

Onset of Oscillatory Convection in a Sparsely Packed Porous Layer Saturated with Viscoelastic Fluid

Geethu Varghese^{#1}, Smita S Nagouda^{#2}, Nalinakshi N^{#3}, Faiz Imam^{*4}

¹Department of Mathematics, CHRIST(Deemed to be University), Bengaluru-560029, India

²Department of Mathematics, CHRIST(Deemed to be University), Bengaluru-560029, India

³Department of Mathematics, Atria Institute of Technology, Bengaluru – 560024, India

⁴Department of Mathematics, BITS Pilani-HyderabadCampus, Hyderabad-500078, India

Abstract

The onset of oscillatory convection in a horizontal sparsely packed porous layer saturated with viscoelastic fluid which is heated from below is analyzed. A linear stability analysis based on the normal mode technique is used to study the eigen value problem with the free-free and isothermal boundaries. Darcy-Rayleigh number on the onset of stationary and oscillatory convection has been derived and effects of various viscoelastic properties and Darcy number have been examined.

Keywords

viscoelastic fluid, Darcy-Rayleigh number, Darcy-Brinkman model, Deborah number, relaxation time, Darcy number.

Mathematics Subject Classification-76A10

I. INTRODUCTION

The study of viscoelastic fluid flow in porous media is seen in various engineering fields such as enhanced oil recovery, paper and textile coating, composite manufacturing process and bioengineering. On one hand, the high viscosity in viscoelastic fluids reduces the chances of occurrence of instability, its elastic nature, while on the other hand, it increases the chances of oscillatory convection. This phenomenon in viscoelastic fluids apart from its rheological importance makes the study of the Rayleigh-Bénard convection for viscoelastic fluids interesting and challenging to the researchers. Some oil sands contain waxy crudes at shallow depth of the reservoirs which are considered to be viscoelastic fluids. In these situations, a viscoelastic model of fluid serves to be more realistic than the Newtonian model. Besides this viscoelastic fluid exhibits unique patterns of instabilities such as the overstability that is not predicted or observed in a Newtonian flow.

Herbert [1] and Green [2] were the first to study the oscillatory convection in an ordinary viscoelastic fluid of the Oldroyd type under the condition of infinitesimal disturbances. Vest and Arpacı [3] studied overstability in a viscoelastic fluid layer heated from below, and obtained the condition for the onset of thermally induced overstability. Bhatia and Steiner [4] studied convective instability in a rotating viscoelastic fluid layer, considering two sets of boundary conditions. They found destabilizing effects of rotation instead of stabilizing as in the case of Newtonian fluid. Bhatia and Steiner [5,6] studied the effect of magnetic field on oscillatory convection in a viscoelastic fluid layer.

Kim *et al.* [7] studied the thermal instability of viscoelastic fluids in porous media, conducted linear analysis for obtaining stability criteria for convective flow, and nonlinear analysis for heat transfer. Malashetty *et al.* [8] have analyzed that the effect of thermal modulation on the onset of convection in a horizontal, anisotropic porous layer saturated by a viscoelastic fluid. Tan and Masuoka [9] studied the stability of a Maxwell fluid in a porous medium using modified Darcy-Brinkman- Maxwell model and found the criterion for the onset of oscillatory convection. Malashetty *et al.* [10] investigated the onset of convection in a viscoelastic fluid-saturated anisotropic porous layer, using a generalized Darcy model, and studied the effects of mechanical and thermal anisotropic parameters. Laroze *et al.* [11] have investigated this problem of viscoelastic flow in a rotating porous medium and obtained the convective thresholds for the case of rigid boundary conditions.

Patil and Vaidyanathan [12] have studied thermal convection in a rotating fluid saturated porous layer under the influence of variable viscosity using the Brinkman model. Shivakumara *et al.* [13] have investigated linear and weakly nonlinear thermal convection in a rotating porous layer and they have shown that decrease in the permeability and increase in the effective viscosity of the fluid have a destabilizing effect on the onset of

stationary convection at high rotation rates. Recently, Falsaperla *et al.* [14] have considered the problem of thermal convection in a rotating horizontal layer of porous medium with Newton-Robin type of temperature boundary conditions.

There are some works related to thermal stability in viscoelastic fluid saturated porous media; Rudraiah *et al.* [15] studied the stability of a viscoelastic fluid in a densely packed saturated porous layer considering an Oldroyd model. Yoon *et al.* [16,17] made a linear stability analysis to study convection in a viscoelastic fluid saturated porous layer, and obtained the expression of Darcy-Rayleigh number for oscillatory case to describe the onset of convection. Bertola and Cafaro [18] studied theoretically the stability of viscoelastic fluid heated from below. Sheu *et al.* [19] analysed chaotic convection for viscoelastic fluids, using truncated Galerkin expansion. Kumar and Bhadauria [20] studied thermal instability in a rotating viscoelastic fluid saturated porous layer, and calculated the heat transfer. Also Kumar and Bhadauria [21] studied linear and nonlinear double diffusive convection in a viscoelastic fluid saturated porous layer. Further, they [22] studied double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid and studied heat and mass transfer across the fluid layer.

Due to geographical and pedagogical processes like sedimentation, compactation, frost action and reorientation of the solid matrix, inhomogeneity and anisotropy are characteristics of most of the natural porous materials. It is to be noted that early studies on convection in a porous medium have usually ignored these aspects of porous materials. There are artificial porous media encountered in numerous systems in industries as well like pelleting used in chemical engineering process, fiber material used for insulating purpose and many more. Despite the practical importance of the topic, very few studies are reported on the Rayleigh-Bénard convection for anisotropic porous material. Epherre [23] performed the first study of the onset of convection in a horizontal layer with an anisotropic permeability. Tyvand [24] studied the problem of thermohaline instability in anisotropic porous media.

Since not much research has been carried out in the case of sparsely packed porous medium until recently, the present work is carried out to examine the effect of viscoelastic properties on oscillatory behavior of convective instabilities in a horizontal sparsely packed porous layer heated from below using linear theory.

II. MATHEMATICAL FORMULATION

Consider an infinite horizontal porous layer saturated with viscoelastic fluid of depth ‘d’ confined with two rigid boundaries. The bottom boundary is heated slowly with a constant temperature T_1 and upper boundary temperature is kept at a lower temperature T_2 . Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards. Let ΔT be the temperature difference between the upper and lower boundaries.

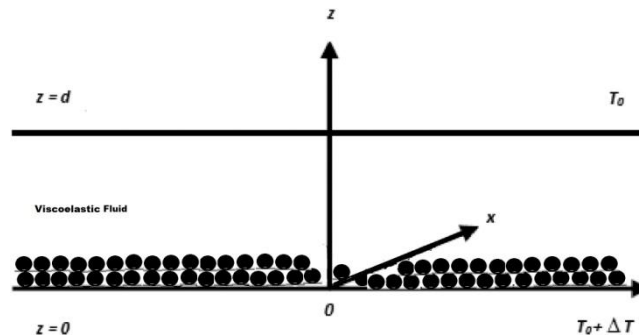


Fig. 1 Physical Configuration of the problem.

The governing equations for an incompressible viscoelastic fluid under the Boussinesq approximation are

Equation of Continuity:

$$\nabla \cdot \vec{U} = 0 \quad (1)$$

Equation of Linear Momentum:

$$\left(\epsilon \frac{\partial}{\partial t} + 1\right) \vec{U} = -\frac{K}{\mu} \left(\lambda \frac{\partial}{\partial t} + 1\right) (\nabla P + \rho g \hat{k} - \bar{\mu} \nabla^2 \vec{U}) \quad (2)$$

Equation of Energy:

$$\left(\frac{\partial}{\partial t} + \vec{U} \cdot \nabla\right) = \alpha \nabla^2 T \quad (3)$$

Equation of State:

$$\rho = \rho_0(1 - \beta(T - T_0)) \quad (4)$$

where, \vec{U} is the velocity vector, P is the pressure, T is the temperature, ϵ is the retardation time, λ is the relaxation time, μ is the viscosity, ρ is the density, β is the thermal expansion coefficient, K is the permeability of porous media, α is the effective thermal diffusivity, g is the gravitational acceleration and $\bar{\mu}$ is the effective viscosity.

A. Basic State:

The basic state of the fluid is quiescent and the quantities in this state are given by

$$\vec{U}_b = (0, 0, 0), \quad P = P_b(z), \quad T = T_b(z), \quad \rho = \rho_b(z) \quad (5)$$

where the suffix ‘b’ represents the quantity in the basic state.

Substituting (5) in (1)-(4) we get the following relations,

$$\frac{dP_b}{dz} + \rho_b g = 0 \quad (6)$$

$$\frac{d^2 T_b}{dz^2} = 0 \quad (7)$$

$$\rho_b = \rho_0 [1 - \beta (T_b - T_0)] \quad (8)$$

The solution of (7), subjected to boundary conditions

$$T_1 = T_0 + \Delta T, \text{ at } z=0$$

$$T_2 = T_0, \text{ at } z=d \quad \text{is given by}$$

$$T_b = \left(-\frac{\Delta T}{d}\right)z + T_1 \quad (9)$$

B. Perturbed State:

Let the basic state be superimposed by infinitesimally small disturbance in the following form.

$$\vec{U} = \vec{U}_b + \vec{U}', \quad \bar{\rho} = \bar{\rho}_b + \bar{\rho}', \quad P = P_b + P', \quad T = T_b + T' \quad (10)$$

where the primes denote perturbed quantities.

Using (10) in (1)-(4) and using the basic state equations, the linearized equations governing small disturbances turn out to be

$$\left(\epsilon \frac{\partial}{\partial t} + 1\right) \vec{w}' = -\frac{K}{\mu} (\nabla \cdot \vec{U}') = 0 \quad (11)$$

$$(12)$$

$$\frac{\partial T'}{\partial t} = w' \frac{\Delta T}{d} + \alpha \nabla^2 T' \quad (13)$$

$$\rho' = -\rho_0 \beta T' \quad (14)$$

Substituting (14) in (12) and considering the k^{th} component,

$$\left(\epsilon \frac{\partial}{\partial t} + 1\right) \vec{w}' = -\frac{K}{\mu} \left(\lambda \frac{\partial}{\partial t} + 1\right) (\nabla P' - \rho_0 \beta T' g - \bar{\mu} \nabla^2 \vec{w}') \quad (15)$$

Non-dimensionalising (13) and (15) using the length, time, velocity and temperature scales as given below

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right); T^* = \frac{T'}{\Delta T}; w^* = \frac{w'}{\alpha/d}; t^* = \frac{t}{d^2/\alpha}; \quad (16)$$

Substituting (16) in (13) and (15) and operating curl twice on (15) and dropping asterisks we obtain,

$$\left(\bar{\epsilon} \frac{\partial}{\partial \tau} + 1\right) (\nabla^2 w) = \left(\bar{\lambda} \frac{\partial}{\partial \tau} + 1\right) (\Lambda Da \nabla^4 w + Ra_D \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \theta) \quad (17)$$

$$\frac{\partial \theta}{\partial t} = w + \nabla^2 \theta \quad (18)$$

where θ is the dimensionless temperature and the non-dimensional parameters are defined as,

Darcy-Rayleigh number, $Ra_D = \frac{K \beta d \Delta T g}{\alpha \gamma}$

Dimensionless retardation time, $\bar{\epsilon} = \frac{\alpha}{d^2} \epsilon$

Darcy Number, $Da = \frac{K}{d^2}$

Ratio of the viscosities, $\Lambda = \frac{\bar{\mu}}{\mu}$

Deborah number or dimensionless relaxation time, $\bar{\lambda} = \frac{\alpha}{d^2} \lambda$

Kinematic Viscosity, $\gamma = \frac{\mu}{\rho_0}$

The appropriate boundary conditions for (17) and (18) are given by

$$w = \theta = 0 \quad \text{at} \quad z = 0, 1. \quad (19)$$

III. STABILITY ANALYSIS

Under the Normal mode analysis, we assume the time dependent periodic disturbances in a horizontal plane of the form

$$w = w(z) e^{ilx + imy + \sigma t} \quad (20)$$

$$\theta = \theta(z) e^{ilx + imy + \sigma t} \quad (21)$$

where l and m are the dimensionless wave numbers in the x and y directions and σ is the growth rate. The stability governing equations can be obtained by introducing (20) and (21) in (17) and (18), noting that the principle of exchange of stabilities is valid and is given by

$$(D^2 - a^2) \theta \left((D^2 - a^2) - \sigma \right) \left((1 + \bar{\epsilon} \sigma) - (1 + \bar{\lambda} \sigma) \Lambda Da (D^2 - a^2) \right) = (1 + \bar{\lambda} \sigma) Ra_D a^2 \theta \quad (22)$$

$$\sigma \theta - w = (D^2 - a^2) \theta \quad (23)$$

where $D = \frac{d}{dz}$ is the differential operator and $a = \sqrt{l^2 + m^2}$ is the horizontal wavenumber.

In view of (20) and (21), the boundary conditions (19) take the form

$$\theta = D^2 \theta = 0 \quad \text{at} \quad z = 0, 1 \quad (24)$$

Both boundaries are isothermal and free of viscous stresses in Darcy-Brinkman law.

The eigenvalue problem given by (22) and (24) involving $Ra_D, a, \bar{\epsilon}, \sigma, \bar{\lambda}$ and Da as parameters is solved upon

assuming

$$\theta = \theta_0 \sin(n\pi z) \quad \text{for} \quad n = 1, 2, 3, \dots \quad (25)$$

where θ_0 denotes the integration constant subject to the boundary conditions. Substituting (25) in (22) results in the following characteristic equation:

$$(-n^2\pi^2 - a^2)(-n^2\pi^2 - a^2 - \sigma)[(1 + \bar{\epsilon}\sigma) - (1 + \bar{\lambda}\sigma)\Lambda Da(-n^2\pi^2 - a^2)] - Ra_D a^2(1 + \bar{\lambda}\sigma) = 0 \quad (26)$$

The algebraic expression can be rearranged into the form of the 2nd order polynomials of σ as

$$\begin{aligned} & [\bar{\epsilon}(n^2\pi^2 + a^2) + \bar{\lambda}\Lambda Da(n^2\pi^2 + a^2)^2]\sigma^2 + [\bar{\epsilon}(n^2\pi^2 + a^2)^2 + \bar{\lambda}(n^2\pi^2 + a^2)^2\Lambda Da(n^2\pi^2 + a^2) \\ & + (n^2\pi^2 + a^2) + (n^2\pi^2 + a^2)\Lambda Da(n^2\pi^2 + a^2) - Ra_D a^2\bar{\lambda}]\sigma + (n^2\pi^2 + a^2)^2 + \Lambda Da(n^2\pi^2 + a^2)^2 \\ & (n^2\pi^2 + a^2) - Ra_D a^2 = 0 \end{aligned} \quad (27)$$

This equation is denoted symbolically as

$$A_1\sigma^2 + A_2\sigma + A_3 = 0. \quad (28)$$

where,

$$A_1 = \bar{\epsilon}(n^2\pi^2 + a^2) + \bar{\lambda}\Lambda Da(n^2\pi^2 + a^2)^2$$

$$A_2 = \bar{\epsilon}(n^2\pi^2 + a^2)^2 + \bar{\lambda}(n^2\pi^2 + a^2)^2\Lambda Da(n^2\pi^2 + a^2) + (n^2\pi^2 + a^2) + (n^2\pi^2 + a^2)\Lambda Da(n^2\pi^2 + a^2) - Ra_D a^2\bar{\lambda}$$

$$A_3 = (n^2\pi^2 + a^2)^2 + \Lambda Da(n^2\pi^2 + a^2)^2(n^2\pi^2 + a^2) - Ra_D a^2$$

A. Stationary Convection:

The onset of stationary convection ($\sigma = 0$) occurs at the following condition:

$$A_3 = 0 \quad (29)$$

From the above condition we obtain the Darcy-Rayleigh number for stationary mode as

$$Ra_{D(stat)} = \frac{(a^2 + n^2\pi^2)^2 [1 + \Lambda Da(n^2\pi^2 + a^2)]}{a^2} \quad (30)$$

Moreover, in the **limiting case** when $Da=0$ we obtain

$$Ra_{D(stat)} = \frac{(a^2 + n^2\pi^2)^2}{a^2} \quad (31)$$

which is the well-known expression in the context of Darcy porous medium convection in a Newtonian Fourier fluid (Nield and Bejan, [25]). We find that $Ra_{D(stat)}$ assumes its minimum value at $4\pi^2$ at critical wavenumber $a_c = \pi$.

B. Oscillatory convection:

From the elementary theory of algebraic equations, it is apparent that a marginally oscillatory mode (i.e. $\sigma = i\sigma_i$) occurs at the following conditions:

$$A_1 A_3 > 0, \quad A_2 = 0 \quad (32)$$

Substituting $\sigma = i\sigma_i$ in (28) and equating the imaginary part to 0, we obtain the Darcy-Rayleigh number for oscillatory modes.

$$Ra_{D(osc)} = \frac{(a^2 + n^2\pi^2)[\bar{\epsilon}(n^2\pi^2 + a^2) + \Lambda Da(n^2\pi^2 + a^2)[\bar{\lambda}(n^2\pi^2 + a^2) + 1] + 1]}{a^2\bar{\lambda}} \quad (33)$$

In the **limiting case** of $Da=0$, one recovers the result of Darcy-Rayleigh number of Do-Young Yoon, Min Chan Kim and Chang Kyun Choi [16], and the associated Darcy-Rayleigh number Ra_D is given by,

$$Ra_{D(osc)} = \frac{(a^2 + n^2\pi^2)^2\bar{\epsilon} + (n^2\pi^2 + a^2)}{a^2\bar{\lambda}} \quad (34)$$

In this case $Ra_{D(osc)}$ assumes its minimum value at the critical wavenumber $a_c^4 = \pi^4 + \frac{\pi^2}{\bar{\epsilon}}$ on $\frac{\partial Ra_D}{\partial a^2} = 0$.

IV. RESULTS AND DISCUSSIONS

In this work, we carry out a study of heat transport for oscillatory convection in a horizontal sparsely packed porous medium saturated with viscoelastic fluid. In order to illustrate the effects of relaxation time ($\bar{\lambda}$), retardation time ($\bar{\epsilon}$) and Darcy number (Da) on heat transport, we plot the marginal stability curves for Darcy-Rayleigh number versus wavenumber where our consideration is confined to the lowest order mode, $n=1$. The oscillatory type of instability is possible only when the relaxation parameter ($\bar{\lambda}$) is greater than the retardation parameter ($\bar{\epsilon}$).

Variations of Darcy-Rayleigh number with wavenumber for different relaxation time when $\bar{\epsilon}=1$, $Da=0.5$ and $\Lambda =0.4$ are plotted in Fig. 2 for representing the exchange of stabilities and overstability at the marginal state. The dimensionless relaxation time $\bar{\lambda}$ means the usual Deborah number used in rheology for characterization of the fluid and material. The larger the Deborah number, the material behaviour enters the non-Newtonian regime, increasingly dominated by elasticity and exhibiting solidlike behaviour. The solid curve represents the Rayleigh number for oscillatory convection as a function of wave number, while the broken curves represent the same for stationary convection. It is clear from (33) that the oscillatory convection depends on both relaxation time, retardation time, Darcy number where in the permeability factor is involved and ratio of the viscosities of the porous medium. The marginal stability curve for oscillatory convection is located below the Newtonian fluid case for $\bar{\epsilon} = \bar{\lambda}$. It is observed that the Darcy-Rayleigh number decreases with increase in the value of relaxation time for fixed values of $\bar{\epsilon}$, Da and Λ indicating that the effect of increasing relaxation time is to destabilize the system. It is observed that the overstability curves lie below that of the steady convection, that is, $Ra_{D(osc)} < Ra_{D(stat)}$ for the same wavenumber. This implies that the oscillatory convection sets in before stationary convection.

Variations of Darcy Rayleigh number with wavenumber for different retardation time when $\bar{\epsilon}=1$, $Da=0.2$ and $\Lambda =0.4$ are plotted in Fig.3. We observe that viscoelastic fluids with higher value of retardation time exhibits overstability at higher Darcy-Rayleigh number, it increases with increasing retardation time, which has a stabilizing effect on the system. It is interesting that the criteria for the overstability are dependent on the values of viscoelastic properties of the saturated liquid.

Variations of Darcy-Rayleigh number with wavenumber for different Darcy number for fixed $\Lambda =0.4$ and $\bar{\epsilon}=\bar{\lambda}=0.5, 1, 1.5$ respectively are plotted from Fig. 4-6. The effect of Darcy number on the onset of convection is also important, as many practically used viscoelastic fluids have large Darcy numbers. The Darcy number represents the scale factor, which describes the extent of division of porous structure(permeability) as compared to the vertical extent of the porous layer. When the permeability is very high, the resistance to the flow becomes effectively controlled by ordinary viscous resistance. We see that as Da decreases, $Ra_{D(osc)}$ decreases and it is found that the decrease in the Darcy number has a destabilizing effect on the system and the effect of increase in Darcy number contracts the dimension of the convective cells. It is also seen that the increase in the value of relaxation as well as retardation time in these cases advances the onset of convection.

Fig. 7 depicts the variations of Darcy Rayleigh number with wavenumber for different Λ with fixed value of $Da=0.8$ and $\bar{\epsilon}=\bar{\lambda}=1.25$. As Λ increases, $Ra_{D(osc)}$ increases and therefore the increase in the ratio of the viscosities has a stabilizing effect on the system.

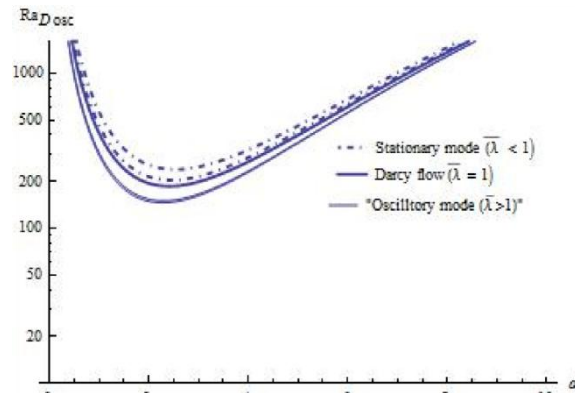


Fig. 2 Oscillatory Darcy- Rayleigh number against wavenumber for given $\bar{\epsilon}=1$, $Da=0.5$ and $\Lambda=0.4$ with different relaxation time.

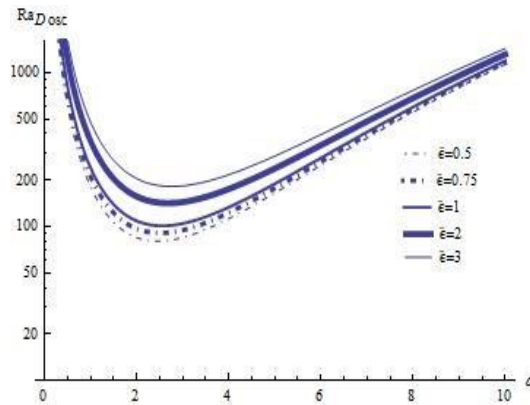


Fig. 3 Oscillatory Darcy- Rayleigh number against wavenumber for given $\bar{\epsilon}=1$, $Da=0.2$ and $\Lambda=0.4$ with different retardation time.

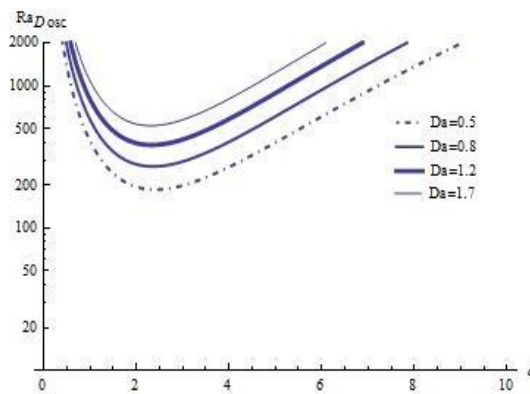


Fig. 4 Oscillatory Darcy-Rayleigh number against wavenumber for given $\Lambda=0.4$ and $\bar{\epsilon}=\bar{\lambda}=0.5$ with different Darcy number

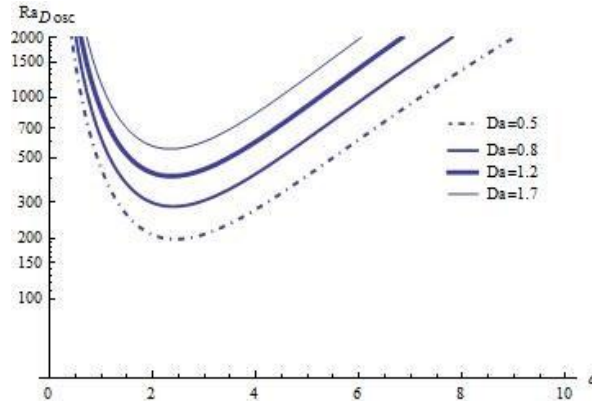


Fig. 5 Oscillatory Darcy- Rayleigh number against wavenumber for given $\Lambda = 0.4$ and $\bar{\epsilon} = \bar{\lambda} = 1$ with different Darcy number

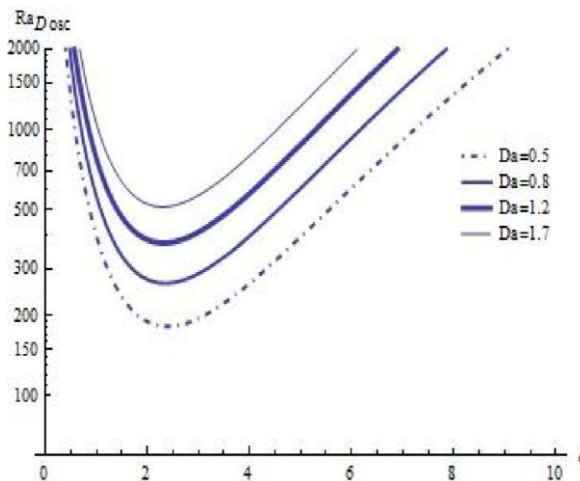


Fig. 6 Oscillatory Darcy-Rayleigh number against wavenumber for given $\Lambda = 0.4$ and $\bar{\epsilon} = \bar{\lambda} = 1.5$ with Darcy number

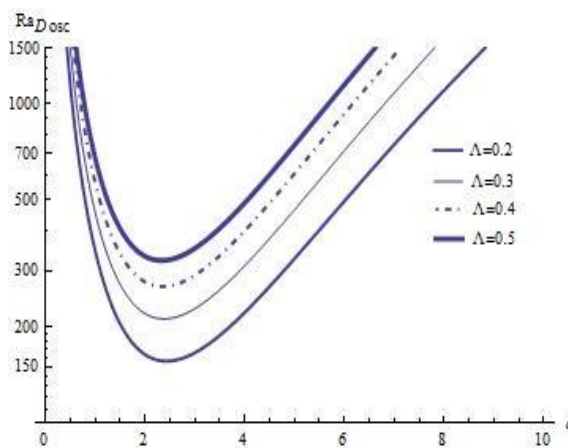


Fig. 7 Oscillatory Darcy- Rayleigh number against wavenumber for given $Da=0.8$ and $\bar{\epsilon} = \bar{\lambda} = 1.25$ with different Λ .

IV. CONCLUSIONS

Viscoelastic fluids show different behaviors of evolving convective instabilities. As the elasticity of viscoelastic fluids allows the periodic instability to be sustained in addition to the stationary modes, these fluids exhibit an oscillatory convection at the threshold of stationary mode. We also see that the oscillatory convection sets in early as compared to the study carried out by Do-Young Yoon, Min Chan Kim and ChangKyun Choi [16] in a horizontal darcian porous layer saturated with viscoelastic fluid. The following conclusions are made:

1. The increase in relaxation time is to advance the onset of convection, and hence enhance the heat transport.
2. The increase in retardation time is to delay the onset of convection, and hence decrease the heat transport.
3. The oscillatory Darcy-Rayleigh number depends on $\bar{\epsilon}$, $\bar{\lambda}$, Λ and Da but in stationary case it is independent of $\bar{\epsilon}$ and $\bar{\lambda}$.
4. The effect of decrease in Darcy number where the permeability factor is involved is to advance the onset of convection.
5. The increase in ratio of the viscosities of the porous medium is to delay the onset of convection.

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NOMENCLATURE

- a: dimensionless horizontal wave number, $a = \sqrt{l^2 + m^2}$
 D : differential operator $[\frac{d}{dz}]$
 g : gravitational acceleration $[m/s^2]$
 K : permeability $[m^2]$
 k : unit vector in vertical direction
 d : depth of porous layer $[m]$
 P : pressure $[Pa]$
 Ra_D : Darcy-Rayleigh number, $\frac{K\beta d \Delta T g}{\alpha \gamma}$
 T : Temperature $[^\circ C]$
 t : time $[s]$
 \vec{U} : velocity vector $[m/s]$
 w: dimensionless velocity component in vertical direction
 (x,y,z): dimensionless Cartesian coordinate
 Da : Darcy number $[\frac{K}{d^2}]$
Greek Letters
 α : effective thermal diffusivity $[m^2/s]$ β
 : thermal expansion coefficient $[^\circ C^{-1}]$
 ϵ : retardation time $[s]$
 θ : dimensionless temperature λ
 : relaxation time $[s]$
 μ : dynamic viscosity $[Pa.s]$
 γ : kinematic viscosity $[m^2/s]$
 ρ : density $[kg/m^3]$
 σ : temporal growth rate
 Λ : ratio of the viscosities of the porous layer

Subscripts

- b : basic state
 (stat): stationary mode
 (osc) : oscillatory mode

Superscripts

- ' : perturbed quantity
 * : dimensionless quantity

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