Five Dimensional Cylindrically Symmetric Solutions and Energy Distribution in *f(R)* Gravity

Ladke L. S. [#1], Mishra R. D. [*2], Tripade V.P. [#3]

[#1]*Principal, NilkanthraoShinde Science and Arts College, Bhadrawati, India*
[*2]*Shriram Commerce and Science College, Kurkheda,India*
[#3]*Mohasinbhai ZaweriMahavidyalaya,Desaiganj (Wadsa),India*

Abstract

    This paper is devoted to study of five dimensional cylindrically symmetric solutions in f(R) theory of gravity using perfect fluid as matter and discussed energy distribution of these solutions by applying Landau-Lifshitz energy momentum complex. To find the solution, the assumption of constant scalar curvature is used. Also stability and constant scalar curvature condition is explored for viable f(R) models. These models satisfy above mentioned conditions.

Keywords : *f(R)gravity,Landua-Lifshitz EMC, Perfect fluid, Equation of state*

I. INTRODUCTION

    General relativity is widely accepted as a fundamental theory to describe the geometric properties of space-time. It is clear from the cosmological observational data that the universe is undergoing an accelerated expansion and fundamental agent driving the acceleration is dark energy. Dark energy is an unknown form of energy which occupies most of the part of entire universe. Accelerated expansion of the universe is confirmed by recent cosmic observation [1, 2, 3, 4]. There are two proposals to understand the accelerating expansion of the universe. One is assumed to be dark energy and another one is the modified theory of gravity. The *f(R)* theory of gravity is one of the modified theory of gravity which is considered most suitable due to cosmological importance of *f(R)* model. The *f(R)* gravity provides a very natural unification of the early-time inflation and late time acceleration. It describes the transition from deceleration to acceleration in the evolution of the universe [5, 6] it also gives a natural gravitational alternative to dark energy. Cosmic acceleration is attained by introducing the term $\dfrac{1}{R}$ which is essential for small curvature. The problem of dark matter can also be addressed by using viable *f(R)* gravity models. Cognolaet. al. [7] firstly used the assumption of constant scalar curvature to find the solutions in *f(R)* gravity. Hollenstein and Francisco S.N. Lobo [8] discussed the exact solutions of *f(R)* gravity coupled to non-linear electrodynamics. The field equations are solved using the assumption of constant scalar curvature which may be zero or non-zero. M.Farasat Shamir [9] used the assumption of constant and non-constant curvature to obtain the dust static cylindrically solutions in *f(R)* gravity. Multamaki and Vilja [10, 11] investigated static spherically symmetric vacuum solutions of the field equations and non-vacuum solutions by taking fluid respectively. Sharif and Arif [12] investigated the static cylindrically symmetric interior solutions in metric *f(R)* gravity. M.T.Rincon-Ramireset,al. [13] studied the cylindrically symmetric solutions in metric *f(R)* gravity with constant curvature R. Azadi et. al. [14] investigated cylindrically symmetric vacuum solutions using weyl coordinates. Godel K [15] and Klepac p [16] described a dust universe with negative cosmological constant for a homogeneous cylindrically symmetric Space – time. Momeni D. and Gholizade [18] studied exact solution in cylindrical symmetry in vacuum for constant scalar curvature. A simple cylindrically symmetric universe filled with dust is presented by Senovilla and Verma [18]

    Energy localization is the open issue which is still unsolved. Einstein was the first who tried to solve this problem but in Einstein energy momentum complex, there is one main drawback that it is not symmetric in its indices hence conservation law of angular momentum cannot be defined. Landau-Lifshitz energy momentum complex is able to develop a conserved angular momentum and satisfies the local conservation law. Energy momentum priscription given by Landau-Lifshitz [19], Papapetrou [20], Bergmann [21], Weinberg [22], Tolman[23], Goldberg [24] were restricted to compute the energy and momenta distribution in quasi co-ordinate

system but energy momentum prescription given by Moller[25] was the first which was utilized for more coordinate system. Nojiri and Odintsov [26] studied modified gravity with negative and positive power curvature unification of inflation and cosmic acceleration and observed that *f(R)* models containing positive power of R account for unification and become dominant at early epoch and negative power of R account for the inflation and play the role of dark energy and support the current expansion. Mortzayavari [27] studied plane symmetric vacuum solutions of the modified field equations for a power model in metric *f(R)* gravity with the assumption of constant Ricci scalar and also determine the energy momentum complexes in *f(R)* gravity for some important models and showed that these models satisfy the stability and constant curvature conditions. M. Sharif and Tasnim Fatima [28] studied Energy-Momentum Distribution: A Crucial Problem in General Relativity. Gamal G.L.,Nashed [29] studied Energy Momentum Complex. Radinschiet. al. [30] studied Landau and Lifshitz energy momentum complex and localization of energy. M. Sharif [31] studied the energy and momentum for different cosmological models using various prescriptions are evaluated. S. K. Tripathyet. al. [32] studied Energy and Momentum of Bianchi Type VI*h*Universes. M. Sharif [33] studied Energy-Momentum Distribution of the Weyl-Lewis-Papapetrou and the Levi-Civita Metrics. Multamakiet. al. [34] explore energy momentum complex in *f(R)* theory of gravity and generalized the Landau-Lifshitz prescription of calculating the energy momentum complex to the framework of *f(R)* gravity. For Bianchi Type-I Universe, Xulu [35] has investigated Weinberg, Landau–Lifshitz and Papapetrou energy–momentum prescriptions and he found that the overall energy is zero. Valerio faraoni and Shahn nadeau [36] studied the stability of modified gravity model and condition for existence and stability of de- sitter space in modified gravity are derived. To explore the hidden knowledge of universe many researchers [37, 38, 39, 40] encouraged to come into the field of higher dimensional theory. Weinberg [41] studied the unification of fundamental forces with gravity which reveals that the space-time should be different from four dimensions. Ladke, L. S. et. al. [42] studied higher dimensional plane symmetric solutions in *f(R)* theory of gravitation using assumption of non-constant and constant scalar curvature condition.

M. Sharif and H.R. Kausar[43] investigate spherically symmetric solution of the field equations in *f(R)* theory of gravity using dust matter. M. Sharif and M. Farasat Shamir [44] studied energy distribution in *f(R)* gravity using Landau-Lifshitz energy momentum complex and evaluated the energy density of plane symmetric solution and constant scalar curvature and the stability condition are also discussed. M.Sharif and Sadiaarif [45] investigated static cylindrically symmetric solution in *f(R)* gravity using assumption of constant scalar curvature and dust matter. They explored the energy distribution of the solution by applying Landau-Lifshitz energy momentum complex and stability as well as constant scalar curvature condition for some viable *f(R)* model.

Motivating by the above literature, in this paper we study five dimensional static cylindrically symmetric solutions in *f(R)* theory of gravity using perfect fluid as matter. To find the energy distribution of the solution having constant scalar curvature, generalized Landau-Lifshitz energy momentum complex is used. We explore the stability and constant scalar curvature conditions for some viable *f(R)* models. The paper is organized as follows in section1, Introduction is given and in section 2, *f(R)* theory of gravity is explained. Section 3 deals with the metric and field equation. In section 4 we find the solution of field equations using constant scalar curvature. In section 5 we calculate the generalized Landau-Lifshitz energy momentum complex for constant scalar curvature solution. Section 6 is devoted to discuss stability of some well-known *f(R)* model. In last section we discuss the result.

II. *F(R)* THEORY OF GRAVITY

The action for $f(R)$ theory of gravity is given by

$$S = \int \left(\frac{1}{16\pi G} f(R) + L_m \right) \sqrt{-g} \; d^5 x, \tag{1}$$

Where $f(R)$ is general function of Ricci scalar R and L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij}, we obtain the field equations in $f(R)$ theory of gravity as

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = kT_{ij}, \quad i,j = 1,2,3,4,5 \tag{2}$$

Where $F(R) \equiv \dfrac{df(R)}{dR}$, $\quad \Box \equiv \nabla^i \nabla_i$

With ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

If we take $f(R) = R$, the field equation (2) in *f(R)* theory of gravity reduce to the field equation of general theory of relativity which is proposed by Einstein.

Contracting field equations (2), we have

$$F(R)R - \frac{5}{2}f(R) + 4\,\Box\,F(R) = kT \tag{3}$$

For non-vacuum, we have

$$F(R)R - \frac{5}{2}f(R) + 4\,\Box\,F(R) = 8\pi T \tag{4}$$
.

From (4), we get

$$5f(R) = 2\big[-8\pi T + 4\,\Box\,F(R) + F(R)R\big] \tag{5}$$
.

Using equations (2) and (5), the field equations take the form

$$5\big[F(R)R_{ij} - \nabla_i\nabla_j\,F(R) - 8\pi T_{ii}\big] = g_{ij}[F(R)R - \Box\,F(R) - 8\pi T]. \tag{6}$$

It follows that the equation (6) is not depend on the index i

Equation (6) can be express as

$$K_i = \frac{F(R)R_{ij} - \nabla_i\nabla_j F(R) - 8\pi T_{ij}}{g_{ij}} \tag{7}$$

This equation is independent of i and hence $K_i - K_j = 0$ for all i and j.

III. METRIC & THE FIELD EQUATIONS

Theline element of cylindrically symmetric space-time

$$ds^2 = a^2 dt^2 - dr^2 - b^2\big(d\phi^2 + \sigma^2 dz^2\big) - c^2 d\psi^2 \tag{8}$$

Where a, b and c are functions of radial co-ordinate r and σ is an arbitrary constant.

The Ricci scalar has the form

$$R = \frac{2\ddot{a}}{a} + \frac{4\ddot{b}}{b} + \frac{2\ddot{c}}{c} + \frac{2\dot{b}^2}{b^2} + \frac{4\dot{a}\dot{b}}{ab} + \frac{4\dot{b}\dot{c}}{bc} + \frac{2\dot{a}\dot{c}}{ac} \tag{9}$$

Where dot denotes derivative with respect to r

The stress energy tensor for perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{10}$$

Which satisfying the equation of state

$$p = \omega\rho, \qquad 0 \le w \le 1 \tag{11}$$

The matter density is given by the scalar function ρ and Pressure p

Where $u_i = \delta^5{}_i$ and u_i is five velocity

Fromequation (7), we have

$K_5 - K_1 = 0$, gives

$$\frac{2\dot{a}\dot{b}F}{ab} + \frac{\dot{a}\dot{c}F}{ac} - \frac{2\ddot{b}F}{b} - \frac{\ddot{c}F}{c} + \frac{\dot{a}\dot{F}}{a} - \ddot{F} - \frac{K}{a^2}(p + \rho) = 0 \tag{12}$$

$K_5 - K_2 = K_5 - K_3 = 0$, gives

$$\frac{\ddot{a}F}{a} + \frac{\dot{a}\dot{b}F}{ab} + \frac{\dot{a}\dot{c}F}{ac} + \frac{\dot{a}\dot{F}}{a} - \frac{\ddot{b}F}{b} - \frac{\dot{b}^2 F}{b^2} - \frac{\dot{b}\dot{c}F}{bc} - \frac{\dot{b}\dot{F}}{b} - \frac{K}{a^2}(p + \rho) = 0 \tag{13}$$

Similarly, $K_5 - K_4 = 0$ yield

$$\frac{\ddot{a}F}{a} + \frac{2\dot{a}\dot{b}F}{ab} + \frac{\dot{a}F}{a} - \frac{\ddot{c}F}{c} - \frac{2\dot{b}\dot{c}F}{bc} - \frac{\dot{c}\dot{F}}{c} - \frac{K}{a^2}(p + \rho) = 0 \tag{14}$$

IV. SOLUTIONS OF THE FIELD EQUATIONS

For the simplicity we take $a = 1$ in above equation we get

$$\frac{2\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{F}}{F} + \frac{K}{F}(\rho + p) = 0 \tag{15}$$

$$\frac{\dot{b}\dot{c}}{bc} + \frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{\dot{b}\dot{F}}{bF} + \frac{K}{F}(\rho + p) = 0 \tag{16}$$

$$\frac{\ddot{c}}{c} + \frac{2\dot{b}\dot{c}}{b\dot{c}} + \frac{\dot{c}\dot{F}}{cF} + \frac{K}{F}(\rho + p) = 0 \tag{17}$$

We solve the field equations with the condition of constant scalar curvature
$R = R_0 = const$, i.e. $\dot{F}(R_0) = \ddot{F}(R_0) = 0$

$$\tag{18}$$

With this condition equations (15), (16) and (17) becomes

$$\frac{2\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{K}{F}(\rho + p) = 0 \tag{19}$$

$$\frac{\dot{b}\dot{c}}{bc} + \frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{K}{F}(\rho + p) = 0 \tag{20}$$

$$\frac{\ddot{c}}{c} + \frac{2\dot{b}\dot{c}}{b\dot{c}} + \frac{K}{F}(\rho + p) = 0 \tag{21}$$

By solving the above equations, we get

$$b = k_2 e^{k_1 r}, \quad c = k_4 e^{k_3 r} \tag{22}$$

Where k_1, k_2, k_3, k_4 are constants

Using equation (11) & (22) in equation (19), we get

$$\rho = \frac{-F_0}{1+\omega}\left(\frac{2k_1^2 + k_3^2}{8\pi}\right)^2 \qquad p = \frac{-\omega F_0}{\omega+1}\left(\frac{2k_1^2 + k_3^2}{8\pi}\right) \tag{23}$$

Where $F_0 < 0$ and k_1, k_3, F_0 are constants.

The line element corresponds to the exterior metric of a cosmic string

$$ds^2 = dt^2 - dr^2 - \left(k_2 e^{k_1 r}\right)^2\left(d\phi^2 + \sigma^2 dz^2\right) - \left(k_4 e^{k_3 r}\right)^2 d\psi^2 \tag{24}$$

From equation (4), we get

$$f(R_0) = \frac{2}{5}\left[-8\pi T + R_0 F(R_0)\right] \tag{25}$$

Where $R_0 = 2(k_1 + k_3)^2$

Substituting the values of density, pressure and scalar curvature, above equation leads to

$$f(R_0) = \left[-\frac{2}{5}\left(2k_1^2 + k_3^2\right)\left(6 + k_2^2 e^{2k_1 r} + k_2^2 e^{2k_1 r}\sigma^2 + k_4^2 e^{2k_3 r} - a_0^2\right) + \frac{4}{5}(k_1 + k_3)^2\right]\dot{f}(R_0) \tag{26}$$

$$f(R_0) = \lambda \dot{f}(R_0) \tag{27}$$

Above equation must be satisfied for the acceptability of any *f(R)* model.

Where $\lambda = -\frac{2}{5}\left(2k_1^2 + k_3^2\right)\left(6 + k_2^2 e^{2k_1 r} + k_2^2 e^{2k_1 r}\sigma^2 + k_4^2 e^{2k_3 r} - a_0^2\right) + \frac{4}{5}(k_1 + k_3)^2$

From this we can deduce that and in the present accelerating period, the expansion could start and cosmological constant can be recognized as *f(R₀)*

V. THE LANDAU-LIFSHITZ ENERGY-MOMENTUM COMPLEX

Now, we will calculate energy density for constant curvature solution using Landau-Lifshitz energy momentum complex

The Generalized Landau-Lifshitz EMC is

$$\tau^{\mu\upsilon} = \tau_{LL}^{\mu\upsilon}\dot{f}(R_0) + \frac{1}{6k}\left[R_0\dot{f}(R_0) - f(R_0)\right]\frac{\partial}{\partial x^\lambda}\left(g^{\mu\upsilon}x^\lambda - g^{\mu\upsilon}x^\upsilon\right) \tag{28}$$

Where $\tau_{LL}^{\mu\upsilon}$ is the Landau-Lifshitz EMC measured in general relativity

The 55-component of eq. (28) is

$$\tau^{55} = \tau_{LL}^{55}\dot{f}(R_5) + \frac{1}{6k}\left[\dot{f}(R_5)R_5 - f(R_5)\right]\frac{\partial}{\partial x^\lambda}\left(g^{55}x^\lambda - g^{5\lambda}x^5\right) \tag{29}$$

This becomes

$$\tau^{55} = \tau_{LL}^{55}\dot{f}(R_5) + \frac{1}{6k}\left[\dot{f}(R_5)R_5 - f(R_5)\right]\left(\frac{\partial g^{55}}{\partial x^\lambda}x^\lambda + 3g^{55}\right) \tag{30}$$

$\tau_{LL}^{\mu\upsilon}$ is the Landau-LifshitzEMC measured in GR

$$\tau_{LL}^{\mu\upsilon} = (-g)\left(T^{\mu\upsilon} + t_{LL}^{\mu\upsilon}\right) \tag{31}$$

Now, τ_{LL}^{55} inequation (29), takes the form

$$\tau_{LL}^{55} = (-g)\left(T^{55} + t_{LL}^{55}\right) \tag{32}$$

Where $t_{LL}^{\mu\upsilon}$ is energy momentum pseudotensor.

$$t_{LL}^{\mu\upsilon} = \frac{1}{2k}\begin{bmatrix}\left(2\Gamma_{\alpha\beta}^\gamma\Gamma_{\gamma\sigma}^\sigma - \Gamma_{\alpha\sigma}^\gamma\Gamma_{\beta\gamma}^\sigma - \Gamma_{\alpha\gamma}^\gamma\Gamma_{\beta\sigma}^\sigma\right)\times\left(g^{\mu\alpha}g^{\upsilon\beta} - g^{\mu\upsilon}g^{\alpha\beta}\right)+ \\ g^{\mu\alpha}g^{\beta\gamma}\left(\Gamma_{\alpha\sigma}^\upsilon\Gamma_{\beta\gamma}^\sigma + \Gamma_{\beta\gamma}^\upsilon\Gamma_{\alpha\sigma}^\sigma - \Gamma_{\gamma\sigma}^\upsilon\Gamma_{\alpha\beta}^\sigma - \Gamma_{\alpha\beta}^\upsilon\Gamma_{\gamma\sigma}^\sigma\right)+ \\ g^{\upsilon\alpha}g^{\beta\gamma}\left(\Gamma_{\alpha\sigma}^\mu\Gamma_{\beta\gamma}^\sigma + \Gamma_{\beta\gamma}^\mu\Gamma_{\alpha\sigma}^\sigma - \Gamma_{\gamma\sigma}^\mu\Gamma_{\alpha\beta}^\sigma - \Gamma_{\alpha\beta}^\mu\Gamma_{\gamma\sigma}^\sigma\right)+ \\ g^{\alpha\beta}g^{\gamma\sigma}\left(\Gamma_{\alpha\gamma}^\mu\Gamma_{\beta\sigma}^\upsilon - \Gamma_{\alpha\beta}^\mu\Gamma_{\gamma\sigma}^\upsilon\right)\end{bmatrix} \tag{33}$$

From equation (33), we can obtain t_{LL}^{55} as

$$t_{LL}^{55} = \frac{1}{2k}\begin{bmatrix}\left(2\Gamma_{\alpha\beta}^\gamma\Gamma_{\gamma\sigma}^\sigma - \Gamma_{\alpha\sigma}\Lambda_{\beta\gamma}^\sigma - \Gamma_{\alpha\gamma}^\gamma\Gamma_{\beta\sigma}^\sigma\right)\times\left(g^{5\alpha}g^{5\beta} - g^{55}g^{\alpha\beta}\right) \\ + g^{5\alpha}g^{\beta\gamma}\left(\Gamma_{\alpha\sigma}^5\Gamma_{\beta\gamma}^\sigma + \Gamma_{\beta\gamma}^5\Gamma_{\beta\gamma}^5\Gamma_{\alpha\sigma}^\sigma - \Gamma_{\gamma\sigma}^5\Gamma_{\alpha\beta}^\sigma - \Gamma_{\alpha\beta}^5\Gamma_{\gamma\sigma}^\sigma\right)+ \\ g^{5\alpha}g^{\beta\gamma}\left(\Gamma_{\alpha\sigma}^5\Gamma_{\beta\gamma}^\sigma + \Gamma_{\beta\gamma}^5\Gamma_{\alpha\sigma}^\sigma - \Gamma_{\gamma\sigma}^5\Gamma_{\alpha\beta}^\sigma - \Gamma_{\alpha\beta}^5\Gamma_{\gamma\sigma}^\sigma\right)+ \\ g^{\alpha\beta}g^{\gamma\sigma}\left(\Gamma_{\alpha\gamma}^5\Gamma_{\beta\sigma}^5 - \Gamma_{\alpha\beta}^5\Gamma_{\gamma\sigma}^5\right)\end{bmatrix} \tag{34}$$

$\alpha,\beta,\gamma,\sigma = 1,2,3,4,5.$

The final expression of t_{LL}^{55} takes the form

$$t_{LL}^{55} = \frac{-5k_1k_3}{8\pi a_0^2} \tag{35}$$

Substituting this value in equation (32) and then inserting the obtained equation in (30) it follows

$$\tau^{55} = -\left(k_2^4 k_4^2 e^{2r(k_1+2k_3)}a_0^2\sigma^2\right)\dot{f}(R_0)\left(\frac{p+\rho}{a_0^4} - \frac{\rho}{a_0^2} - \frac{5k_1k_3}{8\pi a_0^2}\right) + \frac{1}{16\pi a_0^2}\left[\dot{f}(R_0)R_0 - f(R_0)\right] \tag{36}$$

This is the energy density satisfying the condition of constant scalar curvature with validity and stability conditions.

VI. *f(R)* MODELS WITH VALIDITY AND STABILITY CONDITIONS

In this section, we mainly consider four different *f(R)* models to study the energy momentum distribution
First, we consider the *f(R)* model which is given by (Nojiri and Odintsov 2007b, 2008)

$$f(R) = R - (-1)^{n-1}\frac{A}{R^n} + (-1)^{m-1}BR^m \tag{37}$$

Where m , n are positive integer and A , B any real numbers.

Substituting the value of R_0 in equation (37), we get

$$f(R_0) = 2(k_1 + k_3)^2 - (-1)^{n-1} \frac{A}{2^n (k_1 + k_3)^{2n}} + (-1)^{m-1} B\left(2^m (k_1 + k_3)^{2m}\right) \tag{38}$$

$$\dot{f}(R_0) = 1 + (-1)^{n-1} \frac{nA}{2.2^n (k_1 + k_3)^{2n+2}} + m(-1)^{m-1} B\left(\frac{2^m}{2} (k_1 + k_3)^{2m-2}\right) \tag{39}$$

$$\ddot{f}(R_0) = \frac{-n(n+1)(-1)^{n-1} A}{2^n . 4(k_1 + k_3)^{2(n+2)}} \leq 0 \tag{40}$$

Inserting these in equation (27), the constant curvature condition yields

$$2(k_1 + k_3)^2 + \frac{(-1)^n A}{2^n (k_1 + k_3)^{2n}}\left[1 + \frac{n\lambda}{2(k_1 + k_3)^2}\right] - (-1)^m 2^m B(k_1 + k_3)^{2m}\left[1 - \frac{1}{2(k_1 + k_3)^2}\right] - \lambda = 0 \tag{41}$$

Particularly for $B = 0$

$$A = \frac{2^n (k_1 + k_3)^{2n}\left[\lambda - 2(k_1 + k_3)^2\right]}{(-1)^n \left[1 + \dfrac{n\lambda}{2(k_1 + k_3)^2}\right]} \tag{42}$$

For this first model $\ddot{f}(R) \leq 0$ is the stability condion and for this model it is given by equation (40). It is noted that a stability condition is satisfied for value of A in equation (42).

Energy density for the first *f(R)* model takes the form

$$\tau^{55} = -a_0{}^2 \sigma^2 k_2{}^4 k_4{}^2 e^{2r(k_1 + 2k_3)}\left(1 + (-1)^{n-1} \frac{nA}{\left(2(k_1 + k_3)^2\right)^{n+1}} + m(-1)^{m-1} B\left(2(k_1 + k_3)^2\right)^{m-1}\right)$$

$$\left(\frac{p + \rho}{a_0{}^4} - \frac{\rho}{a_0{}^2} - \frac{5k_1 k_3}{8\pi a_0{}^2}\right) + \frac{1}{16\pi a_0{}^2}\left(\frac{(n+1)A(-1)^{n-1}}{\left(2(k_1 + k_3)^2\right)^n} + B(m-1)(-1)^{m-1}\left(2(k_1 + k_3)^2\right)^m\right) \tag{43}$$

Now we discuss the second *f(R)* model is in the form of

$$f(R) = R - A\log\left(\frac{|R|}{K}\right) + (-1)^{m-1} BR^m \tag{44}$$

Which is given by Nojiri and Odintsov2004

Where K is a positive real number and m is a positive integer.

This model is depending on the curvature in the logarithmic form

For constant curvature above model gives

$$f(R_0) = 2(k_1 + k_3)^2 - A\log\frac{2(k_1 + k_3)^2}{K} + (-1)^{m-1} B\left(2(k_1 + k_3)^2\right)^m \tag{45}$$

$$\dot{f}(R_0) = 1 - \frac{A}{2(k_1 + k_3)^2} + mB(-1)^{m-1}\left(2(k_1 + k_3)^2\right)^{m-1} \tag{46}$$

$$\ddot{f}(R_0) = \frac{A}{R^2} + m(m-1)B(-1)^{m-1} R^{m-2} \tag{47}$$

The constant curvature condition (27) yields

$$2(k_1 + k_3)^2 + A\left[\frac{\lambda}{2(k_1+k_3)^2} - \log\frac{2(k_1+k_3)^2}{K}\right] - (-1)^m 2^m B(k_1+k_3)^{2m}\left[1 + \frac{m}{2(k_1+k_3)^2}\right] - \lambda = 0$$

(48)

For $B = 0$, this reduces to

$$A = \frac{\lambda - 2(k_1+k_3)^2}{\dfrac{\lambda}{2(k_1+k_3)^2} - \log\dfrac{2(k_1+k_3)^2}{K}}$$

(49)

Stability condition for this model is $\ddot{f}(R_0) > 0$ and equation (47) is $\ddot{f}(R_0) > 0$ for $B = 0$ and it becomes

$$\frac{A}{4(k_1+k_3)^4} > 0$$

(50)

For the this*f(R)* model energy density is given by

$$\tau^{55} = -a_0^2\sigma^2 k_2^4 k_4^2 e^{2r(k_1+2k_3)}\left(1 - \frac{A}{2(k_1+k_3)^2} + mB(-1)^{m-1}\left(2(k_1+k_3)^2\right)^{m-1}\right)\left(\frac{p+\rho}{a_0^4} - \frac{\rho}{a_0^2} - \frac{5k_1k_3}{8\pi a_0^2}\right) +$$

$$\frac{1}{16\pi a_0^2}\left[A\log\frac{2(k_1+k_3)^2}{K} - A + B(-1)^{m-1}(m-1)\left(2(k_1+k_3)^2\right)^m\right]$$

(51)

Now the third *f(R)* modelis defined as

$$f(R) = R + \varepsilon R^2$$

(52)

Where ε is a positive real number

This is super gravity inspired model (Noakes 1983)

It is also called the inflation model realized by the term R^2. The model has stability criteria which is bounded to $\varepsilon > 0$ i.e $\ddot{f}(R) > 0$ Einstein theory is recovered if $\varepsilon = 0$. stability condition for this model is $\ddot{f}(R) > 0$.

Therefore

$$f(R_0) = 2(k_1+k_3)^2\left[1 + 2\varepsilon(k_1+k_3)^2\right]$$

(53)

$$\dot{f}(R_0) = 1 + 4\varepsilon(k_1+k_3)^2$$

(54)

$$\ddot{f}(R_0) = 2\varepsilon$$

(55)

This model fulfills the condition for constant scalar curvature for the value $(k_1+k_3)^2 = \dfrac{\lambda}{2\left[1 + 2\varepsilon(k_1+k_3)^2 - \lambda\right]}$

and satisfies the stability condition given by Faraoni and Nadeau (2005)

$$\frac{1}{\varepsilon(1+2\varepsilon R_0)} = \frac{1 + 2\varepsilon(k_1+k_3)^2 - \lambda}{\varepsilon\left[1 + 2\varepsilon(k_1+k_3)^2 - \lambda(1-2\varepsilon)\right]} > 0$$

(56)

The 55-component of the generalized EMC becomes.

$$\tau^{55} = -a_0^2\sigma^2 k_2^4 k_4^2 e^{2r(k_1+2k_3)}\left(1 + 4\varepsilon(k_1+k_2)^2\right)\left(\frac{p+\rho}{a_0^4} - \frac{\rho}{a_0^2} - \frac{5k_1k_3}{8\pi a_0^2}\right) + \frac{1}{16\pi a_0^2}4\varepsilon(k_1+k_3)^4$$

(57)

f(R) modelby Nojiri and Odintsov(2003) is

$$f(R) = R - \frac{A}{R} - BR^2$$

(58)

where A and B are real numbers

By using the value of R_0, we get

$$f(R_0) = 2(k_1 + k_3)^2 - \frac{A}{2(k_1 + k_3)^2} - B\left(4(k_1 + k_3)^4\right) \tag{59}$$

$$\dot{f}(R_0) = 1 + \frac{A}{4(k_1 + k_3)^4} - 4B(k_1 + k_3)^2 \tag{60}$$

$$\ddot{f}(R_0) = A + 8B(k_1 + k_3)^6 \tag{61}$$

Constant curvature condition gives the relation between A and B as

$$2(k_1 + k_3)^2 - \frac{A}{2(k_1 + k_3)^2}\left[1 + \frac{\lambda}{2(k_1 + k_3)^2}\right] - 4B(k_1 + k_3)^2\left[(k_1 + k_3)^2 - \lambda\right] - \lambda = 0 \tag{62}$$

The condition for stability i.e. $\ddot{f}(R_0) \leq 0$ yields equation (61) and for positive real number A, this model is acceptable.

Equation no. (29) takes the form

$$\tau^{55} = -a_0{}^2\sigma^2 k_2{}^4 k_4{}^2 e^{2r(k_1 + 2k_3)}\left(1 + \frac{A}{4(k_1 + k_3)^4} - 4B(k_1 + k_3)^2\right)\left(\frac{p + \rho}{a_0{}^4} - \frac{\rho}{a_0{}^2} - \frac{5k_1 k_3}{8\pi a_0{}^2}\right)$$

$$+ \frac{1}{16\pi a_0{}^2}\left(\frac{A}{(k_1 + k_3)^2} - 4B(k_1 + k_3)^4\right) \tag{63}$$

This model holds the negative power of curvature which corresponds to cosmic acceleration of the universe.

VII. CONCLUSION

In this paper we investigate five dimensional static cylindrically symmetric solutions in *f(R)* theory of gravity with perfect fluid as matter. Assuming constant scalar curvature we have obtained the solution. Then generalized Landau-Lifshitz energy momentum complex has been used to discuss the energy distribution of the solution. Energy density obtained is used to explore four *f(R)* models. It is found that stability and constant scalar curvature condition of the *f(R)* models are satisfied for obtained solution. Model having negative power of the scalar curvature support to the cosmic acceleration and model having positive power of scalar curvature can be improved. Large amount of result obtained here by using Landau Lifshitz energy momentum complex suggest that it is a very good tool for energy momentum localization. These models may be useful for studying dark energy and dark matter stages in *f(R)* gravity.

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