Star Related Reverse - Magic Graphoidal Graphs

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Abstract

Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ-magic graphoidal total labelling. The minimum cardinality of such cover is known as graphoidal covering number of G.. In this paper we explained a reverse process of magic graphoidal called reverse-magic graphoidal labelling and proved $[P_n: S_2]$, Double Crowned star $K_{1,n} \odot 2K_1$, $< K_{1,n} : n >$, graph $K_2 + mK_1$, are reverse magic graphoidal.

Keywords

Graphoidal Constant, Graphoidal Cover, Magic Graphoidal, reverse magic graphoidal.

1. INTRODUCTION

B.D. Acharya and E. Sampath Kumar defined Graphoidal cover as partition of edge set of G in to internally disjoint paths (not necessarily open). The maximum cardinality of such cover is known as graphoidal covering number of G.

A graph G is said to be magic if there exist a bijection $f: V \cup E \rightarrow \{1,2,3 \dots \dots . m+n\}$; where *'$n$'* is the number of vertices and *'m'* is the number of edges of a graph. Such that for all edges xy, $f(x) + f(y) + f(xy)$ is a constant. Such a bijection is called a magic labeling of G.

Let $G = (V, E)$ be a graph and let ψ be a graphoidal cover of G. Define $f: V \cup E \rightarrow \{1,2 \dots \dots, m+n\}$ such that for every path $P = \{v_1, v_2, \dots \dots, v_n\}$ in ψ with $f^*(p) = f(v_1) + f(v_n) + \sum_{i=1}^{n-1} f(v_i v_{i+1}) = k$ is a constant, where f^* is the induced labeling on ψ. Then, we say that G admits ψ - magic graphoidal total labeling of G. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labelling of *G.*

Here we introduced a new type of ie. reverse process of magic graphoidal total labeling is called reverse magic graphoidal total labeling.

Definition1.1

A complete bipartite graph $K_{1,n}$ is called a ***star*** and it has $(n+1)$ vertices and n edges

Definition 1.2

The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges

Definition1.3

Let $K_{1,n} \Theta 2K_1$ be the ***Double Crowned Star*** which is the graph obtained from a star $K_{1,n}$ by attaching double edge at each end vertex of $K_{1,n}$.

Definition 1.4

Let $S_2 = (v_1 v_0 v_1)$ be a star and let $[P_n : S_2]$ be the graph obtained from n copies of S_2 and the path $P_n = (u_1, u_2, u_3, \dots \dots \dots \dots, u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_2 by means of an edge, for $1 \le j \le n$

Definition 1.5

The graph $< K_{1,n} : n >$ is obtained by the subdivision of the edges of star $K_{1,n}$

II.MAIN RESULTS

Definition 2. 1

A reverse magic graphoidal labeling of a graph G is one-to-one map f from $V(G) \cup E(G) \rightarrow \{1,2,3,\dots \dots \dots, m + n\}$, where *'n'* is the number of vertices of a graph and *'m'* is the number of the edges of a graph, with the property that , there is an integer constant *'μ'* such that

$f^*(p) = \sum_{i=1}^{n-1} f(v_i\, v_{i+1}) - \{f(v_1) + f(v_n)\} = \mu_{rmgC}$, is a contant

Then the reverse methodology of magic graphoidal labeling is called reverse magic graphoidal labeling (rmgl). Reverse process of magic graphoidal of a graph is called reverse magic graphoidal graph.(rmgg).

Theorem 2.1

The graph $[P_n : S_2]$ is reversed magic graphoidal for $n > 1$

Proof :

Let $G = [P_n : S_2]$

$$V(G) = \begin{cases} v_i, u_i; & 1 \le i \le n \\ v_{ij}; & 1 \le i \le n, \quad j = 1,2 \end{cases}$$

$$E(G) = \{ [(v_i\, v_{i+1}); \quad 1 \le i \le n-1] \cup [(v_i u_i);\ 1 \le i \le n] \cup$$
$$[(u_{i1} u_i);\ 1 \le i \le n] \cup [(u_1 u_{i2}); \quad 1 \le i \le n] \}$$

Define $f: V \cup E \rightarrow \{1,2,3, \dots \dots, m + n\}$ *by*

$f(u_1) = 1$

$f(u_2) = m + n = 8n - 1$

$f(v_{i+1}) = i + 1; \qquad 1 \le i \le n-2$

$f(u_{i+2}) = 8n - 1 - i; \qquad 1 \le i \le n-2$

$f(u_{i1}) = n - 1 + i; \qquad 1 \le i \le n$

$f(u_{i2}) = 7n + 1 - i; \qquad 1 \le i \le n$

$f(u_1 v_1) = 2n$

$f(v_1 v_2) = 2n + 1$

$f(v_2 u_2) = 4n + 2$

$f(v_{i+1}\, v_{i+2}) = 4n + 1 - i; \qquad 1 \le i \le n-2$

$f(v_{i+2}\, u_{i+2}) = 4n + 2 + i; \qquad 1 \le i \le n-2$

$f(u_{i1}u_i) = 6n + 1 - i; \qquad 1 \le i \le n$

$f(u_i u_{i2}) = 2n + 2 + i\,; \qquad 1 \le i \le n$

Let $\psi = \{\, P_1 = [(u_1 v_1 v_2 u_2)]\,, P_2 = [(v_{i+1} v_{i+2} u_{i+2});\ 1 \le i \le n-2],$

$P_3 = [(u_{i1} u_i u_{i2});\ 1 \le i \le n]\}$

$f^*(P_1) = f(u_1 v_1) + f(v_1 v_2) + f(v_2 u_2) - \{f(u_1) + (u_2)\}$

$= 2n + 2n + 1 + 4n + 2 - \{1 + 8n - 1\}$

$= 3 = \mu_{\,rmgc}$ ______________________ (1)

$f^*(P_2) = f(v_{i+1} v_{i+2}) + f(v_{i+2}\, u_{i+2}) - \{f(v_{i+1}) + f(u_{i+2})\} \quad ; \qquad 1 \le i \le n-2$

$= 4n + 1 - i + 4n + 2 + i - \{i + 1 + 8n - 1 - i\}$

$= 3 = \mu_{\,rmgc}$ ______________________ (2)

$f^*(P_3) = f(u_{i1} u_i) + f(u_i u_{i2}) - \{f(u_{i1}) + f(u_{i2})\}$

$= 6n + 1 - i + 2n + 2 + i - \{n - 1 + i + 7n + 1 - i\}$

$= 3 = \mu_{rmgc}$ ______________________ (3)

from (1), (2), and (3); we conclude that G admit ψ − reverse magic graphoidal labeling. The reverse magic graphoidal constant $\mu_{\,rmgc}$ of $[P_n : S_2]$ is always 3.

Theorem 2.2

The graph Double Crowned star $K_{1,n} \odot 2K_1$ is reverse magic graphoidal

Proof :

Let G be the graph $K_{1,n} \odot 2K_1$.

When n is even :

Let $\quad V(G) = \{u, u_i, u_{ij}\ ; \qquad 1 \le i \le n, \quad j = 1,2\}$

And $\quad E(G) = \begin{cases} u u_{2i-1}, u u_{2i}\ ; & 1 \le i \le \frac{n}{2} \\ u_i u_{i1}, u_i u_{i2}\ ; & 1 \le i \le n \end{cases}$

Define $f: V \cup E \rightarrow \{1,2, \dots\dots, m+n\}$ by

Here, $\quad m + n = 6n + 2$

$f(u_{2i-1}) = i; \qquad 1 \le i \le \frac{n}{2}$

$f(u_{2i}) = 6n + 2 - i; \qquad 1 \le i \le \frac{n}{2}$

$f(u_{i1}) = \frac{n}{2} + i; \qquad 1 \le i \le n$

$f(u_{i2}) = \frac{11n}{2} + 2 - i; \quad 1 \le i \le n$

$f(uu_{2i-1}) = \frac{3n}{2} + i; \quad 1 \le i \le \frac{n}{2}$

$f(uu_{2i}) = \frac{9}{2}n + 2 - i; \quad 1 \le i \le \frac{n}{2}$

$f(u_{i1}u_i) = 2n + i; \quad 1 \le i \le n$

$f(u_{i2}u_i) = 4n + 2 - i; \quad 1 \le i \le n$

Let $\psi = \{P_1 = [(u_{2i-1}uu_{2i}); \quad 1 \le i \le \frac{n}{2}], \quad P_2 = [(u_{i1}u_iu_{i2}); \quad 1 \le i \le n]\}$

$f^*(P_1) = f(u_{2i-1}u) + f(uu_{2i}) - \{f(u_{2i-1}) + f(u_{2i})\} \quad 1 \le i \le \frac{n}{2}$

$= \frac{3n}{2} + i + \frac{9n}{2} + 2 - i - \{i + 6n + 2 - i\}$

$= 6n + 2 - \{6n + 2\}$

$= 0 = \mu_{rmgc}$ ________________ (1)

$f^*(P_2) = f(u_{i1}u_i) + f(u_iu_{i2}) - \{f(u_{i1}) + f(u_{i2})\}; \quad 1 \le i \le n$

$= 2n + i + 4n + 2 - i - \{\frac{n}{2} + i + \frac{11n}{2} + 2 - i\}$

$= 6n + 2 - \{6n + 2\}$

$= 0 = \mu_{rmgc}$ ________________ (2)

from (1) and (2) we conclude that G admits $\psi -$ reverse magic graphoidal labeling. Hence $K_{1,n} \odot 2K_1$ is reverse magic graphoidal. The reverse magic graphoidal constant μ_{rmgc} of $K_{1,4} \odot 2K_1$ is ‘0’.

When n is odd

Let $V(G) = \{u, u_i, u_{ij}\}; \quad 1 \le i \le n, \; j = 1,2$

and $E(G) = \begin{cases} uu_i; & 1 \le i \le n \\ u_iu_{i1}, u_iu_{i2}; & 1 \le i \le n \end{cases}$

Define $f: V \cup E \to \{1,2, \ldots .m + n\}$ by

Here, $m + n = 6n + 1$

$f(u) = 1$

$f(u_1) = 6n$

$f(u_{2i}) = i + 1; \quad 1 \le i \le \frac{n-1}{2}$

$f(u_{2i+1}) = 6n - i; \quad 1 \le i \le \frac{n-1}{2}$

$$f(u_{i1}) = \frac{n+1}{2} + i\,; \qquad 1 \le i \le n$$

$$f(u_{i2}) = \frac{11n+1}{2} - i\,; \qquad 1 \le i \le n$$

$$f(uu_1) = 6n+1;$$

$$f(uu_{2i}) = \frac{3n+1}{2} + i; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(uu_{2i+1}) = \frac{9n+1}{2} - i; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(u_i\, u_{i1}) = 2n+i; \qquad 1 \le i \le n$$

$$f(u_i\, u_{i2}) = 4n+1-i; \qquad 1 \le i \le n$$

Let $\psi = \{\ P_1 = [(uu_1)],\quad P_2 = [(u_{2i}uu_{2i+1});\quad 1 \le i \le \frac{n-1}{2}],$

$P_3 = [(u_{i1}u_iu_{i2});\quad 1 \le i \le n]\}$

$$f^*(P_1) = f(uu_1) - \{f(u) + +f(u_1)\}$$

$$= 6n+1 - \{6n-1\}$$

$$= 0 = \mu_{\,rmgc} \quad \text{——————} \quad (1)$$

$$f^*(P_2) = f(uu_{2i}) + f(uu_{2i+1}) - \{f(u_{2i}) + f(u_{2i+1})\}$$

$$= \frac{3n+1}{2} + i + \frac{9n+1}{2}\ i - \{i+1+6n-i\}$$

$$= \frac{12n+2}{2} - \{1+6n\}$$

$$= 6n+1 - \{1+6n\}$$

$$= 0 = \mu_{\,rmgc} \quad \text{——————} \quad (2)$$

$$f^*(P_3) = f(u_iu_{i1}) + f(u_iu_{i2}) - \{f(u_{i1}) + f(u_{i2})\}$$

$$= 2n+i+\ 4n+1-i\,\{\frac{n+1}{2}\ + i + \frac{11n+1}{2} - i\}$$

$$= 6n+1-\{\frac{12n+2}{2}\}$$

$$= 6n+1 - \{6n+1\}$$

$$= 0 = \mu_{rmgc} \quad \text{——————} \quad (3)$$

from (1), (2) , and (3) , when n is odd, G admits ψ-revere magic graphoidal labeling. The reverse magic graphoidal constant μ_{rmgc} of $K_{1,n} \odot 2K_1$ is 0. Hence $K_{1,n} \odot 2K_1$ is reverse magic graphoidal.

Theorem 2.3

The graph $< K_{1,n} : n >$ is reverse magic graphoidal for $n \ge 2$

Proof:

Let G be the graph $< K_{1,n} : n >$.

Let $V(G) = \{v_i, w_i, u; \quad 1 \le i \le n\}$

And $E(G) = \{v_{2i-1}\, w_{2i-1}\, u\, w_{2i}\, v_{2i}\,; \quad 1 \le i \le \frac{n}{2}\}$

Define $f: V \cup E \rightarrow \{1,2, \ldots \ldots, m+n\}$ by

Here, $m + n = 4n + 1$

When n is odd

$$f(u) = 1$$

$$f(v_n) = 2n + 1$$

$$f(v_{2i-1}) = \frac{n+3}{2} - i\,; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2i}) = \frac{3n+1}{2} + i; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(v_{2i-1}\, w_{2i-1}) = \frac{n-1}{2} + 2i; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(w_{2i-1}u) = 3n + 2 - 2i\,; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(uw_{2i}) = 3n + 1 - 2i; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(w_{2i}v_{2i}) = \frac{n+1}{2} + 2i; \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(uw_n) = 3n + 2$$

$$f(w_n v_n) = 4n + 1$$

Let $\psi = \{P_1 = [\,(v_{2i-1}w_{2i-1}uw_{2i}v_{2i}; \quad 1 \le i \le \frac{n-1}{2})],\ P_2 = [(uw_n v_n)]\}$

$$f^*(P_1) = f(v_{2i-1}\, w_{2i-1}) + f(w_{2i-1}u) + f(uw_{2i}) + f(w_{2i}v_{2i}) - \{f(v_{2i-1}) + f(v_{2i})\}$$

$$= \frac{n-1}{2} + 2i + 3n + 2 - 2i + 3n + 1 - 2i + \frac{n+1}{2} + 2i - \{\frac{n+3}{2} - i + \frac{3n+1}{2} + i\}$$

$$= n + 6n + 3 - \{\frac{4n+4}{2}\}$$

$$= 5n + 1 = \mu_{rmgc} \qquad \text{———————} \qquad (1)$$

$$f^*(P_2) = f(uw_n) + f(w_n v_n) - \{f(u) + f(v_n)\}$$

$$= 3n + 2 + 4n + 1 - \{1 + 2n + 1\}$$

$$= 5n + 1 = \mu_{rmgc} \qquad \text{———————} \qquad (2)$$

From (1) & (2); we conclude that G admits ψ- reverse magic graphoidal labeling. When n is odd the reverse magic graphoidal constant μ_{rmgc} of $< K_{1,n} : n >$ is $5n + 1$. Hence $< K_{1,n} : n >$ is reverse magic graphoidal graph.

When n is even

$$f(v_{2i-1}) = i; \qquad 1 \le i \le \frac{n}{2}$$

$$f(v_{2i}) = 4n + 2 - i; \qquad 1 \le i \le \frac{n}{2}$$

$$f(v_{2i-1}w_{2i-1}) = \frac{n-2}{2} + 2i; \qquad 1 \le i \le \frac{n}{2}$$

$$f(w_{2i-1}u) = \frac{7n+6}{2} - 2i; \qquad 1 \le i \le \frac{n}{2}$$

$$f(uw_{2i}) = \frac{7n+4}{2} - 2i; \qquad 1 \le i \le \frac{n}{2}$$

$$f(w_{2i}v_{2i}) = \frac{n}{2} + 2i; \qquad 1 \le i \le \frac{n}{2}$$

$Let\ \psi = \{P = (v_{2i-1}w_{2i-1}uw_{2i}v_{2i}); \qquad 1 \le i \le \frac{n}{2}\}$

$$f^*(P) = f(v_{2i-1}w_{2i}) + f(w_{2i-1}u) + f(uw_{2i}) + f(w_{2i}v_{2i}) - \{f(v_{2i-1}) + f(v_{2i})\}$$

$$= \frac{n-2}{2} + 2i + \frac{7n+6}{2} - 2i + \frac{7n+4}{2} - 2i + \frac{n}{2} + 2i - [i + 4n + 2 - i]$$

$$= \frac{16n+8}{2} - (4n+2)$$

$$= 8n + 4 - (4n+2)$$

$$= 4n + 2$$

$$= 2(n+1) = \mu_{rmgc} \qquad (1)$$

From equation (1), we conclude that G admits ψ - reverse magic graphoidal labeling. When n is even,the reverse magic graphoidal constant μ_{rmgc} of $< K_{1,n} : n >$ is $2(n+1)$. Hence $< K_{1,n} : n >$ is reverse magic graphoidal.

Theorem 2.4

The graph $K_2 + mK_1$ is reverse magic graphoidal; for $m \ge 2$

Proof

Let G be the graph $K_2 + mK_1$.

Let $V(G) = \{v, u, w_i; \qquad 1 \le i \le m\}$

And $E(G) = \{vu, vw_i, uw_i; \qquad 1 \le i \le m\}$

Define $f: V \cup E \rightarrow \{1,2,3, \dots \dots \dots, m+n\}$ by

Here, $m + n = 3m + 3$

$$f(v) = 1$$

$f(u) = 3m + 2$

$f(vu) = 3m + 3$

$f(vw_i) = 1 + i\,;$ $\quad 1 \le i \le m$

$f(uw_i) = 3m + 2 - i;$ $\quad 1 \le i \le m$

Let $\psi = \{P_1 = (uv),$

$P_2 = (vw_i u);$ $\quad 1 \le i \le m\}$

So,

$$f^*(P_1) = f(uv) - \{f(u) + f(v)\}$$

$$= 3m + 3 - \{3m + 2 + 1\}$$

$$= 0 = \mu_{rmgc} \quad \text{———————— (1)}$$

$$f^*(P_2) = f(vw_i) + f(w_i u) - \{f(v) + f(u)\}$$

$$= 1 + i + 3m + 2 - i - \{1 + 3m + 2\}$$

$$= 3m + 3 - \{3m + 3\}$$

$$= 0 = \mu_{rmgc} \quad \text{———————— (2)}$$

From (1) and (2) we conclude that G admits ψ - reverse magic graphoidal labeling. The reverse magic graphoidal constant μ_{rmgc} of $K_2 + mK_1$ is '0' . Hence $K_2 + mK_1$ is reverse magic graphoidal.

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