpcl(B)$ is NVGPCS, this implies that $NVgpcl(A)$ is NVGPOS. This implies that (X,τ) is NVGP extremally disconnected space.

IV. CONCLUSION

We have discussed about the NVC_5-connected space, NVG-connected space and NVGP-connected space, NVGP super connected space and NVGP extremally disconnected space and their characterizations concerning connectedness in these spaces.

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